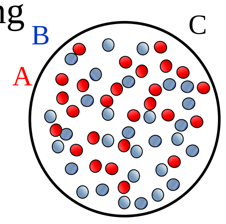
Ideal Gas Mixtures

- Consider a mixture of gases containing more than one ideal gas
  - for example, gas C composed of gases A and B



- How to calculate properties of gas?
  - can use mixture averaged properties,

    e.g., $p=\rho R_{mix}T \qquad R_{mix}=\dfrac{\overline{R}}{MW_{mix}}; \; MW_{mix}=\sum_i \chi_i MW_i$ ***mole fraction*** $\chi_i=n_i/n_{mix}$

    overall gas is ideal
  - or sum up

    partial pressure

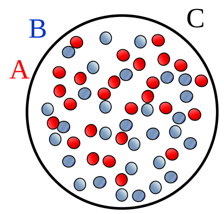
    $p=\sum_i p_i \qquad p_i=\rho_i R_i T$

    $p_i=\chi_i p$

    if each individual gas is ideal

Calculating Properties of Mixtures

- What about other properties?
  - internal energy *U* or enthalpy *H*



e.g., $\Delta H_{C_{1,2}}=m_{mix}\int_{T_1}^{T_2} c_{p_{mix}}\,dT$

$c_{p\,mix}=\sum_i Y_i c_{p_i}$ ***mass fraction*** $Y_i=m_i/m_{mix}$

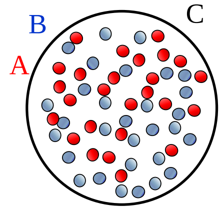
or

$$\Delta H_{C_{1,2}}=\Delta H_{A_{1,2}}+\Delta H_{B_{1,2}}$$

$$\Delta H_{C_{1,2}}=m_A\{h_A(T_2)-h_A(T_1)\}+m_B\{h_B(T_2)-h_B(T_1)\}$$

$$\Delta H_{C_{1,2}}=m_A\int_{T_1}^{T_2} c_{p_A}\,dT+m_B\int_{T_1}^{T_2} c_{p_B}\,dT$$

Ideal Gas Mixture Entropy



- What about entropy S ?
  - use mixture averaged properties

$$\Delta S_{C_{1,2}} = \int_{T_1}^{T_2} c_{p_{mix}} \frac{dT}{T} - R_{mix} \ln\left(\frac{p_2}{p_1}\right)$$

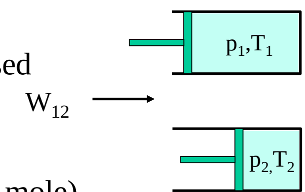
  - or sum up components

$$S_C(T,p) = S_A(T,?) + S_B(T,?)$$

all components have same T, but what pressure should we use for each component? **… their partial pressure**

$$\Delta S_{C_{1,2}} = m_A \left\{ \int_{T_1}^{T_2} c_{p_A} \frac{dT}{T} - R_A \ln\left(\frac{p_{2A}}{p_{1A}}\right) \right\} + m_B \left\{ \int_{T_1}^{T_2} c_{p_B} \frac{dT}{T} - R_B \ln\left(\frac{p_{2B}}{p_{1B}}\right) \right\}$$

Example



- **Given:** air at 1 atm and 300 K compressed to 10 atm and 700 K
- **Find:** change in entropy per unit mass
- **Assume:** air is 79% N_2 and 21% O_2 (by mole)
  N_2, O_2 are TPG and CPG under these conditions
- **Analysis:**

1) mix. avg properties

$$MW_{air} = 0.79(28\,kg/kmol) + 0.21(32\,kg/kmol) = 28.85\,kg/kmol$$

$$R_{air} = \frac{\overline{R}}{MW_{air}} = \frac{8314\,J/kmolK}{28.8\,kg/kmol} = 288\,J/kgK$$

$$c_{p_{air}} \cong \frac{7}{2} R_{air} = 1.01\,kJ/kgK$$

low T diatomic gas

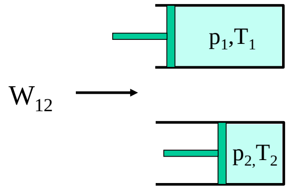
$$\Delta s_{12} = c_{p_{air}} \ln\left(\frac{T_2}{T_1}\right) - R_{air} \ln\left(\frac{p_2}{p_1}\right)$$

$$= 1.01 \frac{kJ}{kgK} \ln\left(\frac{7}{3}\right) - 0.288 \frac{kJ}{kgK} \ln\left(\frac{10}{1}\right)$$

$$\boxed{= 0.191 \frac{kJ}{kgK}}$$

S increased…why?

Example



- **Analysis:**

  2) summation

$$\Delta S_{12,air} = \Delta S_{12,N_2} + \Delta S_{12,O_2}$$

$$m_{air}\Delta s_{12,air} = m_{N_2}\Delta s_{12,N_2} + m_{O_2}\Delta s_{12,O_2}$$

$$\Delta s_{12,air} = Y_{N_2}\Delta s_{12,N_2} + Y_{O_2}\Delta s_{12,O_2}$$

$$= 0.767\frac{kg_{N_2}}{kg_{air}}\left(0.1968\frac{kJ}{kg_{N_2}K}\right)$$

$$+0.233\frac{kg_{O_2}}{kg_{air}}\left(0.1722\frac{kJ}{kg_{O_2}K}\right)$$

$$\boxed{=0.191\frac{kJ}{kgK}}$$ ✓

$$Y_{N_2} = \chi_{N_2}\frac{MW_{N_2}}{MW_{air}} = 0.79\frac{28.01}{28.8} = 0.767$$

$$Y_{O_2} = 1 - Y_{N_2} = 0.233$$

$$\Delta s_{12,N_2} = \frac{8314}{28.01}\frac{J}{kg_{N_2}K}\left\{\frac{7}{2}\ln\left(\frac{7}{3}\right) - \ln\left(\frac{7.9}{0.79}\right)\right\}$$

$$= 0.1968\, kJ/kg_{N_2}K$$

$$\Delta s_{12,O_2} = \frac{8314}{32.0}\frac{J}{kg_{O_2}K}\left\{\frac{7}{2}\ln\left(\frac{7}{3}\right) - \ln\left(\frac{2.1}{0.21}\right)\right\}$$

$$= 0.1722\, kJ/kg_{O_2}K$$