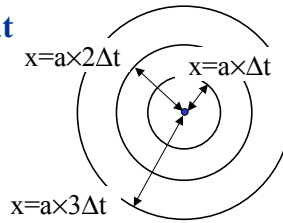


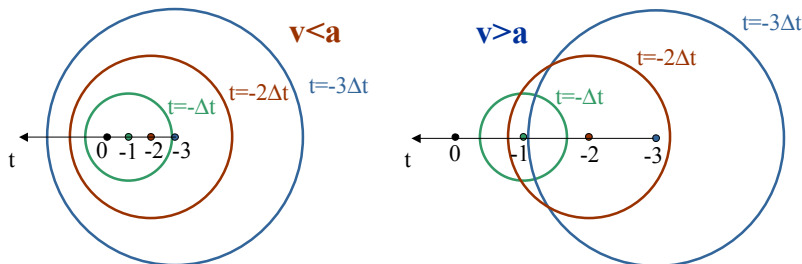
Mach Angle and Mach Number

- Looking for relationship between speed of sound and flow speed (or speed of body moving through fluid)
- Consider small body (point) moving in stagnant fluid
 - continuously “launches” weak pressure disturbances (e.g., from “pushing” fluid)
- Disturbances travel outward spherically at sound speed (a)
- **Look at disturbances generated at equally spaced time intervals**
- Start with body moving with $v \ll a$
 - e.g., nearly stationary or moving through incompressible liquid



Subsonic and Supersonic Motion

- Now compare two bodies, one moving with $v < a$, **subsonic** other body moving with $v > a$, **supersonic**

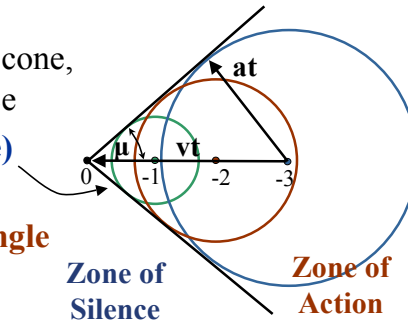


- Subsonic body always behind sound waves launched from previous positions
- Supersonic body moves ahead of previous sound waves

Mach Wave and Mach Angle

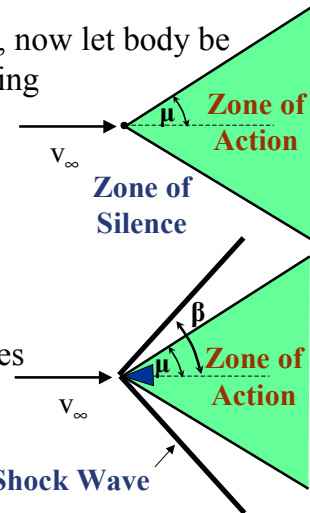
- For supersonic flow, can define region where disturbance has had an effect (been “heard”)
- **Conical region** delineated by **tangents** to sound wave spheres
- Waves coalesce at edge of cone, produce largest disturbance
 - **Mach wave (Mach line)**
- Angle between Mach line and body motion, **Mach angle**

$$\mu = \sin^{-1}\left(\frac{at}{vt}\right) = \sin^{-1}\left(\frac{a}{v}\right)$$



Mach Cone and Shock Waves

- Change frame of reference, now let body be stationary and flow is moving
- **Weak disturbances** from presence of body
 - can only be felt inside Mach cone
 - can not be felt upstream
- Strong pressure disturbances (nonisentropic) can occur, they coalesce to form **shock waves** $\beta \geq \mu$




Mach Number

- So flow/body speed relative to sound speed is fundamental ratio
- **Mach number** $M \equiv \frac{v}{a}$ (VI.3) named for Ernst Mach, Austrian ~1870
- In terms of Mach number, the **Mach angle** is $\mu = \sin^{-1}\left(\frac{1}{M}\right)$ (VI.4)
- Loose demarcation of flow regimes ($M_\infty = v_\infty/a_\infty$)

$M_\infty < 0.3$	“incompressible”	$\Delta\rho < 5\%$ effect
$0.3 < M_\infty < 0.8$	subsonic	ρ changes with v
$0.8 < M_\infty < 1.2$	transonic	shocks for $M > 1$
$1.2 < M_\infty < 3$	supersonic	stronger ρ changes
$3 < M_\infty$	hypersonic	very strong shocks

Adiabatic Flow Ellipse

- Another way to look at M effects
- **Energy equation** $h_o = h + \frac{v^2}{2} = \text{const}$  adiabatic stream tube
- **Stagnation T_o** also constant (thermally & calor. p.g.)

$$T_o = T + \frac{\gamma - 1}{2} \frac{v^2}{\gamma R} = \text{const}$$

$$\frac{2}{\gamma - 1} \gamma R T + v^2 = \text{const}$$

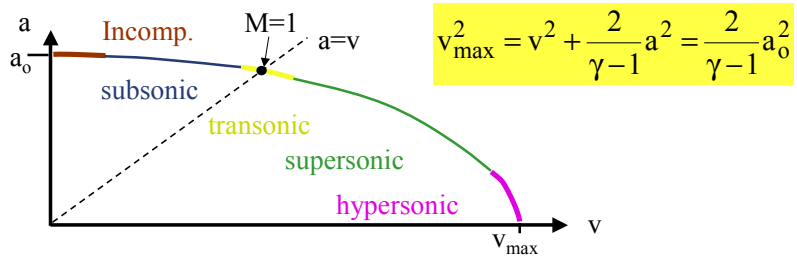
$$\frac{2}{\gamma - 1} a^2 + v^2 = v_{\text{max}}^2 = \frac{2}{\gamma - 1} a_o^2 \quad (\text{VI.5})$$

Stagnation speed of sound
(no kinetic energy left, $v=0$)

Maximum velocity possible
(no thermal energy left, $T=0$)

Adiabatic Flow Ellipse (con't)

- Transition from low speed (a_0) to high speed (v_{\max})



incomp.	$v \ll a, da \ll dv$, little change in $a(T)$
subsonic	$v \lesssim a$, M changes primarily to changes in v
transonic	$ v-a \ll v, a$
supersonic	$v > a$, M changes through substantial changes in v and $a(T)$
hypersonic	$v \gg a, dv \ll da$, M change mostly due to $a(T)$ changes