Energy Conservation for Control Volumes: Integral Form

- **Start with Reynolds Transport Theorem (RTT):**

$$\left.\frac{\mathrm{dB}}{\mathrm{dt}}\right|_{CM} = \frac{\mathrm{d}}{\mathrm{dt}}\int_{CV}\rho\beta dV + \int_{CS}\rho\beta\left(\vec{\mathrm{v}}_{rel}\cdot\hat{n}\right)dA$$

- Energy

$$\mathrm{B} = E_{tot} = U + E_{kinetic} = U + \frac{1}{2}m\mathrm{v}^2$$

$$\beta = e_{tot} = u + \frac{\mathrm{v}^2}{2} \equiv u_o$$

- will put potential energy in work terms
- can include chemical energy in *u*

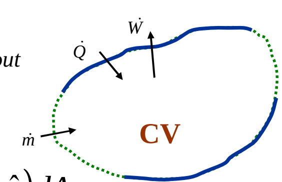
$$\left.\frac{\mathrm{d}\left(E_{tot}\right)}{\mathrm{dt}}\right|_{CM} = \frac{\mathrm{d}}{\mathrm{dt}}\int_{CV}\rho u_o dV + \int_{CS}\rho u_o\left(\vec{\mathrm{v}}_{rel}\cdot\hat{n}\right)dA$$

– use 1st Law Thermodynamics for dE_{CM}

1st Law of Thermodynamics

- Differential form $dE_{CM} = \delta Q_{in} - \delta W_{out}$

$$\left.\frac{\mathrm{d}E}{\mathrm{dt}}\right|_{CM} = \frac{\delta Q_{in}}{\mathrm{dt}} - \frac{\delta W_{out}}{\mathrm{dt}} = \dot{Q}_{in} - \dot{W}_{out}$$



- Into RTT

$$\dot{Q}_{in} - \dot{W}_{out} = \frac{\mathrm{d}}{\mathrm{dt}}\int_{CV}\rho u_o dV + \int_{CS}\rho u_o\left(\vec{\mathrm{v}}_{rel}\cdot\hat{n}\right)dA$$

$\dot{Q}'' = \dot{Q}$ *per unit area*

$\dot{q} = \dot{Q}$ *per unit mass*

- Heat transfer $\dot{Q} = \int_{CS}\dot{Q}''dA + \int_{CV}\dot{q}\,\rho\,dV$

Heat flux at CS (e.g., conduction/convection) **"Body" heating by radiation**

- Work on CV is related to forces (F·x)

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Work and Forces

$$\dot{W} = \vec{F}\cdot\frac{d\vec{x}}{dt} = \vec{F}\cdot\vec{v}$$

- **Body forces**

Since we let work be positive when done **BY** fluid

$$\dot{W}_{body} = - \int_{CV} \rho\vec{f}\cdot\vec{v}dV$$

acceleration caused by field

- **Fluid forces** (stresses)

"Flow Work"

$$\dot{W}_{press} = \int_{CS} p(\vec{v}\cdot\hat{n})dA$$

$$\dot{W}_{shear} = -\int_{CS} \vec{\sigma}\cdot\vec{v}dA$$

- **Forces due to solid bodies crossing CV**
  - lump into "useful" work term, e.g., shaft work $\dot{W}_{shaft}$

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Energy Conservation

- Combine into RTT result (*neglect shear forces to shorten*)

$$\dot{Q}_{in} - \dot{W}_{shaft} + \int_{CV}\rho\vec{f}\cdot\vec{v}dV - \int_{CS} p(\vec{v}\cdot\hat{n})dA = \frac{d}{dt}\int_{CV}\rho u_o dV + \int_{CS}\rho u_o(\vec{v}_{rel}\cdot\hat{n})dA$$

Similar Form

Stagnation Enthalpy

Combine flow work and energy flux

$$\rho u_o + \rho\frac{p}{\rho} = \rho\left(u_o + \frac{p}{\rho}\right) = \rho h_o \qquad h_o = u + \frac{p}{\rho} + \frac{v^2}{2}$$

$$\dot{Q}_{in} - \dot{W}_{shaft} + \int_{CV}\rho\vec{f}\cdot\vec{v}dV - \int_{CS} p(\vec{v}-\vec{v}_{rel})\cdot\hat{n}dA = \frac{d}{dt}\int_{CV}\rho u_o dV + \int_{CS}\rho h_o(\vec{v}_{rel}\cdot\hat{n})dA$$

- For reference frame moving with control volume at *constant velocity and no body forces (+no fluid shear)*

$$\dot{Q}_{in} - \dot{W}_{shaft} = \frac{d}{dt}\int_{CV}\rho u_o dV + \int_{CS}\rho h_o(\vec{v}\cdot\hat{n})dA$$

$\cancel{P}^{0}$I=CO　　**I**n - **O**ut　　**C**hange　　**O**ut - **I**n, of energy in mass

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Enthalpy in CV Energy Equation

- Enthalpy appeared when we combined flux u_o through CS and "flow work" where flow is passing through $\int_{CS} \rho e_o(\vec{v}\cdot\hat{n})dA + \int_{CS} p(\vec{v}\cdot\hat{n})dA = \int_{CS} \rho h_o(\vec{v}\cdot\hat{n})dA$

  so h_o term represents flux of thermal energy and "flow work" through volume

- If steady, adiabatic and no work but flow work

$$\dot{Q}_{in} - \dot{W}_{shaft} = 0 = \int_{CS} \rho h_o(\vec{v}\cdot\hat{n})dA$$

$$h_{o\,in} = h_{o\,out}$$

Stagnation enthalpy = enthalpy flow WOULD achieve if slug of flow was slowed to rest w/o external W or Q

- Note, unsteady term still uses u_o $\quad \frac{d}{dt}\int_{CV} \rho u_o dV$

  energy inside CV is internal and kinetic

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Simplifications of Energy Eqn.

- Neglecting body and viscous forces for simplicity
- **Uniform flow**

$$\dot{Q}_{in} - \dot{W}_{shaft} = \frac{d}{dt}\int_{CV} \rho u_o dV + \sum_{outlets} \dot{m}h_o - \sum_{inlets} \dot{m}h_o$$

$$\dot{Q}_{in} + \sum_{inlets} \dot{m}h_o = \frac{dU_{o,CV}}{dt} + \dot{W}_{shaft,out} + \sum_{outlets} \dot{m}h_o$$

Ins = **C**hange + **O**uts

- **Uniform flow + steady**

$$\dot{Q}_{in} + \sum_{inlets} \dot{m}h_o = \dot{W}_{shaft,out} + \sum_{outlets} \dot{m}h_o$$

Ins = **O**uts