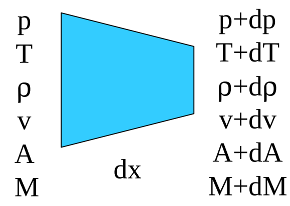
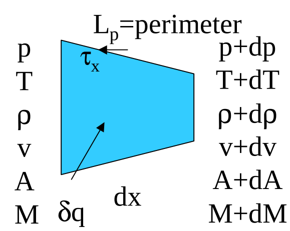
Friction and Heat Transfer

- So far we have examined (1-d) isentropic flows with area change
- Isentropic
  - reversible (e.g., inviscid - *no friction*): **P_s=0**
  - adiabatic (no heat addition or loss): **Q=0**
- What happens if flow not isentropic?
- Reexamine conservation equations



Compressible 1-d Equations

- Nonreacting tpg/cpg, no body forces, viscous work negligible



Mass (VI.9)

$$\frac{d\rho}{\rho}+\frac{1}{2}\frac{dv^2}{v^2}+\frac{dA}{A}=0$$

Momentum (VI.12)

$$\frac{\tau_x}{p}\frac{L_p}{A}dx+\frac{dp}{p}+\frac{\gamma}{2}M^2\frac{dv^2}{v^2}=0$$

Energy (VI.13)

$$\frac{\delta q}{c_pT}-\frac{(\gamma-1)}{2}M^2\frac{dv^2}{v^2}-\frac{dT}{T}=0$$

(or using T_o def'n.) → $\delta q = dh_o = c_p dT_o$

Ideal StateEq. (VI.14)

$$\frac{dp}{p}-\frac{d\rho}{\rho}-\frac{dT}{T}=0$$

Mach Number (VI.15)

$$\frac{dM^2}{M^2}-\frac{dv^2}{v^2}+\frac{dT}{T}=0$$

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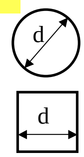
Friction Factor

- Convenient to write the friction induced shear force, τ_x, in terms of a *friction factor*
- **Darcy Friction Factor** $$f=\frac{4\tau_f}{\frac{1}{2}\rho v^2} \quad \textbf{(X.1)}$$
- **Hydraulic Diameter** $$D=\frac{4A}{L_p} \quad \textbf{(X.2)}$$



$$D_{cyl}=\frac{4\left(\pi d^2/4\right)}{\pi d}=d$$

$$D_{sq}=\frac{4\left(d^2\right)}{4d}=d$$

- Into momentum eqn.

$$\frac{\tau_x}{p}\frac{L_p}{A}dx=\frac{1}{2}\frac{\rho v^2}{p}\frac{f}{4}\frac{4}{D}dx=\frac{\gamma}{2}M^2\frac{f dx}{D}$$

$$\gamma\frac{p}{\rho}=a^2$$

$$\frac{\gamma}{2}M^2\frac{f dx}{D}+\frac{dp}{p}+\frac{\gamma}{2}M^2\frac{dv^2}{v^2}=0 \quad \textbf{(X.3)}$$

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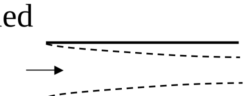
Mach Number Equation

- Combine conservation, state equations
  - can algebraically show **(X.4)**

$$\frac{dM^2}{M^2}=\frac{1+\frac{\gamma-1}{2}M^2}{1-M^2}\left\{\frac{\delta q}{c_pT_o}\left(1+\gamma M^2\right)+\gamma M^2\frac{f dx}{D}-2\frac{dA}{A}\right\}$$

- So we have three ways to change M of flow
  - **area change (dA)**: previously studied
  - **friction**: $f>0$, same effect as –dA
  - **heat transfer**: heating, $\delta q>0$, like –dA
    cooling, $\delta q<0$, like +dA



Mach Number Changes

$$\frac{dM^2}{M^2} = \frac{1+\frac{\gamma-1}{2}M^2}{1-M^2}\left\{\frac{\delta q}{c_p T_o}\left(1+\gamma M^2\right)+\gamma M^2 \frac{f dx}{D} - 2\frac{dA}{A}\right\}$$

- Subsonic flow (M<1): $1-M^2 > 0$
  - friction, heating, converging area ⇒ increase M (dM>0)
  - cooling, diverging area ⇒ decrease M (dM<0)
- Supersonic flow (M>1): $1-M^2 < 0$
  - friction, heating, converging area ⇒ decrease M (dM<0)
  - cooling, diverging area ⇒ increase M (dM>0)

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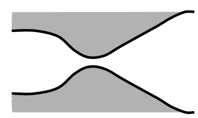
Sonic Flow Trends

- **Friction**
  - accelerates subsonic flow, decelerates supersonic flow
  - always drives flow toward M=1
  - (increases entropy)
- **Heating**
  - same as friction - always drives flow toward M=1
  - (increases entropy)
- **Cooling**
  - opposite - always drives flow away from M=1
  - (decreases entropy)

Nozzles: Sonic Throat

$$\frac{dM^2}{M^2} = \frac{1+\frac{\gamma-1}{2}M^2}{1-M^2}\left\{\frac{\delta q}{c_p T_o}\left(1+\gamma M^2\right) + \gamma M^2 \frac{f dx}{D} - 2\frac{dA}{A}\right\}$$

- Effect on transition point: sub⇔supersonic flow
- As $M \rightarrow 1$, $1-M^2 \rightarrow 0$, need { } term to approach 0
- For isentropic flow, previously showed
  - sonic condition was dA=0, throat



- For friction or heating, need dA>0
  - sonic point in diverging section
- For cooling, need dA<0
  - sonic point in converging section

Mach Number Relations

- Using conservation/state equations can get equations for each TD property as function of dM^2

$$\frac{dT_o}{T_o} = \frac{\delta q}{c_p T_o} \qquad \textbf{(X.5)}$$

$$\frac{dT}{T} = \frac{dT_o}{T_o} - \frac{\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M^2}\frac{dM^2}{M^2}$$

$$\frac{dv^2}{v^2} = \frac{dT_o}{T_o} + \frac{1}{1+\frac{\gamma-1}{2}M^2}\frac{dM^2}{M^2}$$

$$\frac{d\rho}{\rho} = -\frac{1}{2}\frac{dv^2}{v^2} - \frac{dA}{A}$$

$$\frac{dp}{p} = -\frac{\gamma}{2}M^2\left(\frac{dv^2}{v^2} + \frac{f dx}{D}\right)$$

$$\frac{dp_o}{p_o} = \frac{dp}{p} + \frac{\frac{\gamma}{2}M^2}{1+\frac{\gamma-1}{2}M^2}\frac{dM^2}{M^2}$$

$$\frac{ds}{R} = \frac{\gamma}{\gamma-1}\frac{dT_o}{T_o} - \frac{dp_o}{p_o}$$

Mach Number Relations (con't)

- General solution to these equations
  - requires numerical integration of dM^2/M^2 (from X.4) using A(x), *f*(x),q(x); then get other properties from (X.5)
- Can get analytic solutions for one effect at a time
  - already solved **isentropic flow**: only dA change
  - **Fanno flow**: only friction, *f*
  - **Rayleigh flow**: only heat transfer, q