Equilibrium Diagrams and Saturated Liquid/Vapor Systems

- In equilibrium, different phases of matter
  - gas, liquid, solid (also multiple solid phases, e.g., different crystalline structures of steel)
- So far looked at individual phases of simple (homogeneous) substances
- Multiple phases can exist simultaneously in equilibrium
- Consider heating simple compressible substance at constant pressure

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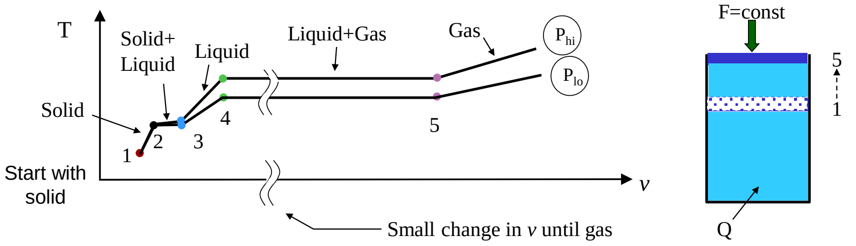
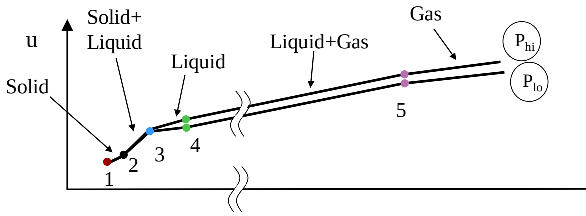
Phase Transitions - Constant Pressure

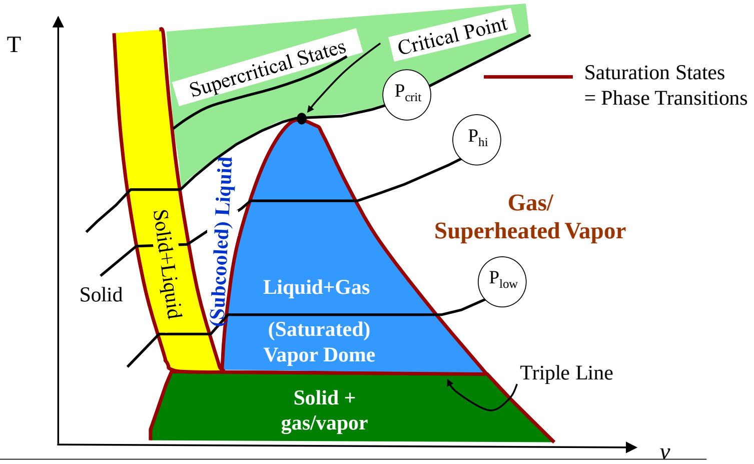




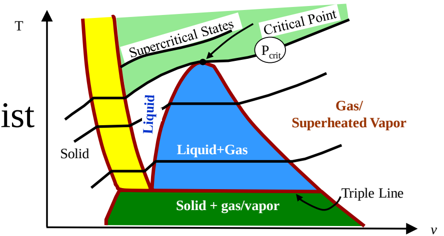
2→3: Melting
4 →5: Evaporation

3→2: Freezing/Fusion
5 →4: Condensation

Typical T-*v* Diagram



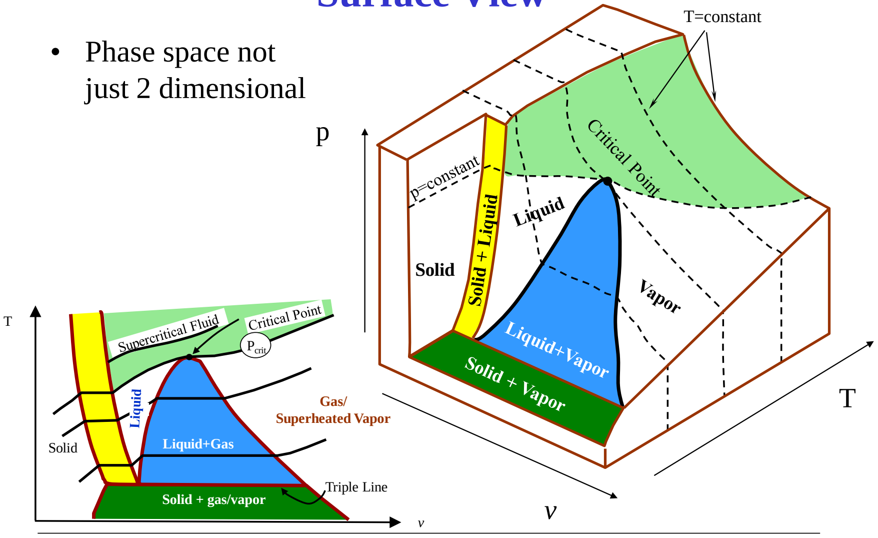
Phases of Compressible Substances



- Triple Line
  - – all 3 phases can coexist at same T and P
- Critical Point
  - – maximum T and p at which distinct liquid and gas phases can coexist
- Supercritical States (or supercritical fluid)
  - – for $p>p_{crit}$), no distinct transition from liquid to gas, fluid can have characteristics of both

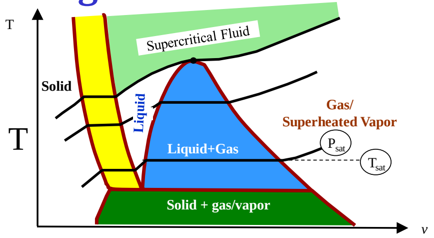
Surface View

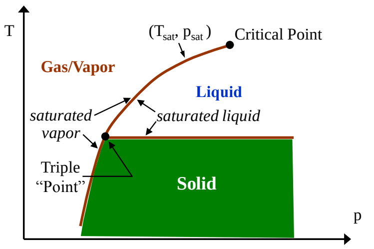
- Phase space not just 2 dimensional



Mixed Phase Regions

- Independent TD properies
  - p and T not independent
  - phase transition at const. p, T (e.g., given T_{sat}, know p_{sat})
- Can define new intensive variable to characterize composition
  - describe how much of each phase present
  - should not depend on total mass





Quality of Saturated Liquid-Vapor

- For saturated liquid-vapor region (vapor dome)
  - new variable is quality, *x*

$$\boxed{x \equiv \frac{\mathrm{m_g}}{\mathrm{m_{mixture}}} = \frac{\mathrm{m_g}}{\mathrm{m_f} + \mathrm{m_g}}}$$

  - *x* gives fraction of mass that is gas (g) **vapor mass fraction, Y_v**
  - (1-*x*) is fraction of mass that is liquid (f) **liquid mass fraction, Y_l**
  - $0 < x < 1$ (0 for liquid, 1 for gas)

Volume of Saturated Liquid-Vapor Mixture

- Get TD properties of saturated mixture by summing up proper amount of property for each phase, e.g.,

$$\mathrm{V_{mix}} = \mathrm{m_g} v_\mathrm{g} + \mathrm{m_f} v_\mathrm{f} = \mathrm{m_{mix}}\left[\frac{\mathrm{m_g} v_\mathrm{g}}{\mathrm{m_{mix}}} + \frac{\mathrm{m_f} v_\mathrm{f}}{\mathrm{m_{mix}}}\right]$$

$$= \mathrm{m_{mix}}\left[x v_\mathrm{g} + (1-x) v_\mathrm{f}\right]$$

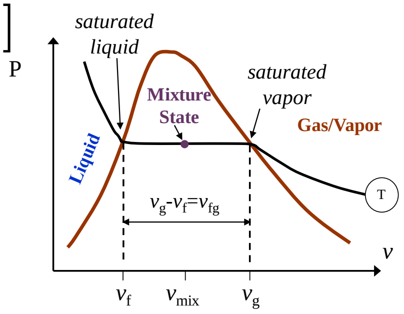
or

$$v_\mathrm{mix} = x v_\mathrm{g} + (1-x) v_\mathrm{f}$$

$$= v_\mathrm{f} + x\left(v_\mathrm{g} - v_\mathrm{f}\right)$$

  - where

g→gas, f → liquid



Saturated Liquid-Vapor Mixture Properties

- For general intensive property ***y*** that obeys sum rule,

$$\boxed{y_{mix} = x y_g + (1-x) y_f}$$

or

$$\boxed{y_{mix} = y_f + x\, y_{fg}}$$

with $y_{fg} \equiv y_g - y_f$

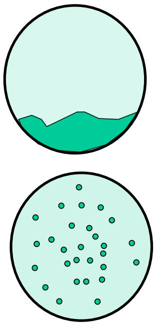
– y_{mix} always bounded by y_f , y_g

– does not work for ρ ($\rho \neq \rho_1 + \rho_2$)

Does T_{mix} obey sum rule?

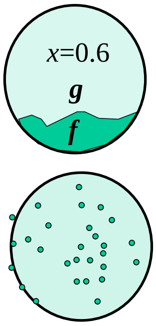
Example 1:

- **Given:** 3 kg of water at 200°C with quality x=0.6
- **Find:** Volume (V) and internal energy (U)
- **Assume:** phase equilibrium



Example 1:

m=3 kg
T=200°C



- **Analysis:**

$$V = m\left[v_f + x v_{fg}\right]$$

Table B.1

$$= 3kg\left[0.0011564 + 0.6(0.1262)\right]\frac{m^3}{kg}$$

$$= 3kg\left[0.07688\, m^3/kg\right]$$

$$= 0.231 m^3$$

Note: v_{mix}>>v_f (~70×)
because v_g>>v_f
or ρ_g<<ρ_f

$$U = m\left[u_f + x u_{fg}\right]$$

$$= 3kg\left[850.58 + 0.6(1744.1)\right] kJ/kg$$

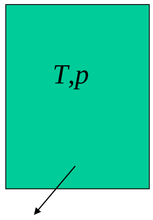
u_g only ~3×u_f

$$= 5691 kJ$$

What is pressure of mixture?

Example 2:

V=const



- **Given:** 5 lb_m of water initially at 700°F and 2000 psia in rigid container
- **Find:** How much energy loss required to cool water to 300°F
- **Assume:** equilibrium initial and final state
- **Analysis:**

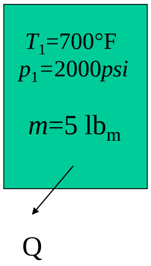
p dV=0

$$Q = \Delta U = m\left(u_1 - u_2\right)$$

Need u_2 and u_1

Example 2:

V=const



T_1=700°F
p_1=2000psi

m=5 $\mathrm{lb_m}$

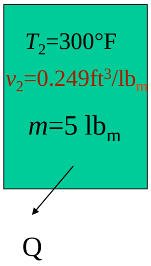
Q

- ***u_1=?***
- T>T_{sat}
  - –superheated vapor

u_1=1147.2 BTU/$\mathrm{lb_m}$

Example 2:

V=const



T_2=300°F
v_2=0.249ft^3/$\mathrm{lb_m}$

m=5 $\mathrm{lb_m}$

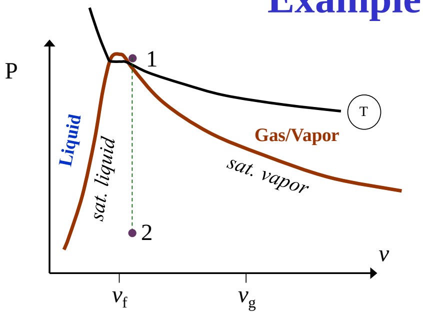
Q

$$v_2 = v_f + x_2 v_{fg}$$

$$u_2 = \qquad = 299\,BTU/lb_m$$

$$\begin{aligned}\Delta U &= m(u_1 - u_2) = Q\\ &= \\ &= 4240\,BTU\,(4.47\,MJ)\end{aligned}$$

Example 2:



V=const

T_1=700°F
p_1=2000psi
v_1=0.25ft^3/lb_m
m=5 lb_m

T_2=300°F
p_2=66.97psi
v_2=0.25ft^3/lb_m
m=5 lb_m

- Do phases always coexist in equilibrium?