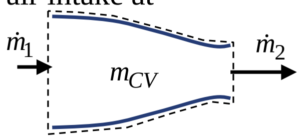
Energy CVAnalysis: Example 1

- **Given:** **Steady flow** in wind tunnel with air intake at 30 m/s, 300 K and exit at 300 m/s



v_1=30 m/s T_1=300 K v_2=300 m/s T_2=?

- **Find**: T_2
- **Assume**: Adiabatic (insulated)
  No work but PV work
  Air is ideal gas
  Uniform flow
- **Analysis:**

Mass Conservation 0 Assume steady-state

$$\dot{m}_1 = \frac{dm_{CV}}{dt} + \dot{m}_2$$

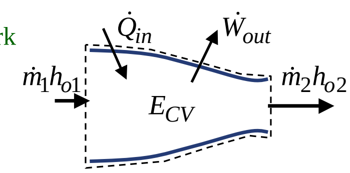
$$\dot{m}_1 = \dot{m}_2$$

Energy CV: Example 1 (con't)

Energy Conservation

Adiabatic 0   0   No "shaft" work 0

$$\dot{m}_1 h_{o1} + \dot{Q}_{in} = \frac{dE_{CV}}{dt} + \dot{m}_2 h_{o2} + \dot{W}_{out}$$



v_1=30 m/s T_1=300 K v_2=300 m/s T_2=?

$$h_{o1} = h_{o2}$$ stagnation enthalpy=constant (for steady, adiab., no work)

$$h_1 + v_1^2/2 = h_2 + v_2^2/2$$

$$h_2 - h_1 = \tfrac{1}{2}\left(v_1^2 - v_2^2\right)$$ assume calor. perfect

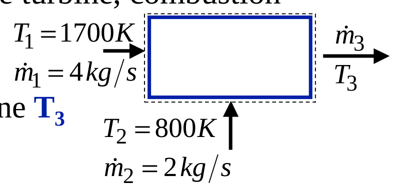
$$c_p\left(T_2 - T_1\right) = \tfrac{1}{2}\left(v_1^2 - v_2^2\right)$$

$$T_2 = T_1 + \frac{v_1^2 - v_2^2}{2c_p} = 300K + \frac{\left(30^2 - 300^2\right)m^2/s^2}{2\left(1000\,J/kgK\right)} = 255K\left(-18^\circ C\right)$$

Note: if v_2=0 $T_2 = T_1 + v_1^2/2c_p$ **Stagnation Temperature** $T_o = T + v^2/2c_p$ thermally & calor. perfect gas

Energy CVAnalysis: Example 2

- **Given:** **Steady flow** in jet engine turbine, combustion products mixed with dilution air (to prevent overheating turbine)



- **Find**: Temperature entering turbine $\mathbf{T_3}$
- **Assume**: Adiabatic, Uniform,
  No work but PV work
  All flows p.g. with same properties (e.g., c_p)
  Neglect kinetic energy of flow (low speed)
- **Analysis:**

Mass Conservation

$$\dot{m}_1 + \dot{m}_2 = \frac{dm_{CV}}{dt} + \dot{m}_3 \qquad \left(\frac{dm_{CV}}{dt} \rightarrow 0\right)$$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

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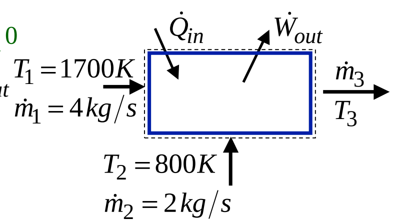
Energy CV: Example 2 (con't)

Energy Conservation



$$\dot{m}_1 h_1 + \dot{m}_2 h_2 + \dot{Q}_{in} = \frac{dE_{CV}}{dt} + \dot{m}_3 h_3 + \dot{W}_{out} \qquad (\dot{Q}_{in} \rightarrow 0,\ \frac{dE_{CV}}{dt} \rightarrow 0,\ \dot{W}_{out} \rightarrow 0)$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

h eq. state
usually Δh

$$\dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_1) h_2 = \dot{m}_3 h_3$$

$$\dot{m}_1 (h_1 - h_2) = \dot{m}_3 (h_3 - h_2)$$

$$\dot{m}_1 c_p (T_1 - T_2) = \dot{m}_3 c_p (T_3 - T_2)$$

$$T_3 = T_2 + \frac{\dot{m}_1}{\dot{m}_3}(T_1 - T_2) = \frac{\dot{m}_1}{\dot{m}_3} T_1 + \frac{\dot{m}_2}{\dot{m}_3} T_2$$

$$T_3 = \frac{4}{6} 1700K + \frac{2}{6} 800K = 1400K$$

Mixed Enthalpy

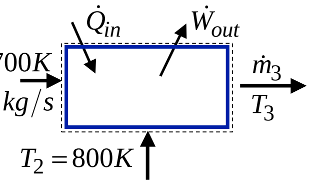
$$h_3 = \frac{\dot{m}_1}{\dot{m}_3} h_1 + \frac{\dot{m}_2}{\dot{m}_3} h_2$$

fraction of total mass in each flow

Energy CV: Example 2 (alternate sol'n.)

Energy Conservation

$$\dot m_1 h_1 + \dot m_2 h_2 + \underbrace{\dot Q_{in}}_{0} = \underbrace{\frac{dE_{CV}}{dt}}_{0} + \dot m_3 h_3 + \underbrace{\dot W_{out}}_{0}$$



$$\dot m_1 h_1 + \dot m_2 h_2 = \dot m_3 h_3$$

h eq. state
usually Δh

or use a reference state

$$\dot m_1 (h_1 - h_{ref}) + \dot m_2 (h_2 - h_{ref}) = \dot m_3 (h_3 - h_{ref})$$

$$\dot m_1 c_p (T_1 - T_{ref}) + \dot m_2 c_p (T_2 - T_{ref}) = \dot m_3 c_p (T_3 - T_{ref})$$

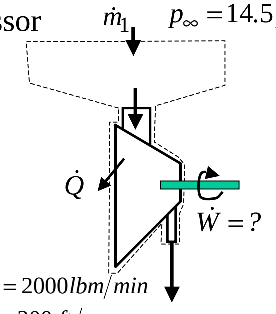
$$\dot m_1 T_1 + \dot m_2 T_2 - \dot m_3 T_3 = T_{ref} \underbrace{(\dot m_1 + \dot m_2 - \dot m_3)}_{0}$$

$$T_3 = \frac{\dot m_1}{\dot m_3} T_1 + \frac{\dot m_2}{\dot m_3} T_2$$

$$T_3 = \frac{4}{6} 1700K + \frac{2}{6} 800K = 1400K$$

Energy CVAnalysis: Example 3

- **Given:** **Air compressor** intakes ambient air (14.5 psia, 80ºF) and outputs 54 psia, 400°F with v_{exit} of 300 ft/s and 2000 lb_m/min. (small jet engine size)
- **Find**: **Power** required to drive compressor if 5% of power lost to heating the surroundings
- **Assume**: Air p.g., steady, uniform
- **Analysis:** pick CS where we know conditions (∞ ⇒ v=0)



Mass Conservation

$$\dot m_1 = \dot m_2 = \dot m$$

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Energy CV: Example 3 (con’t)

Energy Conservation

$$\dot{W} + \dot{m}_1 h_{o1} = \dot{m}_2 h_{o2} + \dot{Q}$$

$$\dot{W} - \dot{Q} = \dot{m}(h_{o2} - h_{o1})$$

$$\dot{W}\left(1 - \dot{Q}/\dot{W}\right) = \dot{m}\left(h_2 + v_2^2/2 - h_1\right)$$

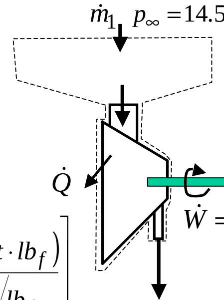
$$\dot{W} = \frac{\dot{m}}{1 - \dot{Q}/\dot{W}}\left[c_p(T_2 - T_1) + v_2^2/2\right]$$

$$\dot{W} = \frac{2000}{1-0.05}\frac{lb_m}{min}\left[.24\frac{BTU}{lb_m R}(320R) + \frac{(300\,ft/s)^2\left(BTU/778\,ft\cdot lb_f\right)}{2\times 32.2\left(lb_m\ ft/s^2\right)/lb_f}\right]$$

$$\dot{W} = \frac{1.2\times10^5}{0.95}\frac{lb_m}{hr}\left[76.8\frac{BTU}{lb_m} + 1.80\frac{BTU}{lb_m}\right]$$

KE effect small (2%) in this case

$$\dot{W} = 9.9\times10^6\ BTU/hr = 3900hp = 2.9MW$$



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Kinetic Energy Importance

- Since KE~v^2
  - can often neglect for “small” v changes
    - $v_2^2 - v_1^2$
  - importance also depends on how much other energies change
- For ΔT=5 K in air (c_p~1 kJ/kgK)
  - Δh comparable to
    - 0→100 m/s
    - 100 →140 m/s
    - 200 →225 m/s