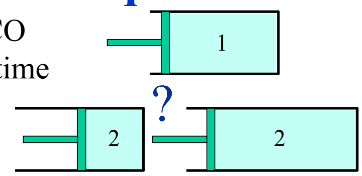
Example: P.G. State Eq'ns.

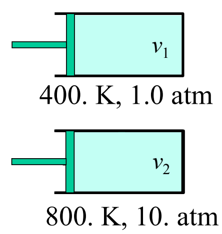


- **Given:** Piston-cylinder containing CO initially at 400. K, 1.0 atm. At later time gas is at 800. K and 10. atm.
- **Find:**
  1. **Change in specific volume ($v_1 \rightarrow v_2$)**
  2. **Change in internal energy (u_2-u_1)=Δu_{12}**
  3. **Change in enthalpy (h_2-h_1)=Δh_{12}**
  4. **Work done (per unit mass) in two "step" process**:
     - first **p=constant** from v_1 to v_2,
     - then ***v*=constant** from p_1 to p_2
  5. **Work done (per mass) if**
     - **p*v*=constant** from v_1 to v_2,
     - then ***v*=constant** to T_2
- **Assume:** Quasi-equilibrium processes (rev.), P.G.

Solution: Volume

- **Analysis:**

***v* change**



P.G. $$pv = RT \Rightarrow \frac{v_2}{v_1} = \frac{RT_2/p_2}{RT_1/p_1} = \frac{T_2}{T_1}\frac{p_1}{p_2} \quad \left(\frac{T_2}{T_1} \to 2,\ \frac{p_1}{p_2} \to 1/10\right)$$

$$v_2/v_1 = \boxed{1/5}$$

***v_1* ?** $$v_1 = RT_1/p_1 \qquad R = \frac{\overline{R}}{\overline{M}} = \frac{8.314\,kJ/kmol\cdot K}{28\,kg/kmol} = 0.297\,kJ/kgK$$

$$v_1 = \frac{297\,J/kgK\,(400K)}{101325Pa} = 1.17\frac{m^2/s^2}{kg/ms^2} = \boxed{1.2\,m^3/kg}$$

1 atm

$$\rho_1 = 0.85\,kg/m^3 \qquad \rho_2 = 4.3\,kg/m^3$$

$$v_2 = \frac{v_1}{5} = \boxed{0.24\,m^3/kg}$$

Solution: Internal Energy

Δu_{12} Method 1:

from Table D.3

T (K)	h (kJ/kg)	u (kJ/kg)	c_p (kJ/kgK)
400	415.81	297.08	1.0475
450	468.36	334.78	1.0546
500	521.31	372.89	1.0637
550	574.77	411.51	1.0746
600	628.80	450.70	1.0868
650	683.48	490.54	1.0997
700	738.80	531.01	1.1128
750	794.78	572.15	1.1259
800	851.40	613.93	1.1388

P.G.

$u = u(T)$

$$\Delta u_{12} = u(T_2) - u(T_1) = u(800K) - u(400K) = 613.93 - 297.08\, kJ/kg = \boxed{316.9\, kJ/kg}$$

Δu_{12} Method 2:

P.G. $$du = c_v dT \Rightarrow \int_{u_1}^{u_2} du = \int_{T_1}^{T_2} c_v dT$$

$$u_2 - u_1 = \int_{T_1}^{T_2} c_v dT$$

if c_v~constant (calorically p.g.)

$$c_v = c_p - R = 1.087\frac{kJ}{kgK} - \underbrace{\frac{8.314\, kJ/kmolK}{28\, kg/kmol}}_{0.297\, kJ/kgK} = 0.790\, kJ/kgK$$

$$\Delta u_{21} = c_v \int_{T_1}^{T_2} dT = c_v(T_2 - T_1) = 0.790\, kJ/kgK\,(800-400)K = \boxed{316\, kJ/kg}$$

using room T c_v value $= 297\, kJ/kg$

c.p.g. can be okay assumption for Δu IF good choice for c_v

Solution: Enthalpy

Δh_{12} Method 1:

from Table D.3

T (K)	h (kJ/kg)	u (kJ/kg)	c_p (kJ/kgK)
400	415.81	297.08	1.0475
450	468.36	334.78	1.0546
500	521.31	372.89	1.0637
550	574.77	411.51	1.0746
600	628.80	450.70	1.0868
650	683.48	490.54	1.0997
700	738.80	531.01	1.1128
750	794.78	572.15	1.1259
800	851.40	613.93	1.1388

P.G.

$h = h(T)$

$$\Delta h_{12} = h(T_2) - h(T_1) = h(800K) - h(400K) = 851.40 - 415.81\, kJ/kg = \boxed{435.6\, kJ/kg}$$

Δh_{12} Method 2:

P.G. $$dh = c_p dT \Rightarrow \int_{u_1}^{u_2} dh = \int_{T_1}^{T_2} c_p dT$$

if c_p~constant (calorically p.g.)

$$\Delta h_{21} = c_p(T_2 - T_1) = 1.087\, kJ/kgK\,(400K) = \boxed{435\, kJ/kg}$$

using room T c_p value $= 416\, kJ/kg$

Δh_{12} Method 3: already know Δu_{12}

$$h = u + pv = u + RT$$

$$\Delta h = \Delta u + R\Delta T$$

$$\Delta h_{12} = \Delta u_{12} + R(T_2 - T_1) = 316 + 0.297(400)\, kJ/kgK = \boxed{435\, kJ/kgK}$$

c.p.g. can be okay assumption for Δh IF good choice for c_p

Solution: Work Path 1

W/m :

only compr. expansion W & quasi-equil.

$$W_{12} = \int_1^2 p\,dV$$

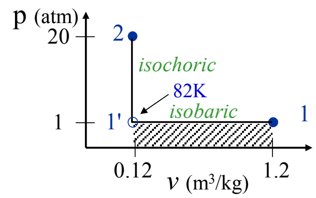
$$\frac{W_{12}}{m} = \int_1^2 p\frac{dV}{m} = \int_1^2 p\,dv$$

$$= \int_1^{1'} p\,dv + \int_{1'}^2 p\,dv \quad (\rightarrow 0)$$

$$= p_1\int_{v_1}^{v_2} dv = p_1\left(v_2 - v_1\right)$$

$$= 101.325\frac{kN}{m^2}(0.24 - 1.2)\frac{m^3}{kg}$$

$$= \boxed{-97.3\,kJ/kg}$$



$T_{1'}$=?

$$T_{1'} = \frac{p_{1'}v_{1'}}{R} = \frac{p_1 v_2}{R} = \frac{1.013\times10^5\frac{N}{m^2}\,0.24\frac{m^3}{kg}}{297\,Nm/kgK}$$

$T_{1'}$=82K

pdv < 0….
we did work to system (compressed it)

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Solution: Work Path 2

W/m :

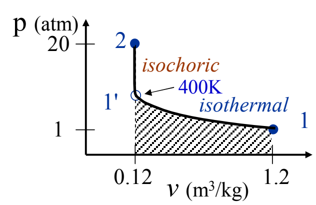
$$\frac{W}{m} = \int_1^{1'} p\,dv + \int_{1'}^2 p\,dv \quad (\rightarrow 0)$$

$$= RT_1\int_1^{1'}\frac{dv}{v}$$

$$= RT_1\ln\frac{v_2}{v_1}$$

$$= 0.297\frac{kJ}{kgK}(400K)\ln\frac{1}{5}$$

$$= \boxed{-191\,kJ/kg}$$



1→1' pv=constant
$v_{1'} = v_2$
P.G. $pv = RT$
⇒ 1→1' **isothermal**
1'→2 isochoric

$|W_{12,\,path\,1}| < |W_{12,\,path\,2}|$ - But both start and end at the same states?

But $T_{1',path\,1}$ (82K) < $T_{1',path\,2}$(400K) ⇒ **different amount of Q required!**

Will explore this later in Energy Conservation