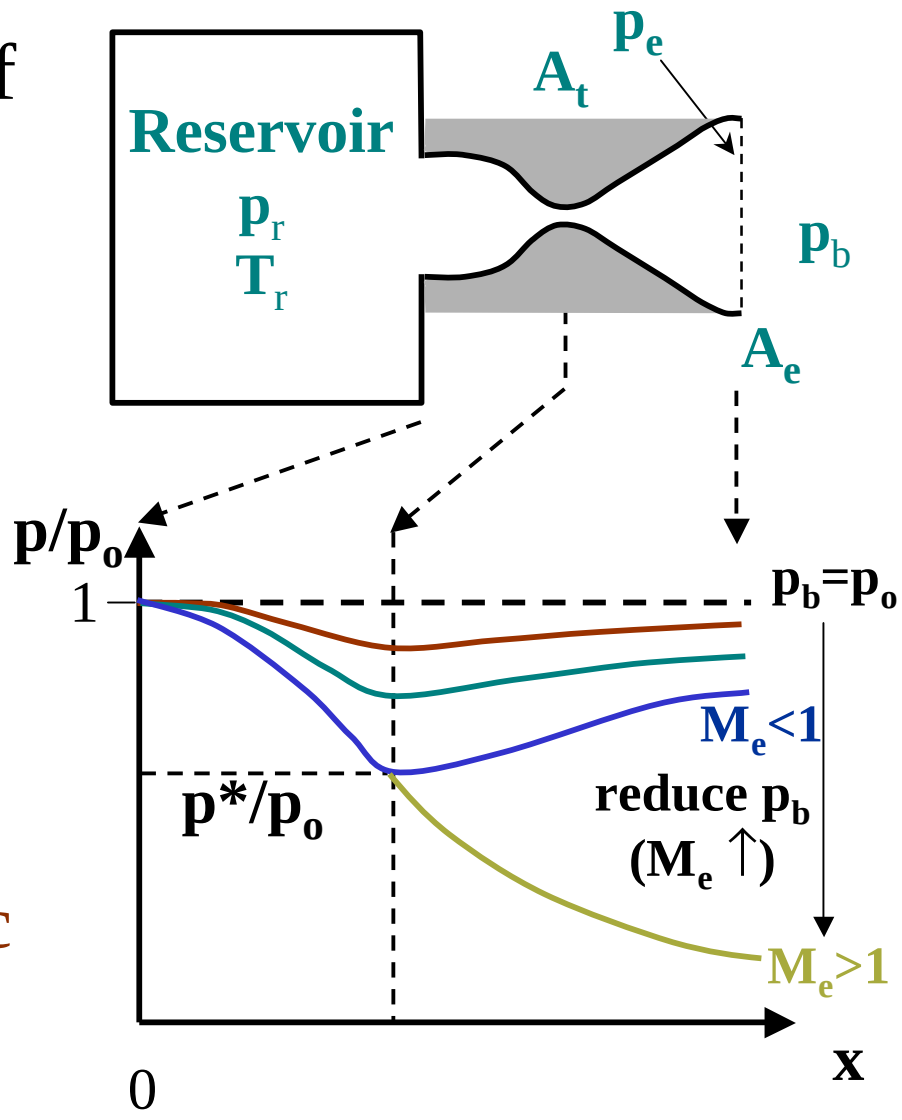
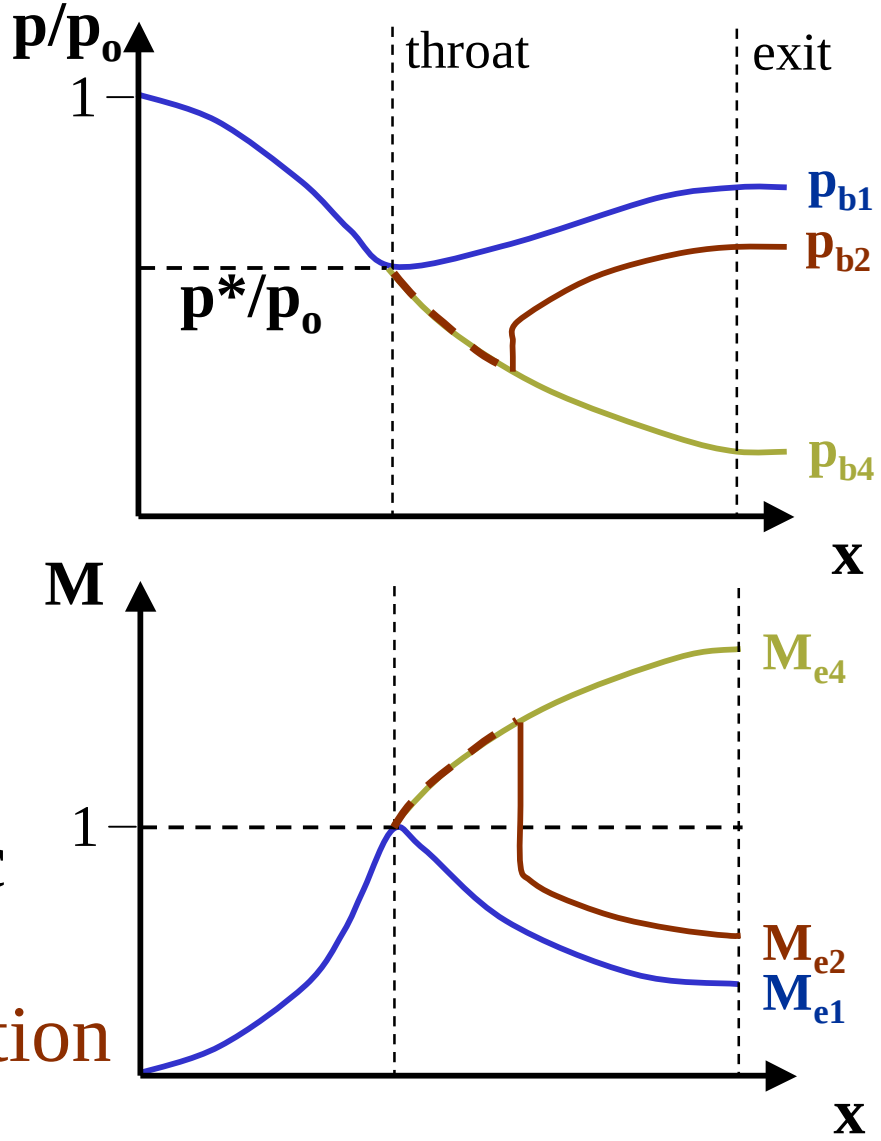
Normal Shocks in CD Nozzles

- Recall previous analysis of converging-diverging nozzles
  - e.g., as back pressure is reduced
- Once p_b lowered enough to get sonic flow at throat:
  2 ***isentropic*** solutions
  - higher $p_b(=p_e)$, subsonic
  - lower $p_b(=p_e)$, supersonic



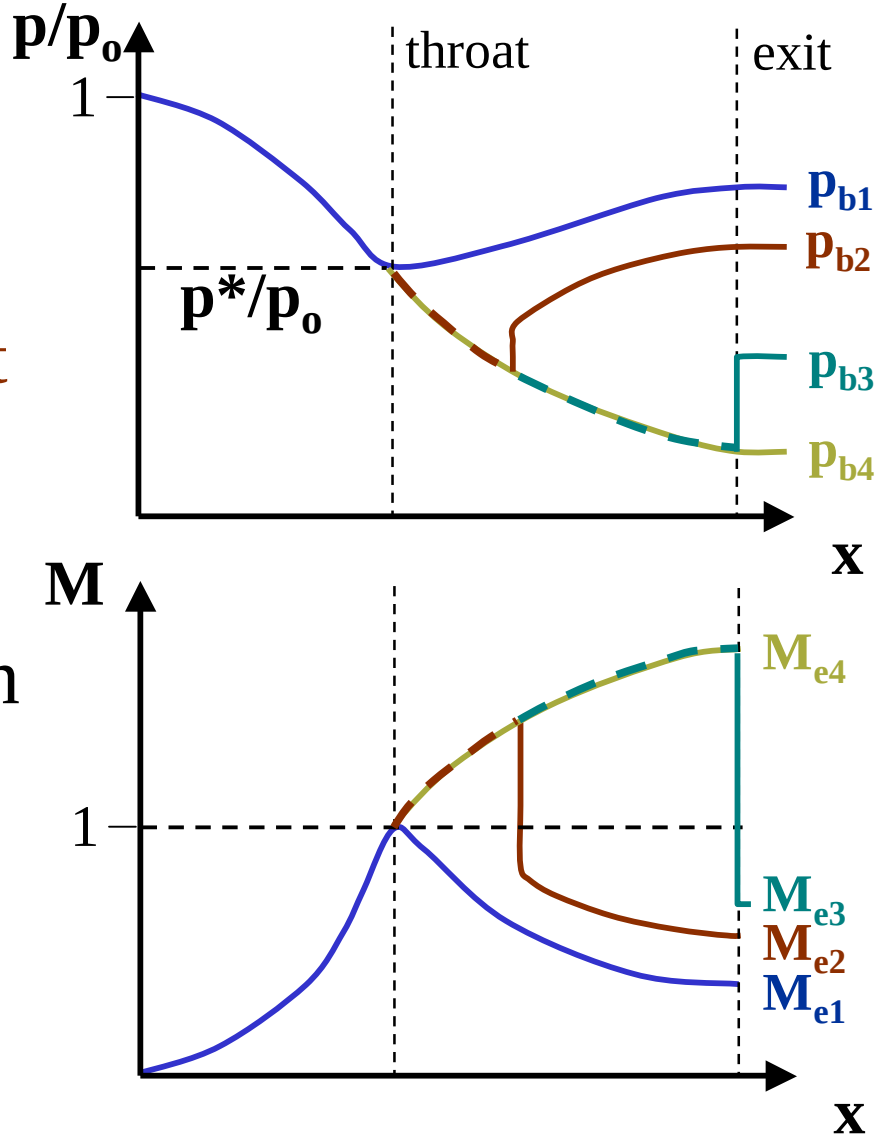
Nonisentropic Solutions

- What happens for p_b in between the isentropic solutions?
  - nonisentropic flow
- For $p_b < p_{b1}$
  - flow starts to go supersonic after throat
- For $p_{b1} > p_b > p_{b4}$, p must increase above supersonic isen. case to match p_b
  $\Rightarrow$ shock in diverging section



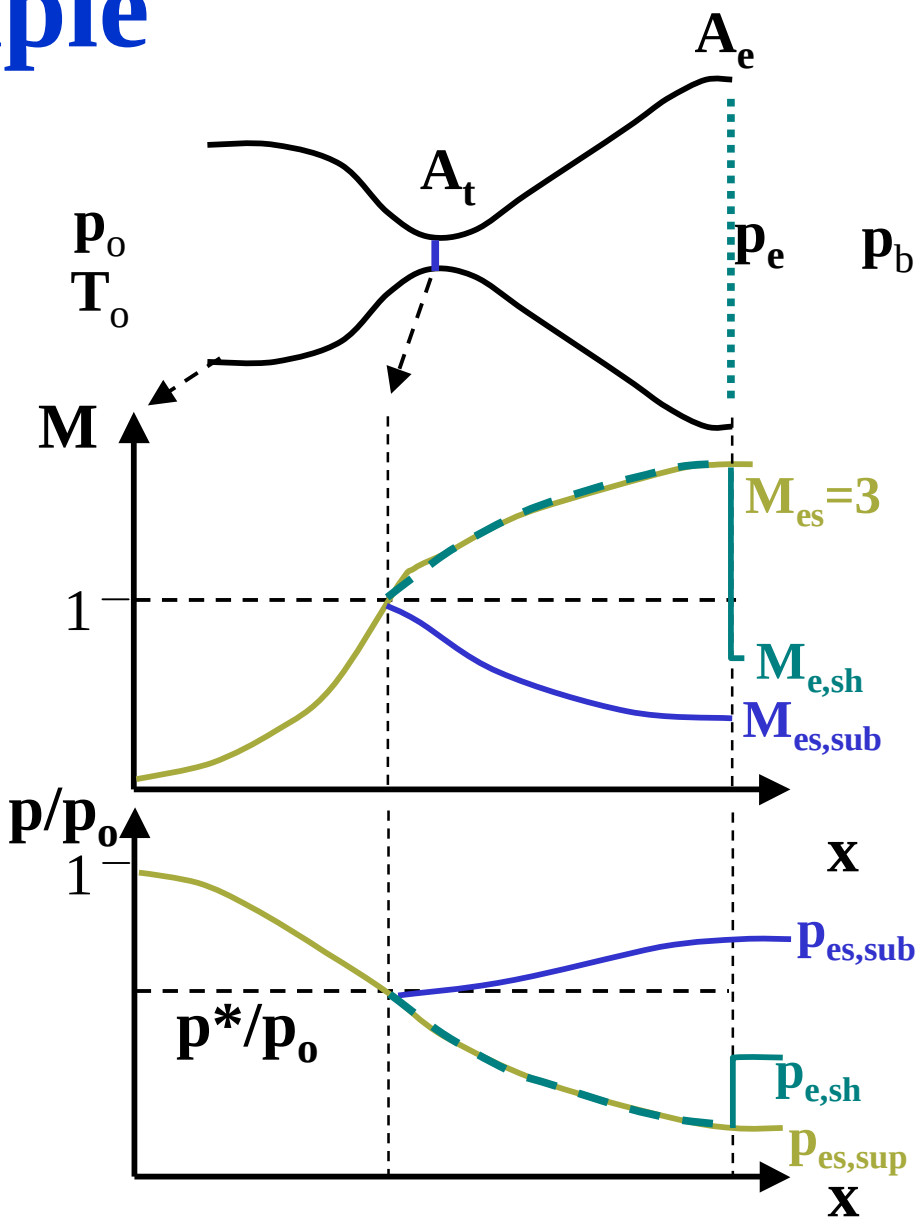
Shocks Inside Nozzle

- Over what range of back pressures will there be shock in nozzle
  - until shock occurs at exit plane of nozzle
- So, question becomes - what is exit pressure when normal shock sits at exit?
  - answer found by **combining isentropic and shock solutions**



Example

- **Given:** CD nozzle designed to produce M_e=3 for isentropic flow
- **Find:** What range of back pressure, p_b will produce shock in nozzle (**throat** →**exit**)?
- **Assume:** TPG/CPG with γ=1.4
- **Analysis:** Exit pressure, p_e, will have to match back pressure



Solution: Shock at Exit

- **Analysis (con't):**
  - **"Shock" at throat**

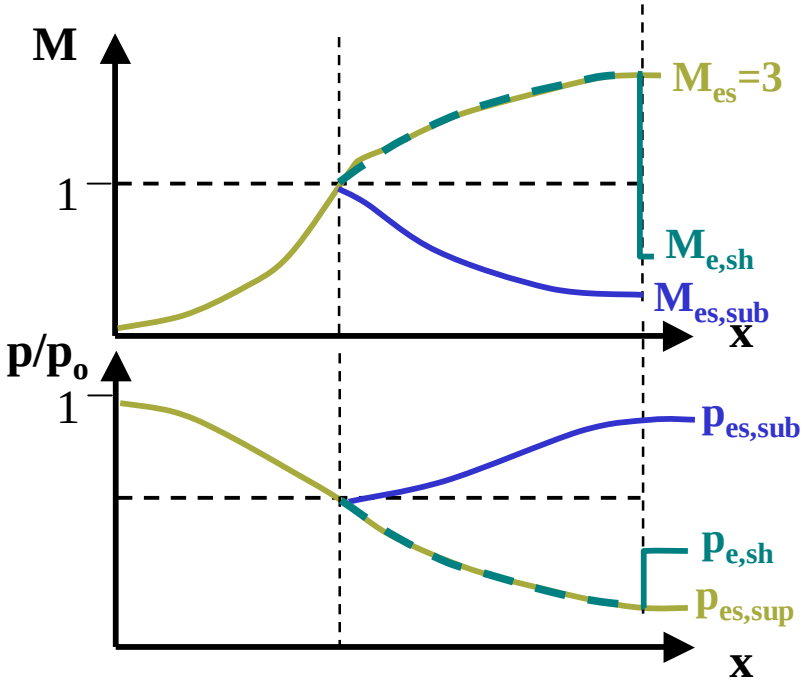
    (Use isentropic relations/tables)

    $$M_{es,sup} = 3 \Rightarrow M_{es,sub} = 0.138$$

    $$(\text{same } A_e/A^* = 4.235)$$

    $$p/p_o\big|_{M=0.138} = 0.9867$$

    $$\Rightarrow p_b = p_{es,sub} = 98.67\%p_o$$



  - **Shock at exit**

    (supersonic isentropic flow up to exit)

    $$p/p_o\big|_{M_{es}=3.0} = 0.0272 \Rightarrow p_{es,sup} = 2.72\%p_o$$

    (normal shock at M=3, shock relations/tables)

    $M_1=3 \mid M_2=M_e$

    $$M_{e,sh} = M_2\big|_{M_1=3} = 0.475 \text{ and } p_2/p_1\big|_{M_1=3} = 10.33$$

    $$\Rightarrow p_b = p_{es,sup}(p_2/p_1) = 28.1\%p_o$$

Over- and Underexpanded Nozzles

- What happens if back pressure goes below value where shock is at exit, $< p_{b3}$
  - isentropic flow up to exit, **supersonic exhaust**
  - shocks (and expansions) **outside nozzle** (not normal shocks)
- $p_{b4} < p_b < p_{b3}$
  - **Overexpanded** exhaust
- $p_b < p_{b4}$
  - **Underexpanded** exhaust