Speed of Sound

- Consider adiabatic, 1-D propagation of weak (infinitessimal) pressure wave traveling through initially stationary (nonmoving), simple compressible substance.

- Can think of piston given small “push”
  - Fluid to right must “find out” it needs to move.

- Want to know how fast the wave propagates (wave speed=?)

Control Volume & Reference Frame

- Analyze using control volume approach
  - pick “easiest” reference frame

  **“Lab” Reference Frame: Unsteady**

  **“Wave” Reference Frame: Steady**
Conservation Laws

- Assume steady, uniform, 1-D, inviscid, no body forces
- Mass
  \[ \dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m} \]
  \[ \rho c A = (\rho + \rho_c)(c - \frac{dc}{c}) A \]
  \[ dv = c \frac{dp}{\rho + dp} \]
- Momentum (x-dir)
  \[ A[(\rho + dp) - \rho] = m[(c - \frac{dc}{c}) + c] \]
  \[ \text{Adp} = (\rho c A) \text{dv} \]
  \[ \text{dp} = \rho c \text{dv} \]
- Combine (eliminate dv)
  \[ \text{dp} = \rho c^2 \frac{dp}{\rho + dp} \]
  \[ c^2 = \frac{dp}{dp} \left(1 + \frac{dp}{\rho}\right) \]

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Speed of Sound

- For weak wave
  - wave speed \( \equiv \) sound speed, \( a \)
  - \( dp/\rho \ll 1 \)
  - already assumed reversible and adiabatic
  \[ \Rightarrow \text{isentropic} \]
  \[ c^2 = a^2 = \frac{dp}{dp} \]
  (VI.1)
  \[ \text{speed of sound} \]

- Derivation valid for all simple compressible substances
- For incompressible substance, \( dp \to 0 \) and \( a \to \infty \)
Sound Speed for Ideal Gases

\[ a^2 = \frac{\partial p}{\partial \rho} \]

- Gibbs equations for ideal gases
  \[ ds = c_v \frac{dT}{T} + p \frac{d(l/\rho)}{l} \quad \text{or} \quad ds = c_p \frac{dT}{T} - \frac{1}{\rho^2} \frac{dp}{\rho} \]
- Use \( ds=0 \) (isentropic) and \( d(1/\rho) = -dp/\rho^2 \)
  \[ \frac{dT}{T} = \frac{\rho}{\rho^2 c_v} dp = \frac{1}{\rho c_p} dp \]
  \[ \frac{\partial p}{\partial \rho} = c_p \frac{p}{\rho} = \gamma \frac{p}{\rho} \]
- Ideal gas equation of state, \( p/\rho = RT \)

(\( \gamma \)↑: \( a \)↑)

Sound Speed for Ideal Gases (con’t)

\[ a = \sqrt{\gamma RT} \]

- \( \gamma \)↑: \( a \)↑
- MW ↑: \( a \) ↓ (R ↓)
- T ↑: \( a \) ↑

For air at “moderate” temperatures (\( \gamma = 1.4 \))
- \( a(\text{ft/s}) = 49 \sqrt{T(\text{R})} \)
- \( a(\text{m/s}) = 20 \sqrt{T(\text{K})} \)

<table>
<thead>
<tr>
<th>Substance</th>
<th>( a ) (m/s) @ 15°C (60°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂(gas)</td>
<td>1294 (=4245 ft/s)</td>
</tr>
<tr>
<td>Air (gas)</td>
<td>340 (=1120 ft/s)</td>
</tr>
<tr>
<td>Water (liquid)</td>
<td>1490</td>
</tr>
<tr>
<td>Ice (solid)</td>
<td>3200</td>
</tr>
<tr>
<td>Aluminum (solid)</td>
<td>5150</td>
</tr>
</tbody>
</table>