

Energy Conservation for Control Volumes: Integral Form

- Start with Reynolds Transport Theorem (RTT):

$$\left. \frac{dB}{dt} \right|_{CM} = \frac{d}{dt} \int_{CV} \rho \beta dV + \int_{CS} \rho \beta (\vec{v}_{rel} \cdot \hat{n}) dA$$

- Energy

$$B = E_{tot} = U + E_{kinetic} = U + \frac{1}{2} m v^2$$

- will put potential energy in work terms

$$\beta = e_{tot} = u + \frac{v^2}{2} \equiv u_o$$

- can include chemical energy in u

$$\left. \frac{d(E_{tot})}{dt} \right|_{CM} = \frac{d}{dt} \int_{CV} \rho u_o dV + \int_{CS} \rho u_o (\vec{v}_{rel} \cdot \hat{n}) dA$$

– use 1st Law Thermodynamics for dE_{CM}

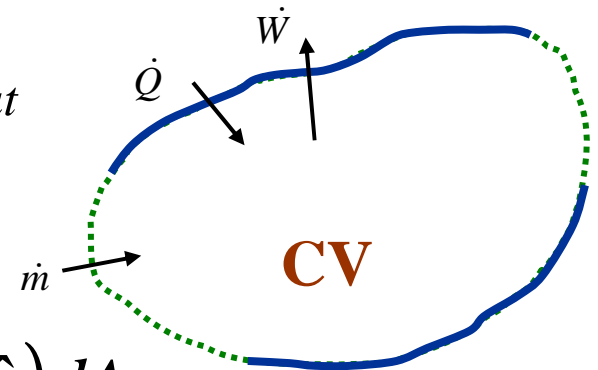
1st Law of Thermodynamics

- Differential form $dE_{CM} = \delta Q_{in} - \delta W_{out}$

$$\left. \frac{dE}{dt} \right|_{CM} = \frac{\delta Q_{in}}{dt} - \frac{\delta W_{out}}{dt} = \dot{Q}_{in} - \dot{W}_{out}$$

- Into RTT

$$\dot{Q}_{in} - \dot{W}_{out} = \frac{d}{dt} \int_{CV} \rho u_o dV + \int_{CS} \rho u_o (\vec{v}_{rel} \cdot \hat{n}) dA$$



$\dot{Q}'' = \dot{Q}$ per unit area

- Heat transfer $\dot{Q} = \int_{CS} \dot{Q}'' dA + \int_{CV} \dot{q} \rho dV$ $\dot{q} = \dot{Q}$ per unit mass

Heat flux at CS (e.g., conduction/convection) “Body” heating by radiation

- Work on CV is related to forces ($F \cdot x$)

Work and Forces

$$\dot{W} = \vec{F} \cdot \frac{d\vec{x}}{dt} = \vec{F} \cdot \vec{v}$$

Since we let work be positive when done **BY** fluid

- **Body forces**

$$\dot{W}_{body} = \int_{CV} \rho \vec{f} \cdot \vec{v} dV$$

acceleration caused by field

- **Fluid forces** (stresses)

“Flow Work”

$$\dot{W}_{press} = \int_{CS} p(\vec{v} \cdot \hat{n}) dA$$

$$\dot{W}_{shear} = - \int_{CS} \vec{\sigma} \cdot \vec{v} dA$$

- **Forces due to solid bodies crossing CV**

– lump into “useful” work term, e.g., shaft work

$$\dot{W}_{shaft}$$

Energy Conservation

- Combine into RTT result (*neglect shear forces to shorten*)

$$\dot{Q}_{in} - \dot{W}_{shaft} + \int_{CV} \rho \vec{f} \cdot \vec{v} dV - \int_{CS} p(\vec{v} \cdot \hat{n}) dA = \frac{d}{dt} \int_{CV} \rho u_o dV + \int_{CS} \rho u_o (\vec{v}_{rel} \cdot \hat{n}) dA$$

Combine flow work and energy flux

Similar Form

$$\rho u_o + \rho \frac{p}{\rho} = \rho \left(u_o + \frac{p}{\rho} \right) = \rho h_o \quad h_o = u + \frac{p}{\rho} + \frac{v^2}{2}$$

Stagnation Enthalpy

$$\dot{Q}_{in} - \dot{W}_{shaft} + \int_{CV} \rho \vec{f} \cdot \vec{v} dV - \int_{CS} p(\vec{v} - \vec{v}_{rel}) \cdot \hat{n} dA = \frac{d}{dt} \int_{CV} \rho u_o dV + \int_{CS} \rho h_o (\vec{v}_{rel} \cdot \hat{n}) dA$$

- For reference frame moving with control volume at *constant velocity and no body forces (+no fluid shear)*

$$\dot{Q}_{in} - \dot{W}_{shaft} = \frac{d}{dt} \int_{CV} \rho u_o dV + \int_{CS} \rho h_o (\vec{v} \cdot \hat{n}) dA$$

0
PI=CO

In - Out

Change

Out - In, of energy in mass

Enthalpy in CV Energy Equation

- Enthalpy appeared when we combined flux u_o through CS and “flow work” where flow is passing through

$$\int_{CS} \rho e_o (\vec{v} \cdot \hat{n}) dA + \int_{CS} p (\vec{v} \cdot \hat{n}) dA = \int_{CS} \rho h_o (\vec{v} \cdot \hat{n}) dA$$

so h_o term represents flux of thermal energy and “flow work” through volume

- If steady, adiabatic and no work but flow work

$$\dot{Q}_{in} - \dot{W}_{shaft} = 0 = \int_{CS} \rho h_o (\vec{v} \cdot \hat{n}) dA$$

Stagnation enthalpy = enthalpy flow **WOULD** achieve if slug of flow was slowed to rest w/o external W or Q

$$h_{o\,in} = h_{o\,out}$$

- Note, unsteady term still uses u_o

$$\frac{d}{dt} \int_{CV} \rho u_o dV$$

energy inside CV is internal and kinetic

Simplifications of Energy Eqn.

- Neglecting body and viscous forces for simplicity

- **Uniform flow**

$$\dot{Q}_{in} - \dot{W}_{shaft} = \frac{d}{dt} \int_{CV} \rho u_o dV + \sum_{outlets} \dot{m} h_o - \sum_{inlets} \dot{m} h_o$$

$$\dot{Q}_{in} + \sum_{inlets} \dot{m} h_o = \frac{dU_{o,CV}}{dt} + \dot{W}_{shaft,out} + \sum_{outlets} \dot{m} h_o$$

$$\text{Ins} = \text{Change} + \text{Outs}$$

- **Uniform flow + steady**

$$\dot{Q}_{in} + \sum_{inlets} \dot{m} h_o = \dot{W}_{shaft,out} + \sum_{outlets} \dot{m} h_o$$

$$\text{Ins} = \text{Outs}$$