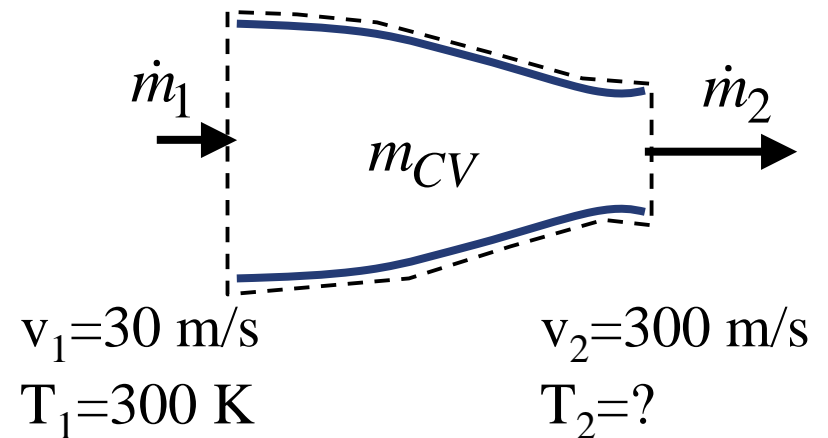


Energy CV Analysis: Example 1

- **Given: Steady flow** in wind tunnel with air intake at 30 m/s, 300 K and exit at 300 m/s

- **Find: T_2**

- **Assume:** Adiabatic (insulated)
 No work but PV work
 Air is ideal gas
 Uniform flow



- **Analysis:** 0 Assume steady-state

Mass Conservation $\dot{m}_1 = \frac{dm_{CV}}{dt} + \dot{m}_2$

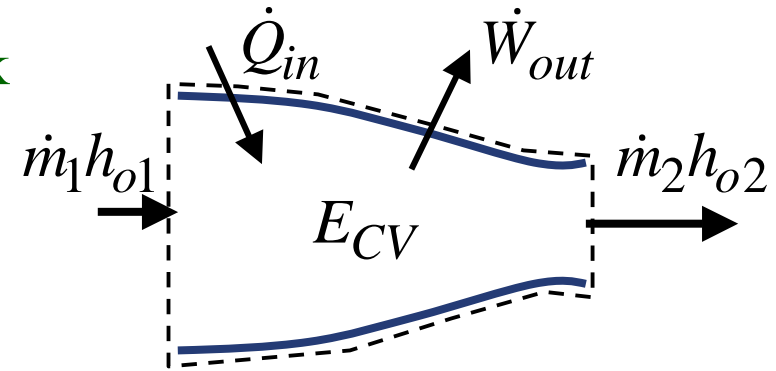
$$\dot{m}_1 = \dot{m}_2$$

Energy CV: Example 1 (con't)

Energy Conservation

Adiabatic $\dot{Q}_{in} = 0$ No "shaft" work $\dot{W}_{out} = 0$

$$\dot{m}_1 h_{o1} + \dot{Q}_{in} = \frac{dE_{CV}}{dt} + \dot{m}_2 h_{o2} + \dot{W}_{out}$$



$h_{o1} = h_{o2}$ stagnation enthalpy=constant
(for steady, adiab., no work)

$v_1 = 30 \text{ m/s}$
 $T_1 = 300 \text{ K}$

$v_2 = 300 \text{ m/s}$
 $T_2 = ?$

$$h_1 + v_1^2/2 = h_2 + v_2^2/2$$

$$h_2 - h_1 = \frac{1}{2}(v_1^2 - v_2^2)$$

assume calor. perfect

$$c_p(T_2 - T_1) = \frac{1}{2}(v_1^2 - v_2^2)$$

$$T_2 = T_1 + \frac{v_1^2 - v_2^2}{2c_p} = 300\text{K} + \frac{(30^2 - 300^2)\text{m}^2/\text{s}^2}{2(1000\text{J}/\text{kgK})} = 255\text{K} (-18^\circ\text{C})$$

Note: if $v_2=0$ $T_2 = T_1 + v_1^2/2c_p$ **Stagnation Temperature** $T_o = T + v^2/2c_p$

thermally & calor. perfect gas

Energy CV Analysis: Example 2

- Given: Steady flow** in jet engine turbine, combustion products mixed with dilution air $T_1 = 1700K$ (to prevent overheating turbine) $\dot{m}_1 = 4 kg/s$



- Find:** Temperature entering turbine T_3

$$T_2 = 800K \uparrow$$

$$\dot{m}_2 = 2 kg/s$$

- Assume:** Adiabatic, Uniform,
 No work but PV work
 All flows p.g. with same properties (e.g., c_p)
 Neglect kinetic energy of flow (low speed)

- Analysis:**

Mass Conservation

$$\dot{m}_1 + \dot{m}_2 = \frac{dm_{CV}}{dt} + \dot{m}_3$$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Energy CV: Example 2 (con't)

Energy Conservation

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q}_{in} = \frac{dE_{CV}}{dt} + \dot{m}_3 h_3 + \dot{W}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$\dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_1) h_2 = \dot{m}_3 h_3$$

$$\dot{m}_1 (h_1 - h_2) = \dot{m}_3 (h_3 - h_2)$$

$$\dot{m}_1 c_p (T_1 - T_2) = \dot{m}_3 c_p (T_3 - T_2)$$

$$T_3 = T_2 + \frac{\dot{m}_1}{\dot{m}_3} (T_1 - T_2) = \frac{\dot{m}_1}{\dot{m}_3} T_1 + \frac{\dot{m}_2}{\dot{m}_3} T_2$$

$$T_3 = \frac{4}{6} 1700K + \frac{2}{6} 800K = 1400K$$

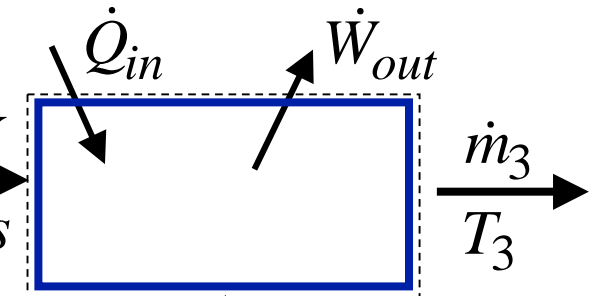
h eq. state
usually Δh

$$T_1 = 1700K$$

$$\dot{m}_1 = 4 \text{ kg/s}$$

$$T_2 = 800K$$

$$\dot{m}_2 = 2 \text{ kg/s}$$



Mixed Enthalpy

$$h_3 = \frac{\dot{m}_1}{\dot{m}_3} h_1 + \frac{\dot{m}_2}{\dot{m}_3} h_2$$

fraction of total mass
in each flow

Energy CV: Example 2 (alternate sol'n.)

Energy Conservation

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q}_{in} = \frac{dE_{CV}}{dt} + \dot{m}_3 h_3 + \dot{W}_{out}$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

h eq. state
usually Δh

or use a reference state

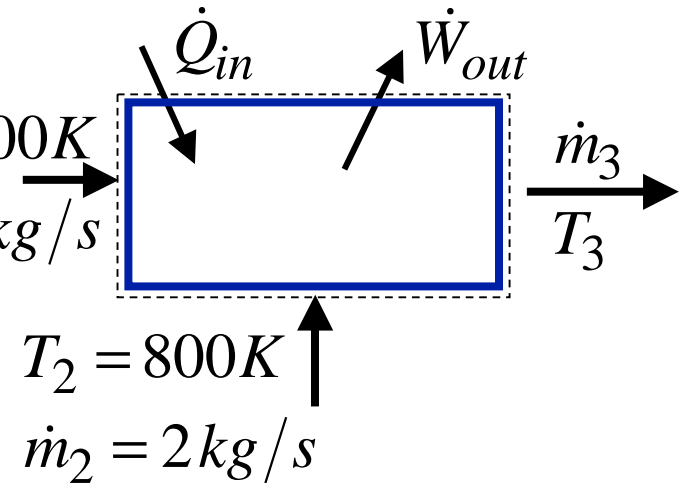
$$\dot{m}_1 (h_1 - h_{ref}) + \dot{m}_2 (h_2 - h_{ref}) = \dot{m}_3 (h_3 - h_{ref})$$

$$\dot{m}_1 c_p (T_1 - T_{ref}) + \dot{m}_2 c_p (T_2 - T_{ref}) = \dot{m}_3 c_p (T_3 - T_{ref})$$

$$\dot{m}_1 T_1 + \dot{m}_2 T_2 - \dot{m}_3 T_3 = T_{ref} (\dot{m}_1 + \dot{m}_2 - \dot{m}_3)$$

$$T_3 = \frac{\dot{m}_1}{\dot{m}_3} T_1 + \frac{\dot{m}_2}{\dot{m}_3} T_2$$

$$T_3 = \frac{4}{6} 1700K + \frac{2}{6} 800K = 1400K$$

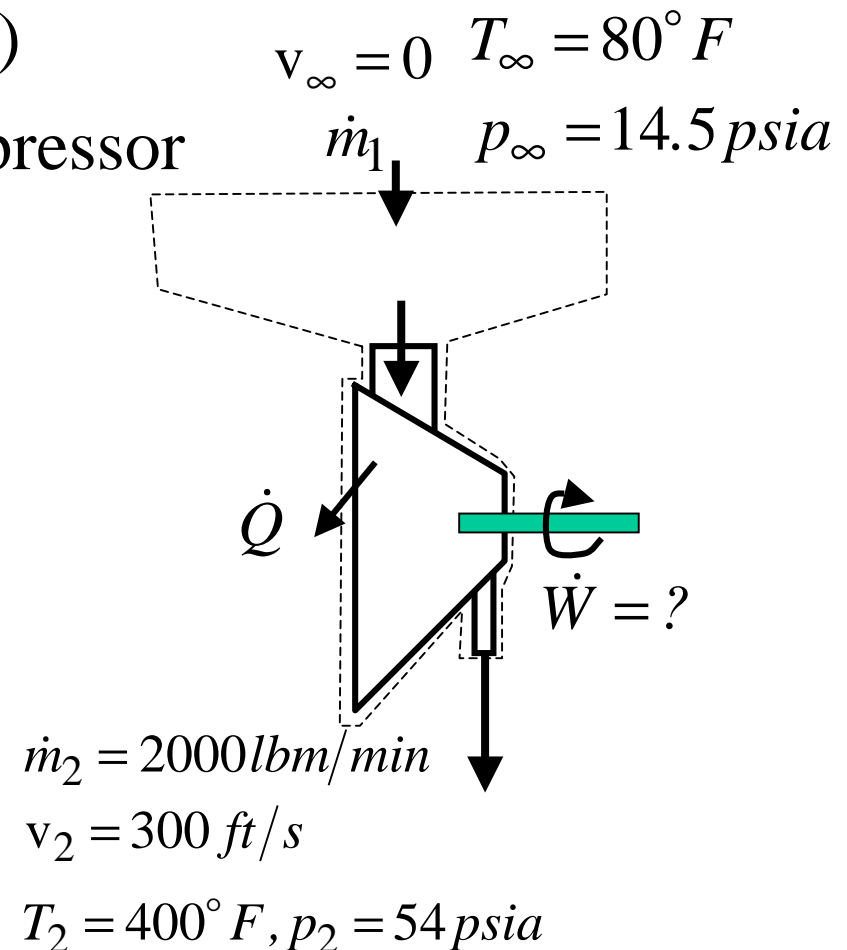


Energy CV Analysis: Example 3

- **Given:** **Air compressor** intakes ambient air (14.5 psia, 80°F) and outputs 54 psia, 400°F with v_{exit} of 300 ft/s and 2000 lb_m/min. (small jet engine size)
- **Find:** **Power** required to drive compressor if 5% of power lost to heating the surroundings
- **Assume:** Air p.g., steady, uniform
- **Analysis:** pick CS where we know conditions ($\infty \Rightarrow v=0$)

Mass Conservation

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$



Energy CV: Example 3 (con't)

Energy Conservation

$$\dot{W} + \dot{m}_1 h_{o1} = \dot{m}_2 h_{o2} + \dot{Q}$$

$$\dot{W} - \dot{Q} = \dot{m}(h_{o2} - h_{o1})$$

$$\dot{W}(1 - \dot{Q}/\dot{W}) = \dot{m}(h_2 + v_2^2/2 - h_1)$$

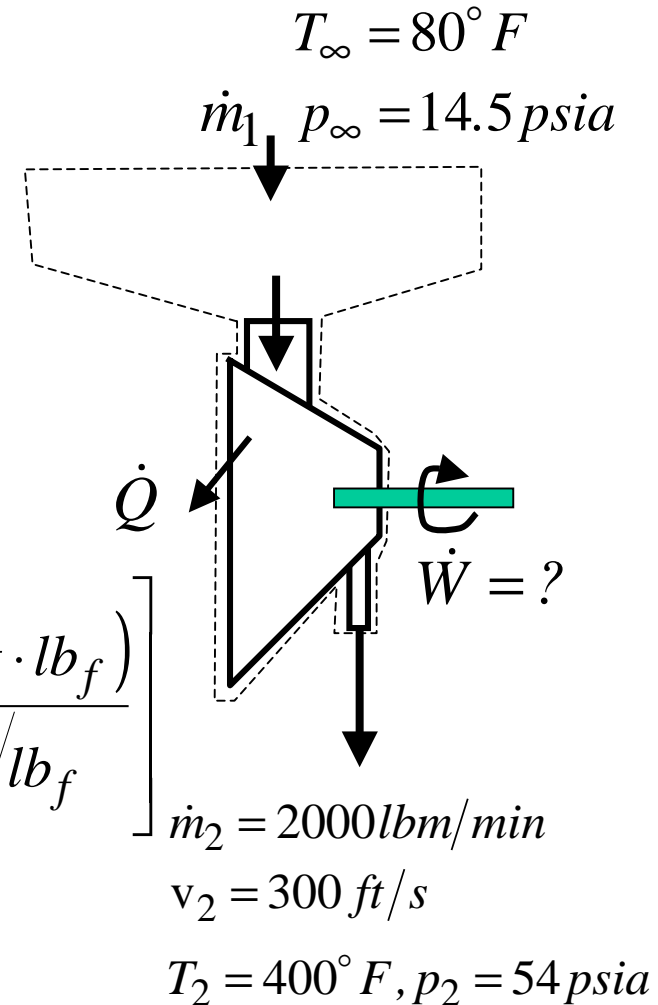
$$\dot{W} = \frac{\dot{m}}{1 - \dot{Q}/\dot{W}} \left[c_p (T_2 - T_1) + v_2^2/2 \right]$$

$$\dot{W} = \frac{2000 \text{ lb}_m}{1 - 0.05 \text{ min}} \left[.24 \frac{\text{BTU}}{\text{lb}_m \text{R}} (320\text{R}) + \frac{(300 \text{ ft/s})^2 (\text{BTU}/778 \text{ ft} \cdot \text{lb}_f)}{2 \times 32.2 (\text{lb}_m \text{ ft/s}^2) / \text{lb}_f} \right]$$

$$\dot{W} = \frac{1.2 \times 10^5 \text{ lb}_m}{0.95 \text{ hr}} \left[76.8 \frac{\text{BTU}}{\text{lb}_m} + 1.80 \frac{\text{BTU}}{\text{lb}_m} \right]$$

$$\dot{W} = 9.9 \times 10^6 \text{ BTU/hr} = 3900 \text{ hp} = 2.9 \text{ MW}$$

KE effect small (2%) in this case



Kinetic Energy Importance

- Since $KE \sim v^2$
 - can often neglect for “small” v changes
 - $v_2^2 - v_1^2$
 - importance also depends on how much other energies change
- For $\Delta T = 5$ K in air ($c_p \sim 1$ kJ/kgK)
 - Δh comparable to
 - $0 \rightarrow 100$ m/s
 - $100 \rightarrow 140$ m/s
 - $200 \rightarrow 225$ m/s