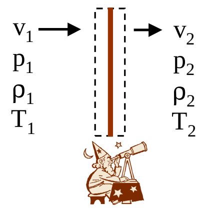
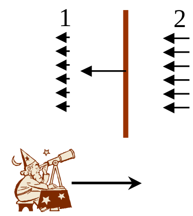
Moving Normal Shocks

- So far, considered changes across shock wave for the case of the shock not moving
  – observer "sitting" on the shock, moving with shock
- What happens to properties if we consider the shock to be moving
  – observer not moving at same speed as shock





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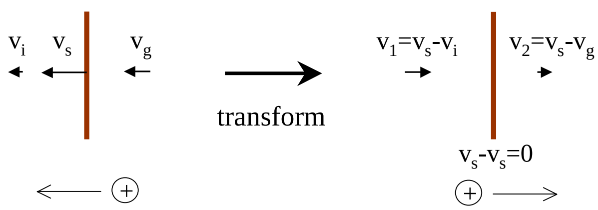
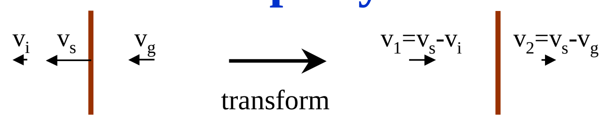
Coordinate Transformation

- First, convert moving shock to stationary shock
  – Galilean transform
  – switch directions (+) and add shock speed, v_s



- Now shock problem looks same as stationary (steady) problem that we already solved

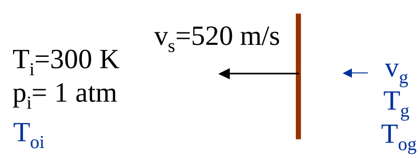
Shock Property Ratios



- **Static properties**
  - property you would measure if moving with flow
  - so, unaffected by transformation
  - e.g., still use $T_2/T_1 = \left(1+\frac{\gamma-1}{2}M_1^2\right) \Big/ \left(1+\frac{\gamma-1}{2}M_2^2\right)$
    with $M_1=v_1/a_1$; $a_1^2=\gamma RT_1$; $T_1=T_i$; etc.
- **Stagnation properties**
  - depend on velocity; not same after transform
  - find using static properties and M_i, M_g

Example: Known Shock Speed

- **Given:** Normal shock moving at 520 m/s **into still air** (300 K, 1 atm)



- **Find:**
  1. Temperature behind shock (after shock passes)
  2. Velocity of gas behind shock (in “lab” reference frame)
  3. Stagnation temperature before and after shock (in lab ref. frame)
- **Assume:** Air TPG/CPG with γ=1.4

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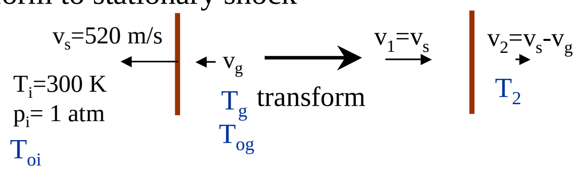
Solution: Known Shock Speed

- **Analysis:** Transform to stationary shock



  - find M_1 in stationary frame

$$M_1 = \frac{v_1}{a_1} = \frac{v_1}{20\sqrt{T_1}} = \frac{v_1}{20\sqrt{T_i}} = \frac{520}{20\sqrt{300}} = 1.50$$

  - M_2 from B.1 or (VII.11)

$$M_2 = \sqrt{\left(M_1^2 + \frac{2}{\gamma-1}\right) \Big/ \left(\frac{2\gamma}{\gamma-1} M_1^2 - 1\right)} = 0.70$$

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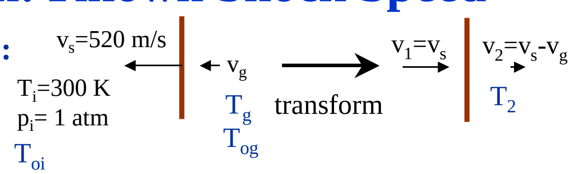
Solution: Known Shock Speed

- **Analysis (con't):**



  - T_2 from B.1 or (VII.9)

$$T_g = T_2 = 300\text{K}\left(1 + \frac{1.4-1}{2} 1.5^2\right) \Big/ \left(1 + \frac{1.4-1}{2} 0.7^2\right) = 396\text{K}$$

  - v_g from B.1 or (VII.8)

$$v_g = v_s - v_2 = v_s\left(1 - \frac{v_2}{v_1}\right) = v_s\left(1 - \frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}}\right) = 241\text{m/s}$$

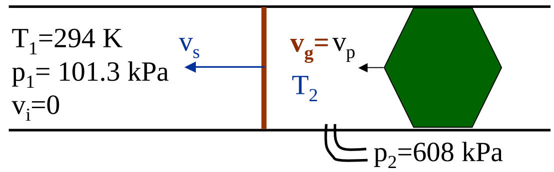
  - T_{oi}, T_{og}

$$T_o = T + \frac{v^2}{2c_p};\ T_{oi} = 300\text{K};\ T_{og} = 396\text{K} + \frac{(241\text{m/s})^2}{2(1005\text{J/kgK})} = 425\text{K}$$

T_o not same for moving shock

Example: Known Postshock Pressure

- **Given:** Supersonic projectile (or equivalently piston) pushing gas ahead in tube filled with **initially still air**
  - leading shock produced
  - p_2 measured



- **Find:**
  1. shock speed (v_s) (lab reference frame)
  2. projectile speed (v_p) (lab reference frame)
- **Assume:** Air TPG/CPG with γ=1.4

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Solution: Known Postshock Pressure

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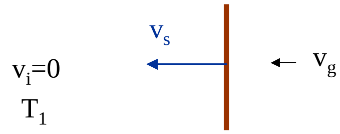
Example: Postshock Speed Known

- **Given:** Normal shock **moving into still gas** (at T_1) produces known gas speed (v_g) behind shock



- **Find:**
  - Expression for shock speed v_s in terms of v_g
- **Assume:**
  - TPG/CPG

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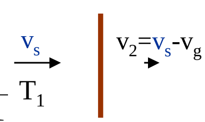
Solution: Postshock Speed Known

- **Analysis:** In stationary ref. frame
  - velocity ratio



$$\frac{v_1}{v_2}=\frac{M_1}{M_2}\sqrt{\frac{T_1}{T_2}}=\frac{M_1}{M_2}\sqrt{\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}}=\sqrt{\frac{\frac{1}{M_2^2}+\frac{\gamma-1}{2}}{\frac{1}{M_1^2}+\frac{\gamma-1}{2}}}=\ldots=\frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}$$

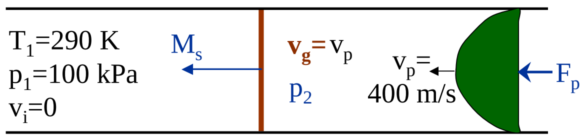
(VII.18)
another normal shock relation

$$\frac{v_s}{v_s-v_g}=\frac{(\gamma+1)(v_s/a_1)^2}{(\gamma-1)(v_s/a_1)^2+2}\Rightarrow v_s^2-\left(v_g\frac{\gamma+1}{2}\right)v_s-a^2=0$$

$$v_s=\frac{\gamma+1}{4}v_g+\frac{1}{2}\sqrt{\left(\frac{\gamma+1}{2}\right)^2 v_g^2+4\gamma RT_1}\quad \textbf{(VII.19)}$$

Numerical Example: Known v_g

- **Given:** Piston impulsively set into motion at 400 m/s in $25 cm^2$ tube filled with **initially still air** @ 290K, 100 kPa
  - leading shock produced



- **Find:**
  1. Mach number of shock, M_s (relative to unshocked gas)
  2. Force, F_p, required to keep piston moving
- **Assume:** Air TPG/CPG with γ=1.4, no friction on piston

Solution: Known v_g