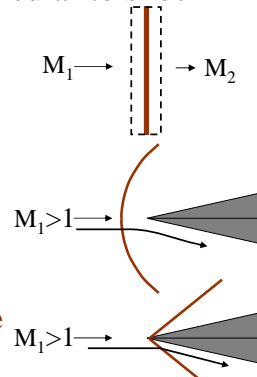


Supersonic Flow Turning

- For normal shocks, flow is perpendicular to shock
 - no change in flow direction
- How does supersonic flow change direction, i.e., make a turn
 - either slow to subsonic ahead of turn (can then make gradual turn) = **bow shock**
 - go through non-normal wave with sudden angle change, i.e., **oblique shock** (also expansions: see later)
- Can have straight/curved, 2-d/3-d oblique shocks
 - will examine straight, 2-d oblique shocks



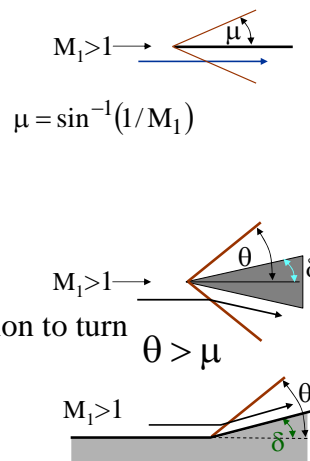
Oblique Shocks -1

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Oblique Shock Waves

- Recall Mach wave
 - consider infinitely thin body
 - no flow turn required
 - infinitesimal wave
- Oblique shock
 - consider finite-sized wedge, half-angle, δ
 - flow must undergo compression to turn
 - if attached shock \Rightarrow **oblique shock** at angle θ
 - similar for concave corner



Oblique Shocks -2

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Equations of Motion

- Governing equations
 - same approach as for normal shocks
 - use conservation equations and state equations
- Conservation Equations
 - mass, energy and momentum
 - this time 2 momentum equations - 2 velocity components for a 2-d oblique shock
- Assumptions
 - steady flow (stationary shock), inviscid except inside shock, adiabatic, no work but flow work

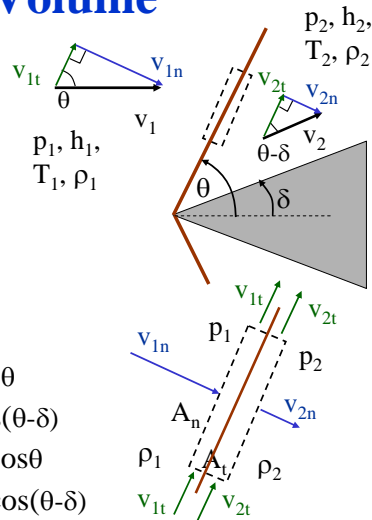
Oblique Shocks -3

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Control Volume

- Pick control volume along shock
- Divide velocity into two components
 - one tangent to shock, \mathbf{v}_t
 - one normal to shock, \mathbf{v}_n
- Angles from geometry
 - $v_{1n} = v_1 \sin \theta$; $v_{1t} = v_1 \cos \theta$
 - $v_{2n} = v_2 \sin(\theta - \delta)$; $v_{2t} = v_2 \cos(\theta - \delta)$
 - $M_{1n} = M_1 \sin \theta$; $M_{1t} = M_1 \cos \theta$
 - $M_{2n} = M_2 \sin(\theta - \delta)$; $M_{2t} = M_2 \cos(\theta - \delta)$



Oblique Shocks -4

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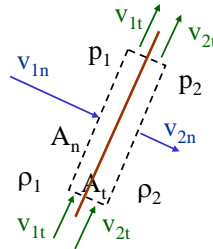
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Conservation Equations

- Mass** $0 = \int \rho(\vec{v}_{rel} \cdot \vec{n})dA$

$$\rho_1 v_{1n} A_n + \rho_1 v_{1t} \frac{A_t}{2} + \rho_2 v_{2t} \frac{A_t}{2} = \rho_2 v_{2n} A_n + \rho_1 v_{1t} \frac{A_t}{2} + \rho_2 v_{2t} \frac{A_t}{2}$$

$$\rho_1 v_{1n} = \rho_2 v_{2n} \quad (1)$$



- Momentum** $-\int p \cdot \vec{n} dA = \int \rho \vec{v}(\vec{v}_{rel} \cdot \vec{n})dA$

tangent $(p_1 + p_2) \frac{A_t}{2} - (p_1 + p_2) \frac{A_t}{2} = v_{1t}(-\rho_1 v_{1n} A_n) + v_{2t}(\rho_2 v_{2n} A_n)$

$$v_{1t} = v_{2t}$$

normal $p_1 A_n - p_2 A_n = v_{1n}(-\rho_1 v_{1n} A_n) + v_{2n}(\rho_2 v_{2n} A_n)$

$$p_1 - p_2 = \rho_1 v_{1n}^2 + \rho_2 v_{2n}^2 \quad (2)$$

Oblique Shocks -5

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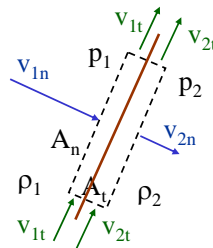
Conservation Equations (con't)

- Energy**

$$\rho_1 v_{1n} A_n \left(h_1 + \frac{v_1^2}{2} \right) = \rho_2 v_{2n} A_n \left(h_2 + \frac{v_2^2}{2} \right)$$

$$h_1 + \frac{v_{1n}^2}{2} + \frac{v_{1t}^2}{2} = h_2 + \frac{v_{2n}^2}{2} + \frac{v_{2t}^2}{2}$$

$$h_1 + \frac{v_{1n}^2}{2} = h_2 + \frac{v_{2n}^2}{2} \quad (3)$$



- Eq's. (1)-(3) are same equations used to characterize normal shocks (VII.1-3) with $v_n \rightarrow v$

- So oblique shock acts like normal shock in direction normal to wave

– v_t constant, but $M_{t1} \neq M_{t2}$ $\frac{M_{t2}}{M_{t1}} = \frac{v_{t2}/a_2}{v_{t1}/a_1} \Rightarrow \frac{M_{t2}}{M_{t1}} = \sqrt{\frac{T_1}{T_2}}$ (VII.20)

Oblique Shocks -6

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Oblique Shock Relations

- To find conditions across shock, use M relations from normal shocks, e.g., (VII.5-17) but replace

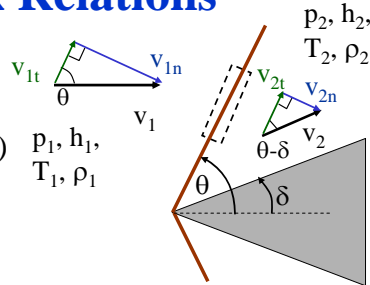
$$M_1 \rightarrow M_1 \sin \theta = M_{1n}$$

$$M_2 \rightarrow M_2 \sin(\theta - \delta) = M_{2n}$$

- Mach Number (after shock)**

$$\text{from (VII.11)} \quad M_2^2 = \left(M_1^2 + \frac{2}{\gamma - 1} \right) / \left(\frac{2\gamma}{\gamma - 1} M_1^2 - 1 \right)$$

$$M_2^2 \sin^2(\theta - \delta) = \left(M_1^2 \sin^2 \theta + \frac{2}{\gamma - 1} \right) / \left(\frac{2\gamma}{\gamma - 1} M_1^2 \sin^2 \theta - 1 \right) \quad \text{(VII.21)}$$



Oblique Shocks - 7

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Oblique Shock Relations (con't)

- Static Properties**

(from VII.12)

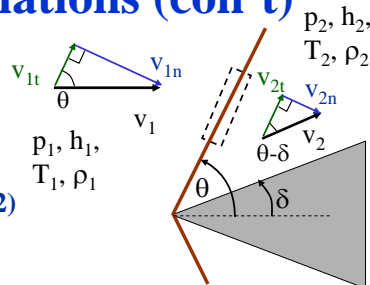
$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma + 1} M_1^2 \sin^2 \theta - \frac{\gamma - 1}{\gamma + 1} \quad \text{(VII.22)}$$

(from VII.18)

$$\frac{v_{1n}}{v_{2n}} = \frac{\rho_2}{\rho_1} = \frac{(\gamma + 1) M_1^2 \sin^2 \theta}{(\gamma - 1) M_1^2 \sin^2 \theta + 2} \quad \text{(VII.23)}$$

(from VII.9)

$$\frac{T_2}{T_1} = \frac{\left(1 + \frac{\gamma - 1}{2} M_1^2 \sin^2 \theta \right) \left(\frac{2\gamma}{\gamma - 1} M_1^2 \sin^2 \theta - 1 \right)}{M_1^2 \sin^2 \theta (\gamma + 1)^2 / 2(\gamma - 1)} \quad \text{(VII.24)}$$



Oblique Shocks - 8

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Oblique Shock Relations (con't)

- Stagnation Properties

T_o (from energy conservation)

$$T_{o2} = T_{o1}$$

p_o (from VII.14) $p_{o2}/p_{o1} = (T_1/T_2)^{\gamma/\gamma-1} (p_2/p_1)$ since function of static property ratios, don't have to factor in $p_{ot} \cdot p_{on}$

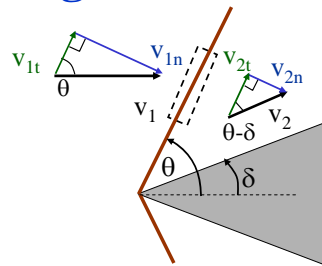
(from VII.13)

$$\frac{p_{o2}}{p_{o1}} = \left[\frac{\frac{\gamma+1}{2} M_1^2 \sin^2 \theta}{1 + \frac{\gamma-1}{2} M_1^2 \sin^2 \theta} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{2\gamma}{\gamma+1} M_1^2 \sin^2 \theta - \frac{\gamma-1}{\gamma+1} \right]^{\frac{1}{1-\gamma}}$$

(VII.25)

Wave/Shock Angle

- Equations above are functions of M_1 , θ (shock angle) and δ (turning angle)
- Is there a relationship between them?



(from VII.23) (from geometry) **Alternate Eq'n.**

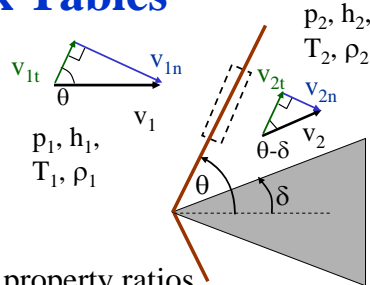
$$\frac{v_{1n}}{v_{2n}} = \frac{(\gamma+1)M_1^2 \sin^2 \theta}{(\gamma-1)M_1^2 \sin^2 \theta + 2} = \frac{v_{1t} \tan \theta}{v_{2t} \tan(\delta - \theta)} \quad \tan \delta = \frac{(1/\tan \theta)(M_1^2 \sin^2 \theta - 1)}{\frac{\gamma+1}{2} M_1^2 - (M_1^2 \sin^2 \theta - 1)}$$

$$\tan \delta = \frac{(2/\tan \theta)(M_1^2 \sin^2 \theta - 1)}{M_1^2 (\gamma + \cos 2\theta) + 2} \quad \text{(VII.26)}$$

So to find oblique shock solution, need 2 indep. parameters, e.g., M_1 and δ

Use of Shock Tables

- Since just replacing
 - $M_1 \rightarrow M_1 \sin \theta$
 - $M_2 \rightarrow M_2 \sin(\theta - \delta)$
 - can also use normal shock tables
 - use $M_1' = M_1 \sin \theta$ to look up property ratios
 - $M_2 = M_2' / \sin(\theta - \delta)$, with M_2' from normal shock tables
- **Warning**
 - do not use p_1/p_{o2} from tables
 - only gives p_{o2} associated with v_{2n} , not v_{2t}



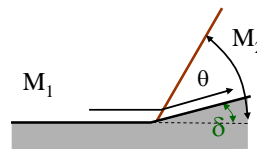
Oblique Shocks -11

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Oblique Solution Summary

- If given M_1 and shock angle, θ
 1. Find δ from VII.26 or use oblique shock charts (Appendix C in John)
 2. Calculate $M_{1n} = M_1 \sin \theta$
 3. Use normal shock tables or Mach relations, e.g., VII.22-25 to get property ratios
 4. Get M_2 from $M_2 = M_{2n} / \sin(\theta - \delta)$ or VII.21
- If given M_1 and turning angle, δ
 1. Find θ from (iteration) VII.26 or use oblique shock charts (e.g., Appendix C in John)
 2. Steps 2-4 above



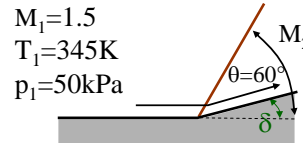
Oblique Shocks -12

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Example #1 – Known Shock Angle

- **Given:** Uniform Mach 1.5 air flow ($p=50$ kPa, $T=345$ K) approaching sharp concave corner. Oblique shock produced with shock angle of 60°



- **Find:**
 1. T_{o2}
 2. p_2
 3. δ (turning angle)
- **Assume:** TPG/CPG with $\gamma=1.4$, steady, adiabatic, no work, inviscid except for shock,....

Oblique Shocks -13

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Example #1 (con't)

- **Analysis:**

– **T_o**

$$T_{o2} = T_{o1} = T_1 \left(1 + \frac{\gamma-1}{2} M_1^2 \right)$$

$$= 345K \left(1 + 0.2(1.5)^2 \right) = 500K$$

– **p_2**
 – calculate normal component

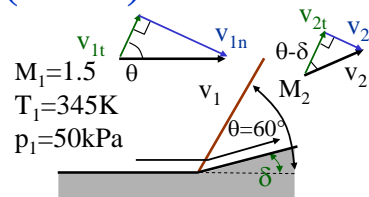
$$M_{1n} = M_1 \sin \theta = 1.5 \sin 60^\circ = 1.30 \quad (B.1) \Rightarrow M_{2n} = 0.786$$

$$p_2 = (p_2/p_1)p_1 = (1.805)50kPa = 90.3kPa \quad \begin{matrix} p_2/p_1 = 1.805 \\ T_2/T_1 = 1.191 \end{matrix}$$

– **δ** (from VII.26)

$$\tan \delta = \frac{(1/\tan 60^\circ)(1.5^2 \sin^2 60^\circ - 1)}{\frac{2.4}{2} 1.5^2 - (1.5^2 \sin^2 60^\circ - 1)} \Rightarrow \delta = 11.2^\circ$$

NOTE: $M_2 = M_{2n} / \sin(\theta - \delta) = 1.04 > 1$ Supersonic flow okay after oblique shock



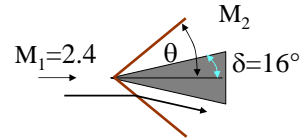
Oblique Shocks -14

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Example #2 – Known Turn Angle

- **Given:** Uniform Mach 2.4, cool, nitrogen flow passing over 2-d wedge with 16° half-angle.



- **Find:**

$$\theta, p_2/p_1, T_2/T_1, p_{o2}/p_{o1}, M_2$$

- **Assume:** N_2 is TPG/CPG with $\gamma=1.4$, steady, adiabatic, no work, inviscid except for shock,....

Example #2 (con't)

- **Analysis:**

– θ (from VII.26)

$$\tan 16^\circ = \frac{(2/\tan \theta)(2.4^2 \sin^2 \theta - 1)}{2.4^2(1.4 + \cos 2\theta) + 2}$$

iterate $\theta = 39.4^\circ$ or from John App. C: oblique shock charts

– **use shock relations** calculate normal component

$$M_{1n} = M_1 \sin \theta = 2.4 \sin 39.4^\circ = 1.52$$

$$(B.1) \Rightarrow M_{2n} = 0.6935; p_2/p_1 = 2.535; T_2/T_1 = 1.335; p_{o2}/p_{o1} = 0.9227$$

$$M_2 = M_{2n}/\sin(\theta - \delta) = 1.75 \quad \text{Supersonic after shock}$$

Example #2 (con't)

- Analysis (con't):

- can find second solution for θ

$$\tan 16^\circ = \frac{(2/\tan \theta)(2.4^2 \sin^2 \theta - 1)}{2.4^2(1.4 + \cos 2\theta) + 2}$$

in addition to 39.4° $\theta = 82.1^\circ$

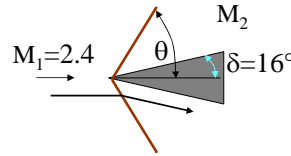
- use shock relations calculate normal component

$$M_{1n} = M_1 \sin \theta = 2.4 \sin 82.1^\circ = 2.38$$

(B.1) $\Rightarrow M_{2n} = 0.5256$; $p_2/p_1 = 6.425$; $T_2/T_1 = 2.018$; $p_{02}/p_{01} = 0.5499$

$$M_2 = M_{2n} / \sin(\theta - \delta) = 0.575 \quad \text{Now subsonic after shock}$$

- VII.26 generally has 2 solutions for θ : Strong and Weak oblique shocks



Oblique Shocks -17

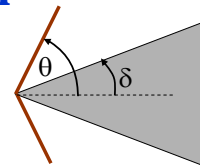
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Alternate Approach

- Possible to find direct relationship* for θ as function of M_1 and δ

$$\tan \theta = \frac{M_1^2 - 1 + 2\lambda \cos\left(\frac{4\pi\alpha + \cos^{-1} \chi}{3}\right)}{3\left(1 + \frac{\gamma-1}{2} M_1^2\right) \tan \delta} \quad \text{(VII.27)}$$



Removes iteration requirement...just longer equations

$$\lambda = \sqrt{(M_1^2 - 1)^2 - 3\left(1 + \frac{\gamma-1}{2} M_1^2\right)\left(1 + \frac{\gamma+1}{2} M_1^2\right) \tan^2 \delta}$$

$$\chi = \frac{1}{\lambda^3} \left[(M_1^2 - 1)^3 - 9\left(1 + \frac{\gamma-1}{2} M_1^2\right)\left(1 + \frac{\gamma-1}{2} M_1^2 + \frac{\gamma+1}{4} M_1^4\right) \tan^2 \delta \right]$$

$$\alpha = \begin{cases} = 0 & \text{strong shock solution} \\ = 1 & \text{weak shock solution} \end{cases}$$

Oblique Shocks -18

*e.g., Anderson, Modern Comp. Flow, 3rd ed., p. 142

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