Stagnation Properties and Mach Number

- Rewrite stagnation properties in terms of Mach number for thermally and calorically perfect gases
- **Stagnation Temperature**
  - from energy conservation: no work but flow work and **adiabatic**

  $\Rightarrow$ **T_o** (and **h_o**) **constant** for **adiabatic** flow

$$T_o = T + \frac{1}{c_p}\frac{v^2}{2} = T + \frac{\gamma-1}{2}\frac{v^2}{\gamma R}$$

$$\frac{T_o}{T} = 1 + \frac{\gamma-1}{2}\frac{v^2}{\gamma R T} = 1 + \frac{\gamma-1}{2}\frac{v^2}{a^2}$$

$$\frac{T_o}{T} = 1 + \frac{\gamma-1}{2}M^2 \qquad \textbf{(VI.6)}$$

- **Stagnation Pressure**
  - from entropy conservation: reversible and adiabatic **$\Rightarrow$isentropic (Δs=0)**

  $\Rightarrow$ **p_o** (and **s_o**) **constant** if ***also* reversible**

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{\gamma/\gamma-1}$$

from state eq. for isen. process

$$\frac{p_o}{p} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1} \qquad \textbf{(VI.7)}$$

Compressible p_o and Bernoulli Equation

- Incompressible flow, **Bernoulli eqn.** also gives a stagnation pressure (static + dynamic pressure)

$$p_o = p + \frac{1}{2}\rho v^2$$

- Expand compressible p_o in **Taylor series**

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \ldots$$

$$\frac{p_o}{p} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/\gamma-1} = 1 + \frac{\gamma}{\gamma-1}\frac{\gamma-1}{2}M^2 + \frac{\gamma}{2(\gamma-1)}\left(\frac{\gamma}{\gamma-1}-1\right)\left(\frac{\gamma-1}{2}M^2\right)^2 + \ldots$$

$$\frac{p_o}{p} = 1 + \frac{\gamma}{2}M^2 + \frac{\gamma}{2}\left(\frac{M^2}{2}\right)^2 + \ldots = 1 + \frac{\rho}{2}\frac{v^2}{p} + \frac{\rho}{8}\frac{v^2}{p}M^2 + \ldots$$

use $M^2 = \dfrac{v^2}{\gamma p/\rho}$

$$p_o = p + \frac{1}{2}\rho v^2 + \frac{1}{2}\rho v^2\frac{M^2}{4} + \ldots$$

Bernoulli **higher terms negligible for small M (<0.3)** **$0.3^2/4=0.0225$**

Stagnation Density and Tables

- **Stagnation Density**
  - from T_o, p_o and ideal gas law ($\rho = p/RT$)
  - **ρ_o constant** for **isentropic** flow

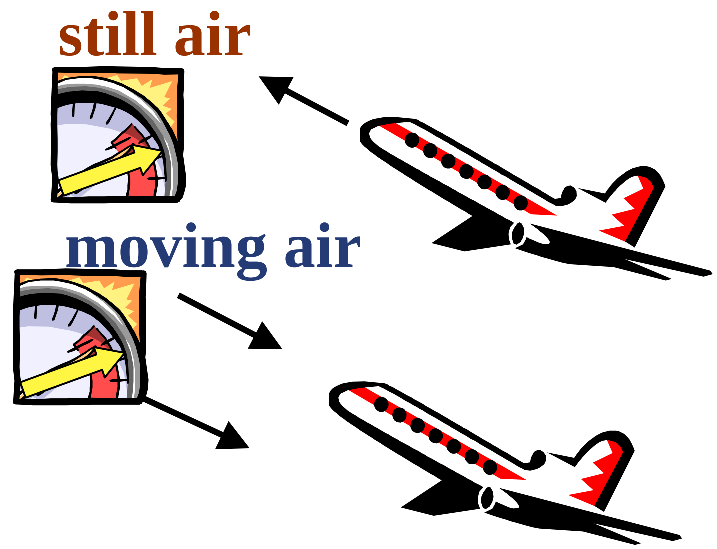
$$\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{1/\gamma-1}$$

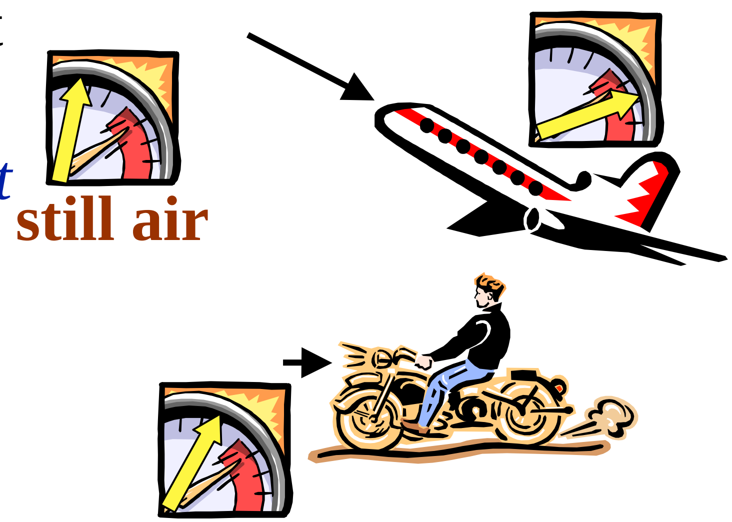
$$\frac{\rho_o}{\rho} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{1/\gamma-1} \qquad \textbf{(VI.8)}$$

- **Tables**
  - T_o/T and p_o/p tabulated in text (John) as function of M listed as T_t/T and p_t/p (t for *total* T and total p)
  - γ=5/3 (Table A.3): atoms (Ar, He, …) at “not too high” T
  - γ =1.4 (Table A.1): diatomics (N_2, O_2, …) at “moderate” T
  - γ =1.3 (Table A.2): more atoms or higher T
  - make your own?

Stagnation versus Static Properties

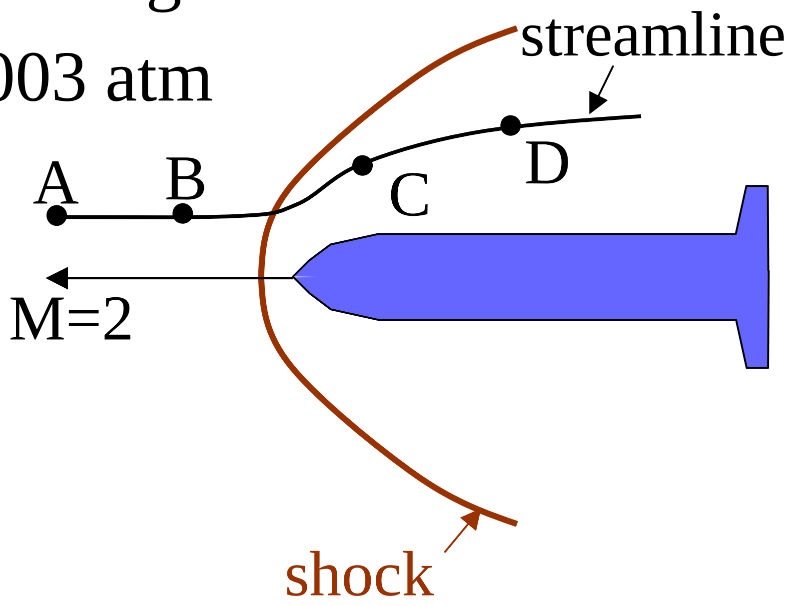
- **Static Properties**
  - represent the properties you would measure if you were moving with the flow (at the local flow velocity)
  - always defined in the flow's reference frame
- **Stagnation Properties**
  - always defined by conditions at a point
  - represent the (static) properties you'd measure if you first brought the fluid *at that point to a stop* (**isentropically**) *with respect to a chosen observer*
  - depends on observer's reference frame





Stagnation Properties: Example

- **Supersonic** projectile (M=2) flying through still air
- **Static** conditions: T_∞=250 K, p_∞=0.003 atm



- **Find**:
  1. **T_o at A** (T_{oA}) relative to observer on projectile
  2. **T_{oD}** (same observer) <, >, = T_{oA} ?
  3. **p_{oB}** (same observer)
  4. **p_{oC}** (same observer) <, >, = p_{oB} ?

Stagnation Properties: Example 2

- Projectile flying through still air at 170 m/s
- Static conditions: T_∞=288 K, p_∞=1 atm
- Nose of projectile = point B
- Find:
  1. **p_{oA}** (relative to observer on projectile)
  2. **p_B**
  3. **T_B**
- Hint, use $a = \sqrt{\gamma RT}$