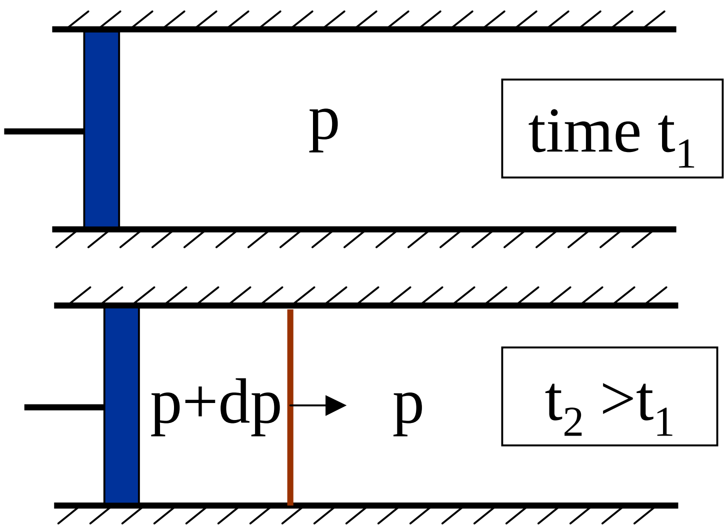
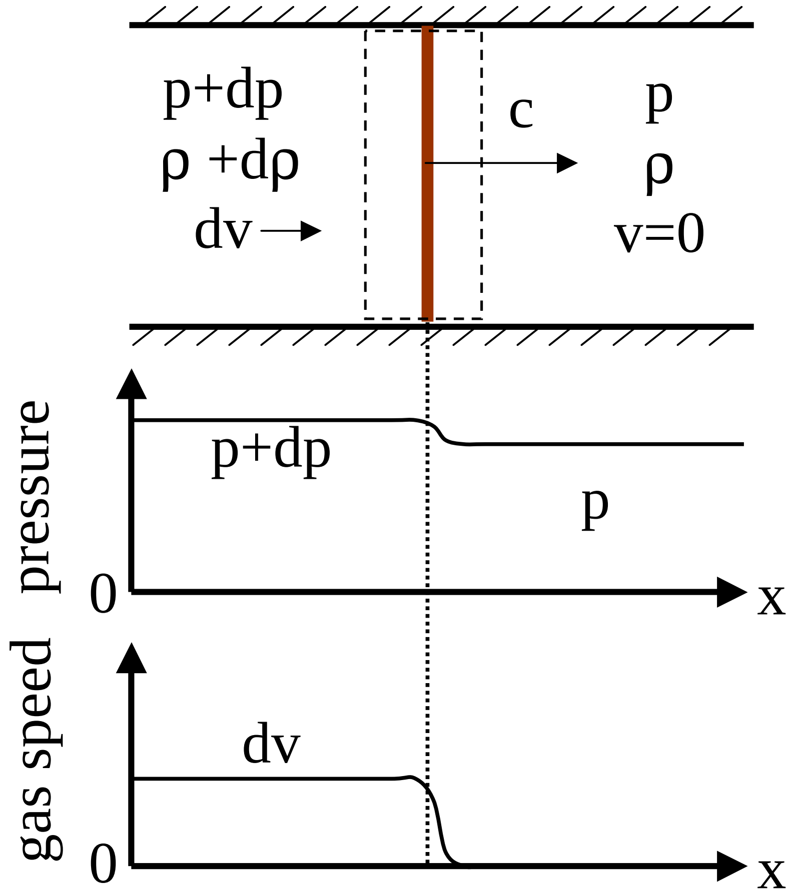
Speed of Sound

- Consider **adiabatic**, **1-D** propagation of **weak** (infinitessimal) pressure wave traveling through **initially stationary** (nonmoving), simple compressible substance
- Can think of piston given small "push"
  - Fluid to right must "find out" it needs to move
- Want to know how fast the wave propagates (**wave speed=?**)

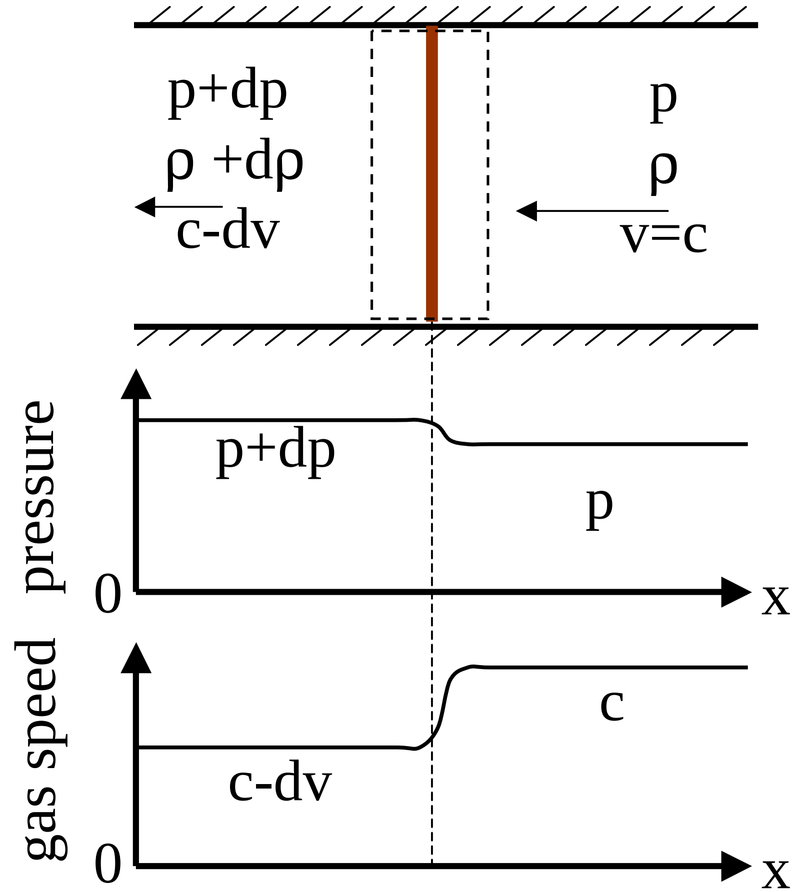


Control Volume & Reference Frame

- Analyze using control volume approach
  - pick "easiest" reference frame



"Lab" Reference Frame: Unsteady



"Wave" Reference Frame: Steady

Conservation Laws

- Assume steady, uniform, 1-D, inviscid, no body forces
- **Mass**

$$\dot{m}_{in} = \dot{m}_{out} = \dot{m}$$

$$\rho c A = (\rho + d\rho)(c - dv)A$$

$$\boxed{dv = c\frac{d\rho}{\rho + d\rho}}$$

- **Momentum** (x-dir) $A[(p + dp) - p] = \dot{m}[-(c - dv) + c]$

$$A\,dp = (\rho c A)dv$$

$$\boxed{dp = \rho c\, dv}$$

- **Combine** (eliminate dv)

$$dp = \rho c^2 \frac{d\rho}{\rho + d\rho}$$

$$c^2 = \frac{dp}{d\rho}\left(1 + \frac{d\rho}{\rho}\right)$$

Speed of Sound

$$c^2 = \frac{dp}{d\rho}\left(1+\frac{d\rho}{\rho}\right)$$

- For *weak* wave
  - wave speed ≡ sound speed, **a**
  - $d\rho/\rho << 1$
  - already assumed reversible and adiabatic
    ⇒ **isentropic**

$$c^2 = a^2 \equiv \left.\frac{\partial p}{\partial \rho}\right|_s \qquad \textbf{(VI.1)}$$

speed of sound

- Derivation valid for all simple compressible substances
- For incompressible substance, $d\rho \rightarrow 0$ and $a \rightarrow \infty$

Sound Speed for Ideal Gases

$$a^2 = \left.\frac{\partial p}{\partial \rho}\right|_s$$

- Gibbs equations for ideal gases

$$ds = c_v \frac{dT}{T} + p\,d(1/\rho) \quad \text{or} \quad ds = c_p \frac{dT}{T} - \frac{1}{\rho} dp$$

- Use ds=0 (isentropic) and $d(1/\rho) = -d\rho/\rho^2$

$$\frac{dT}{T} = \frac{p}{\rho^2 c_v} d\rho = \frac{1}{\rho c_p} dp$$

$$\left.\frac{\partial p}{\partial \rho}\right|_s = \frac{c_p}{c_v}\frac{p}{\rho} = \gamma \frac{p}{\rho}$$

- Ideal gas equation of state, $p/\rho = RT$

(VI.2) $a = \sqrt{\gamma R T}$

Calorically perfect gas *NOT* assumed, only equilibrium

Sound Speed for Ideal Gases (con't)

$$a = \sqrt{\gamma R T}$$

- $\gamma \uparrow$: **a** $\uparrow$
- **MW** $\uparrow$: **a** $\downarrow$ ($R \downarrow$)
- **T** $\uparrow$: **a** $\uparrow$

Substance	**a (m/s)** @ 15°C (60°F)
H_2(gas)	1294 (=4245 ft/s)
Air (gas)	340 (=1120 ft/s)
Water (liquid)	1490
Ice (solid)	3200
Aluminum (solid)	5150

- For air at "moderate" temperatures (γ=1.4)
  - $a(\text{ft/s}) = 49\sqrt{T(R)}$
  - $a(\text{m/s}) = 20\sqrt{T(K)}$