Normal Shocks in CD Nozzles

- Recall previous analysis of converging-diverging nozzles
  - e.g., as back pressure is reduced
- Once $p_b$ lowered enough to get sonic flow at throat: 2 isentropic solutions
  - higher $p_b (= p_e)$, subsonic
  - lower $p_b (= p_e)$, supersonic
Nonisentropic Solutions

- What happens for \( p_b \) in between the isentropic solutions?
  - nonisentropic flow
- For \( p_b < p_{b1} \)
  - flow starts to go supersonic after throat
- For \( p_{b1} > p_b > p_{b4} \), \( p \) must increase above supersonic isentropic case to match \( p_b \)
  \( \Rightarrow \) shock in diverging section
Shocks Inside Nozzle

- Over what range of back pressures will there be shock in nozzle
  – until shock occurs at exit plane of nozzle

- So, question becomes - what is exit pressure when normal shock sits at exit?
  – answer found by combining isentropic and shock solutions
Example

• **Given:** CD nozzle designed to produce $M_e = 3$ for isentropic flow

• **Find:** What range of back pressure, $p_b$ will produce shock in nozzle (throat $\rightarrow$ exit)?

• **Assume:** TPG/CPG with $\gamma = 1.4$

• **Analysis:** Exit pressure, $p_e$, will have to match back pressure
Solution: Shock at Exit

- **Analysis (con’t):**
  - “Shock” at throat
    (Use isentropic relations/tables)
    \[ M_{es,\text{sup}} = 3 \Rightarrow M_{es,\text{sub}} = 0.138 \]
    (same \( A_e/A^* = 4.235 \))
    \[ \frac{p}{p_o}_{M=0.138} = 0.9867 \]
    \[ \Rightarrow p_b = p_{es,\text{sub}} = 98.67\% p_o \]
  - Shock at exit
    (supersonic isentropic flow up to exit)
    \[ \frac{p}{p_o}_{M_{es}=3.0} = 0.0272 \Rightarrow p_{es,\text{sup}} = 2.72\% p_o \]
    (normal shock at \( M=3 \), shock relations/tables)
    \[ M_{e,\text{sh}} = M_2 |_{M_1=3} = 0.475 \text{ and } \frac{p_2}{p_1} |_{M_1=3} = 10.33 \]
    \[ \Rightarrow p_b = p_{es,\text{sup}} \left( \frac{p_2}{p_1} \right)_{M_1=3} = 28.1\% p_o \]
Over- and Underexpanded Nozzles

- What happens if back pressure goes below value where shock is at exit, \(<p_b < p_{b3}\)
  - isentropic flow up to exit, \(\text{supersonic exhaust}\)
  - shocks (and expansions) \(\text{outside nozzle}\)
    (not normal shocks)
- \(p_{b4} < p_b < p_{b3}\)
  - \(\text{Overexpanded exhaust}\)
- \(p_b < p_{b4}\)
  - \(\text{Underexpanded exhaust}\)