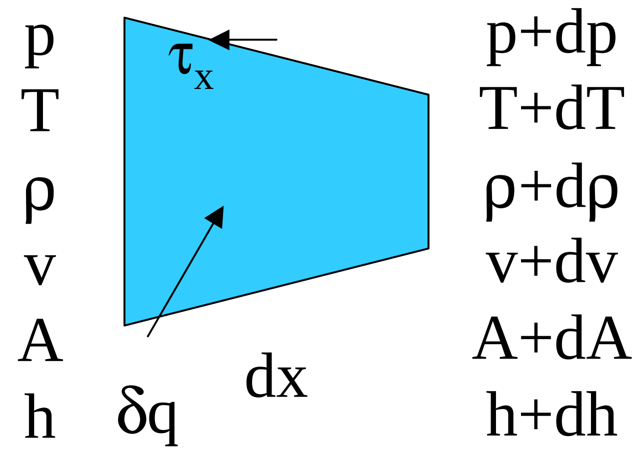
Conservation Equations: Quasi-1D, Steady

- Already derived differential forms of *steady* conservation eqs.
  - no body forces
  - neglect viscous work



Mass

$$\frac{d\rho}{\rho}+\frac{1}{2}\frac{dv^2}{v^2}+\frac{dA}{A}=0 \qquad \text{(VI.9)}$$

Recall, 1-D valid only for dA/dx small

Momentum

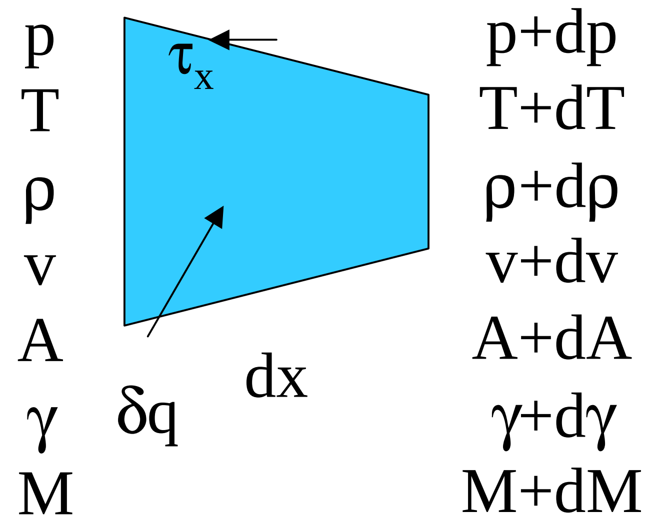
$$\frac{\tau_x}{p}\frac{L_p}{A}dx+\frac{dp}{p}+\frac{1}{2}\frac{v^2}{p/\rho}\frac{dv^2}{v^2}=0 \qquad \text{(VI.10)}$$

Energy

$$\frac{\delta q}{RT}-\frac{1}{2}\frac{v^2}{RT}\frac{dv^2}{v^2}-\frac{dh}{RT}=0 \qquad \text{(VI.11)}$$

Compressible Ideal Gas Equations

- In addition, limit to nonreacting **ideal gases** (nonreact.⇒R=const.)



Mass

$$\frac{d\rho}{\rho}+\frac{1}{2}\frac{dv^2}{v^2}+\frac{dA}{A}=0 \qquad \textbf{(VI.9)}$$

Momentum

$$\frac{\tau_x}{p}\frac{L_p}{A}dx+\frac{dp}{p}+\frac{\gamma}{2}M^2\frac{dv^2}{v^2}=0 \qquad \textbf{(VI.12)}$$

Energy

$$\frac{\delta q}{c_pT}-\frac{(\gamma-1)}{2}M^2\frac{dv^2}{v^2}-\frac{dT}{T}=0 \qquad \textbf{(VI.13)}$$

Use: $p/\rho=RT$

$dh=c_p dT$

$c_p/R=\gamma/(\gamma-1)$

$M^2=v^2/\gamma RT$

Ideal Gas Eq. State

$$\frac{dp}{p}-\frac{d\rho}{\rho}-\frac{dT}{T}=0 \qquad \textbf{(VI.14)}$$

Mach Number

$$\frac{dM^2}{M^2}-\frac{dv^2}{v^2}+\frac{dT}{T}+\frac{d\gamma}{\gamma}=0 \qquad \textbf{(VI.15)}$$

Analytic Solutions

- To solve quasi-1D flows, just need to solve these **five equations**
  - with **five unknowns***: M, ρ, v, p, T
  - for given "**inputs**":
    - ***area change***, dA
    - ***shear stress/friction***, τ_x
    - ***heat transfer***, δq
  - only get analytic solutions if two inputs are zero
- Start with only **area change**

$$\frac{d\rho}{\rho}+\frac{1}{2}\frac{dv^2}{v^2}+\frac{dA}{A}=0$$

$$\frac{\tau_x}{p}\frac{L_p}{A}dx+\frac{dp}{p}+\frac{\gamma}{2}M^2\frac{dv^2}{v^2}=0$$

$$\frac{\delta q}{c_pT}-\frac{(\gamma-1)}{2}M^2\frac{dv^2}{v^2}-\frac{dT}{T}=0$$

$$\frac{dp}{p}-\frac{d\rho}{\rho}-\frac{dT}{T}=0$$

$$\frac{dM^2}{M^2}-\frac{dv^2}{v^2}+\frac{dT}{T}+\frac{d\gamma}{\gamma}=0$$

***must also know** $\gamma=\gamma(T)$