

Friction and Heat Transfer

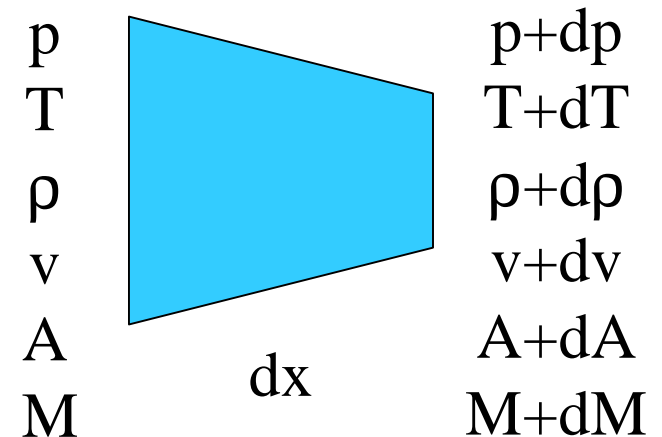
- So far we have examined (1-d) isentropic flows with area change

- Isentropic

- reversible (e.g., inviscid - *no friction*): $P_s=0$

- adiabatic (no heat addition or loss): $Q=0$

- What happens if flow not isentropic?
- Reexamine conservation equations



Compressible 1-d Equations

- Nonreacting tpg/cpg, no body forces, viscous work negligible

Mass
(VI.9)

$$\frac{d\rho}{\rho} + \frac{1}{2} \frac{dv^2}{v^2} + \frac{dA}{A} = 0$$

Momentum
(VI.12)

$$\frac{\tau_x}{p} \frac{L_p}{A} dx + \frac{dp}{p} + \frac{\gamma}{2} M^2 \frac{dv^2}{v^2} = 0$$

Energy
(VI.13)

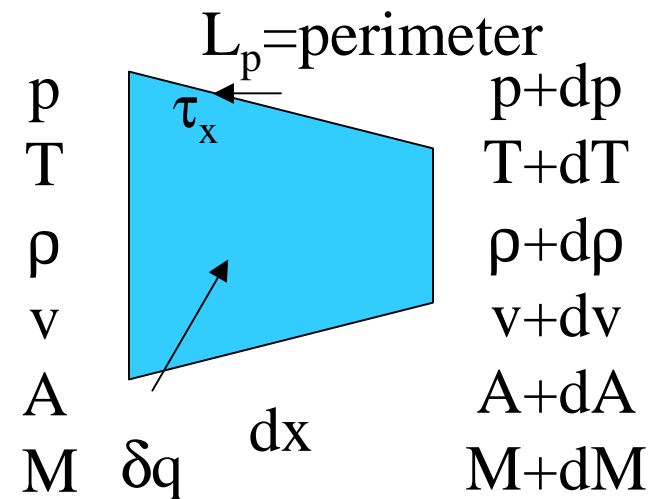
$$\frac{\delta q}{c_p T} - \frac{(\gamma-1)}{2} M^2 \frac{dv^2}{v^2} - \frac{dT}{T} = 0$$

Ideal State Eq.
(VI.14)

$$\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0$$

Mach Number
(VI.15)

$$\frac{dM^2}{M^2} - \frac{dv^2}{v^2} + \frac{dT}{T} = 0$$



(or using T_o def'n.)

$$\delta q = dh_o = c_p dT_o$$

Friction Factor

- Convenient to write the friction induced shear force, τ_x , in terms of a *friction factor*

- Darcy Friction Factor**

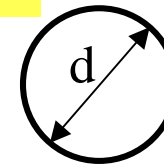
(X.1)

$$f = \frac{4\tau_f}{\frac{1}{2}\rho v^2}$$

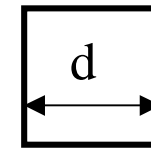
- Hydraulic Diameter**

(X.2)

$$D = \frac{4A}{L_p}$$



$$D_{cyl} = \frac{4(\pi d^2/4)}{\pi d} = d$$



$$D_{sq} = \frac{4(d^2)}{4d} = d$$

- Into momentum eqn.

$$\frac{\tau_x}{p} \frac{L_p}{A} dx = \frac{1}{2} \frac{\rho v^2}{p} \frac{f}{4} \frac{4}{D} dx = \frac{\gamma}{2} M^2 \frac{f dx}{D}$$

$$\gamma \frac{p}{\rho} = a^2$$

$$\frac{\gamma}{2} M^2 \frac{f dx}{D} + \frac{dp}{p} + \frac{\gamma}{2} M^2 \frac{dv^2}{v^2} = 0$$

(X.3)

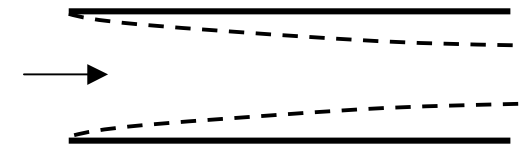
Mach Number Equation

- Combine conservation, state equations
 - can algebraically show

(X.4)

$$\frac{dM^2}{M^2} = \frac{1 + \frac{\gamma - 1}{2} M^2}{1 - M^2} \left\{ \frac{\delta q}{c_p T_o} (1 + \gamma M^2) + \gamma M^2 \frac{f dx}{D} - 2 \frac{dA}{A} \right\}$$

- So we have three ways to change M of flow
 - **area change (dA)**: previously studied
 - **friction: $f > 0$** , same effect as $-dA$
 - **heat transfer**: heating, $\delta q > 0$, like $-dA$
cooling, $\delta q < 0$, like $+dA$



Mach Number Changes

$$\frac{dM^2}{M^2} = \frac{1 + \frac{\gamma - 1}{2} M^2}{1 - M^2} \left\{ \frac{\delta q}{c_p T_o} (1 + \gamma M^2) + \gamma M^2 \frac{f dx}{D} - 2 \frac{dA}{A} \right\}$$

- **Subsonic flow ($M < 1$):** $1 - M^2 > 0$
 - friction, heating, converging area \Rightarrow increase M ($dM > 0$)
 - cooling, diverging area \Rightarrow decrease M ($dM < 0$)
- **Supersonic flow ($M > 1$):** $1 - M^2 < 0$
 - friction, heating, converging area \Rightarrow decrease M ($dM < 0$)
 - cooling, diverging area \Rightarrow increase M ($dM > 0$)

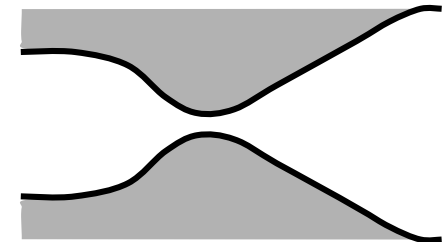
Sonic Flow Trends

- **Friction**
 - accelerates subsonic flow, decelerates supersonic flow
 - always drives flow toward $M=1$
 - (increases entropy)
- **Heating**
 - same as friction - always drives flow toward $M=1$
 - (increases entropy)
- **Cooling**
 - opposite - always drives flow away from $M=1$
 - (decreases entropy)

Nozzles: Sonic Throat

$$\frac{dM^2}{M^2} = \frac{1 + \frac{\gamma - 1}{2} M^2}{1 - M^2} \left\{ \frac{\delta q}{c_p T_o} (1 + \gamma M^2) + \gamma M^2 \frac{f dx}{D} - 2 \frac{dA}{A} \right\}$$

- Effect on transition point: sub \leftrightarrow supersonic flow
- As $M \rightarrow 1$, $1 - M^2 \rightarrow 0$, need $\left\{ \right\}$ term to approach 0
- For isentropic flow, previously showed
 - sonic condition was $dA=0$, throat
- For friction or heating, need $dA > 0$
 - sonic point in diverging section
- For cooling, need $dA < 0$
 - sonic point in converging section



Mach Number Relations

- Using conservation/state equations can get equations for each TD property as function of dM^2

$$\frac{dT_o}{T_o} = \frac{\delta q}{c_p T_o} \quad \text{(X.5)}$$

$$\frac{dT}{T} = \frac{dT_o}{T_o} - \frac{\frac{\gamma-1}{2} M^2}{1 + \frac{\gamma-1}{2} M^2} \frac{dM^2}{M^2}$$

$$\frac{dv^2}{v^2} = \frac{dT_o}{T_o} + \frac{1}{1 + \frac{\gamma-1}{2} M^2} \frac{dM^2}{M^2}$$

$$\frac{d\rho}{\rho} = -\frac{1}{2} \frac{dv^2}{v^2} - \frac{dA}{A}$$

$$\frac{dp}{p} = -\frac{\gamma}{2} M^2 \left(\frac{dv^2}{v^2} + \frac{f dx}{D} \right)$$

$$\frac{dp_o}{p_o} = \frac{dp}{p} + \frac{\frac{\gamma}{2} M^2}{1 + \frac{\gamma-1}{2} M^2} \frac{dM^2}{M^2}$$

$$\frac{ds}{R} = \frac{\gamma}{\gamma-1} \frac{dT_o}{T_o} - \frac{dp_o}{p_o}$$

Mach Number Relations (con't)

- General solution to these equations
 - requires numerical integration of dM^2/M^2 (from X.4) using $A(x), f(x), q(x)$; then get other properties from (X.5)
- Can get analytic solutions for one effect at a time
 - already solved **isentropic flow**: only dA change
 - **Fanno flow**: only friction, f
 - **Rayleigh flow**: only heat transfer, q