

Entropic State Eqns. – Ideal Gases

- Recall Gibbs eqn. $ds = \frac{du}{T} + \frac{p}{T} dv$
- for ideal gas $= \frac{c_v dT}{T} + \frac{R}{v} dv$
- integrate $\int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} \frac{c_v dT}{T} + \int_{v_1}^{v_2} \frac{R}{v} dv$

$$s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v dT}{T} + R \int_{v_1}^{v_2} \frac{dv}{v}$$

$$\Delta s_{12} \equiv s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v dT}{T} + R \ln \frac{v_2}{v_1}$$

$$\Delta s_{12} \equiv s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_v dT}{T} - R \ln \frac{\rho_2}{\rho_1}$$

Entropic State Eqns. – Ideal Gases

- Using h version $ds = \frac{dh}{T} - \frac{v}{T} dp$
- for ideal gas $= \frac{c_p dT}{T} - \frac{R}{p} dp$
- integrate $\int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} \frac{c_p dT}{T} - R \int_{p_1}^{p_2} \frac{dp}{p}$

$$\Delta s_{12} \equiv s_2 - s_1 = \int_{T_1}^{T_2} \frac{c_p dT}{T} - R \ln \frac{p_2}{p_1}$$

– $s(T,p)$ behavior?

\swarrow T dependence, $\Delta s \uparrow$ due to $T \uparrow$ \searrow p dependence, $\Delta s \downarrow$ due to $p \uparrow$
and dependence is separable!!
also true for $s(T,p)$

Entropic State Eqns. – Ideal Gases

- So can generally write $\Delta s_{12} = \Delta s_{12}^o(T) - R \ln \frac{p_2}{p_1}$
 - with $\Delta s_{12}^o(T) \equiv \int_{T_1}^{T_2} \frac{c_p dT}{T}$
 - can calculate and/or find in tables
 - e.g., $\Delta s_{12}^o(T) = s^o(T_2) - s^o(T_1)$
 - easy to calculate

$$s^o(T) \equiv \int_{T_{ref}}^T \frac{c_p dT}{T}$$

T_{ref} typically 0 K (or R)

TABLE D.1 IDEAL-GAS PROPERTIES OF AIR- Mol. Wt.: 28.967

T , K	h , kJ/kg	u , kJ/kg	s^o , kJ/kg · K
0	0.0	0.0	0.0
100	99.93	71.23	4.6004
120	119.97	85.53	4.7805
140	140.01	99.83	4.9366
160	160.05	114.12	5.0721
180	180.09	128.42	5.1904
200	200.13	142.72	5.2950
220	220.17	157.03	5.3908

CPG (+Ideal) Equations

- Simplifying for $c_p = \text{const.}$

$$\Delta s_{12} = c_p \int_{T_1}^{T_2} \frac{dT}{T} - R \ln \frac{p_2}{p_1} = c_p \int_{T_1}^{T_2} \frac{dT}{T} - R \ln \frac{\rho_2}{\rho_1}$$

$$\Delta s_{12} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{\rho_2}{\rho_1}$$
 - normalizing $s(T,p)$ $s(T,\rho)$

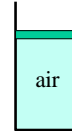
$$\frac{\Delta s_{12}}{R} = \frac{c_p}{R} \ln \frac{T_2}{T_1} - \ln \frac{p_2}{p_1} = \ln \left(\frac{T_2}{T_1} \right)^{c_p/R} - \ln \frac{p_2}{p_1}$$

$$\frac{\Delta s_{12}}{R} = \ln \left(\frac{T_2}{T_1} \right)^{\gamma/\gamma-1} - \ln \frac{p_2}{p_1}$$

entropy more sensitive to T changes

Air Example

- **Given:** Air initially @ 27 C, 1 atm; later same air compressed to 327 C, 50 atm
- **Find:** Entropy change for cpg and non-cpg cases
- **Assume:** tpg



Air Example: Solution

• **Analysis (cpg):**
State Eqn. $\Delta s_{12} = R \left\{ \ln \left(\frac{T_2}{T_1} \right)^{\frac{\gamma}{\gamma-1}} - \ln \frac{p_2}{p_1} \right\}$

need values for R, γ

$$R_{air} = 287 \text{ J/kgK}$$

$$\gamma_{air, 300-600K} \sim 1.4$$

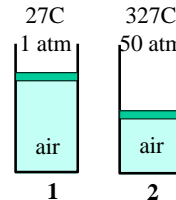
must use absolute T

$$\Delta s_{12} = 287 \frac{\text{J}}{\text{kgK}} \left\{ \ln \left(\frac{600\text{K}}{300\text{K}} \right)^{3.5} - \ln \frac{50}{1} \right\}$$

$$= \left\{ \underset{T \uparrow \Rightarrow \Delta s \uparrow}{0.696} - \underset{p \uparrow \Rightarrow \Delta s \downarrow}{1.122} \right\} \frac{\text{kJ}}{\text{kgK}}$$

$$= -0.426 \frac{\text{kJ}}{\text{kgK}}$$

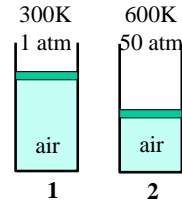
factor of 50 increase in p
 only about twice the effect
 of $2\times$ increase in T



Air Example: Solution

- Analysis (non-cpg):

State Eqn. $\Delta s_{12} = [s_{air}^o(T_2) - s_{air}^o(T_1)] - R_{air} \ln \frac{p_2}{p_1}$



from Table D.1

$$\Delta s_{12} = [6.409 - 5.702] \frac{kJ}{kgK} - 1.122 \frac{kJ}{kgK} \ln \frac{50}{1}$$

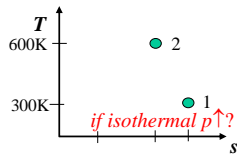
p term same, even if not cpg

was 0.696

$$= \{0.707 - 1.122\} \frac{kJ}{kgK}$$

$$= -0.415 \frac{kJ}{kgK}$$

<2% difference in $\Delta s^o(T)$ term for cpg air calculation over 300-600K



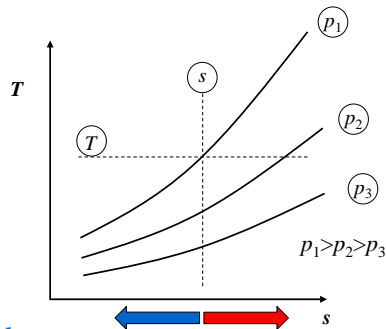
$\Delta s < 0 \Rightarrow ???$

T-s Diagrams

- Often drawing a process on a T-s state diagram will be very helpful to understanding the process

- isothermal process?
- isentropic process?
- isobaric process?
 - what do lines of constant p look like?
 - can show for **tpg** slope increases with s, T
 - lower pressure lines?
 - $p \downarrow \Rightarrow \Delta s \uparrow$

Note: p lines diverge to the right



Isentropic Relations - TPG

- What can we say for $\Delta s=0$?

- For tpg $0 = \int_{T_1}^{T_2} \frac{c_p dT}{T} - R \ln \frac{p_2}{p_1} \Rightarrow \ln \frac{p_2}{p_1} = \frac{1}{R} \int_{T_1}^{T_2} \frac{c_p dT}{T}$

$$\left. \frac{p_2}{p_1} \right|_{isen} = e^{\frac{1}{R} \int_{T_1}^{T_2} \frac{c_p(T) dT}{T}} = e^{\left(\frac{s_2^o}{R} - \frac{s_1^o}{R} \right)} \Rightarrow \left. \frac{p_2}{p_1} \right|_{isentropic} = \frac{p_{r2}}{p_{r1}}$$

$e^{s^o(T)/R}$ available in book tables as p_r

similarly

TABLE D.1 IDEAL-GAS PROPERTIES OF AIR—MASS BASIS; SI UNITS
Mol. Wt.: 28.967

T , K	h , kJ/kg	u , kJ/kg	s^o , kJ/kg · K	c_{p0} , kJ/kg · K	P_r	v_r
0	0.0	0.0	0.0			
100	99.93	71.23	4.6004	1.0000	0.02977	964.30
120	119.97	85.53	4.7805	1.0000	0.05574	616.28
140	140.01	99.83	4.9366	1.0000	0.09603	418.07

$$\left. \frac{v_2}{v_1} \right|_{isentropic} = \frac{v_{r2}(T)}{v_{r1}(T)}$$

Ideal Gas Entropy State Relations - 9

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Isentropic Relations – TPG and CPG

- For isen. & cpg $0 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$

$$\Rightarrow \left. \frac{p_2}{p_1} \right|_{isen} = \left(\frac{T_2}{T_1} \right)^{c_p/R} = \left(\frac{T_2}{T_1} \right)^{\gamma/(\gamma-1)} \quad \text{since } \gamma > 1 \quad \left. \frac{p_2}{p_1} \right|_{isen} > \left(\frac{T_2}{T_1} \right)$$

– using $p = \rho RT$

$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1} \right)^{1/(\gamma-1)} = \left(\frac{p_2}{p_1} \right)^{1/\gamma}$$

isentropic process

$$\Rightarrow \frac{p}{\rho^\gamma} = p v^\gamma = \text{const}$$

$$\left. \frac{\partial p}{\partial \rho} \right|_s = \gamma \rho^{\gamma-1} \left(\frac{p}{\rho^\gamma} \right) = \gamma \frac{p}{\rho} = \gamma RT$$

will need this later

Ideal Gas Entropy State Relations - 10

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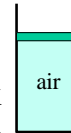
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Automobile Engine Example

- **Given:** Air compressed in auto engine cylinder with compression ratio of 10 ($=V_1/V_2$), a typical value

$$T_1=300\text{K}$$

$$p_1=1 \text{ atm}$$



- **Find:**

$$T_2, p_2$$

- **Assume:** tpg, cpg ($\gamma=1.4$)

compression fast enough that no heat transfer to cylinder walls BUT slow enough to be reversible

Auto Engine Example: Solution

- **Analysis:**

Mass Conserv. $m_1 = m_2 \Rightarrow \frac{V_1}{V_2} = \frac{m/\rho_1}{m/\rho_2} = \frac{\rho_2}{\rho_1}$

2nd Law

rev+adiab

\Rightarrow isentropic $\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma-1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$$T_2/T_1 = 10^{0.4} = 2.51$$

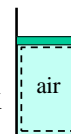
$$T_2=754\text{K}$$

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = 10^{1.4} = 25.1$$

$$p_2=25\text{atm}$$

$$T_1=300\text{K}$$

$$p_1=1 \text{ atm}$$



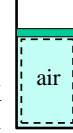
Auto Engine Example: Solution

- What if don't assume cpg:

still have $\frac{v_1}{v_2} = \frac{V_1}{V_2}$

Use Table D.1

$T_1=300K$
 $p_1=1 \text{ atm}$



$$10 = \frac{v_1}{v_2} \Big|_{isen} = \frac{v_r(300K)}{v_r(T_2)} \Rightarrow v_r(T_2) = \frac{62.4}{10} \Rightarrow T_2 \sim 730K$$

vs. 754 K

T (K)	C_p kJ/kgK	p_r	v_r
300	1.0000	1.3800	62.396
700	1.0700	28.682	7.0051
720	1.0758	31.882	6.4822
740	1.0819	35.351	6.0087

$$\frac{p_2}{p_1} \Big|_{isen} = \frac{p_r(T_2)}{p_r(T_1)} = \frac{33.6}{1.38} \Rightarrow p_2 = 24.3 \text{ atm}$$

vs. 25 atm

Stagnation Example

- **Given:** Air @ 300 K, 1 atm,
700 m/s

$T=300K$
 $p=1 \text{ atm}$
 $v=700 \text{ m/s}$

T_o
 p_o

A diagram showing a flow stream on the left with velocity 'v' and on the right it is decelerated to zero velocity at a stagnation point. The stagnation conditions are labeled as T_o and p_o.

- **Find:**

Pressure (p_o) if flow is slowed reversibly and adiabatically to zero velocity (**stagnation pressure**)

- **Assume:** tpg, cpg ($\gamma=1.4$) – and no external work

Stagnation Example: Solution

- Analysis:**

1st Law already showed

if $Q=0, W=0$ $h_o = h + v^2/2$

$T=300\text{K}$
 $p=1\text{ atm}$
 $v=700\text{ m/s}$

and if cpg $T_o = T + \frac{v^2}{2c_p} = T + \frac{\gamma-1}{2} \frac{v^2}{\gamma R}$

stagnation temperature

2nd Law

rev+adiab \Rightarrow isentropic $\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\gamma/(\gamma-1)}$

$R = 287\text{ J/kgK}$

$\gamma_{air} \sim 1.4$

$\frac{T_o}{T} = 1 + \frac{\gamma-1}{2} \frac{v^2}{\gamma RT} = 1 + 0.2 \frac{700^2\text{ m}^2/\text{s}^2}{1.4(287)300\text{ J/kg}} = 1.81$

$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\gamma/(\gamma-1)} = 1.81^{3.5} = 8.0$

$T_o = 544\text{K}$
 $p_o = 8.0\text{atm}$

What if process irrev.?

