Stagnation Properties and Mach Number

• Rewrite stagnation properties in terms of Mach number for thermally and calorically perfect gases

  **Stagnation Temperature**
  - from energy conservation:
    - no work but flow work and adiabatic
    \[ T_o = T + \frac{1}{c_p} \left( \frac{1}{2} v^2 - \frac{\gamma-1}{2} \frac{v^2}{\gamma R} \right) \]
    \[ \frac{T_o}{T} = 1 + \frac{\gamma-1}{2} \frac{v^2}{\gamma R T} \]
    \[ \frac{T_o}{T} = 1 + \frac{\gamma-1}{2} M^2 \quad (VI.6) \]
  - \( T_o \) (and \( h_o \)) constant for adiabatic flow

  **Stagnation Pressure**
  - from entropy conservation:
    - reversible and adiabatic
    \[ p_o = \frac{(T_o/T)^{\gamma-1}}{M^2} \quad \text{from state eq. for isentropic process} \]
    \[ p_o = \left( \frac{T_o}{T} \right)^{\gamma-1} \] (VI.7)
  - \( p_o \) (and \( s_o \)) constant if also reversible

Compressible \( p_o \) and Bernoulli Equation

• Incompressible flow, **Bernoulli eqn.** also gives a stagnation pressure (static + dynamic pressure)

  \[ p_o = p + \frac{1}{2} p v^2 \]

• Expand compressible \( p_o \) in **Taylor series**

  \[ (1 + x)^n = 1 + nx + \frac{n(n-1)}{2} x^2 + \ldots \]

  \[ \frac{p_o}{p} = \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma-1} = 1 + \frac{\gamma-1}{2} \frac{M^2}{\gamma-1} \left( \frac{1}{2} M^2 \right)^2 + \ldots \]

  \[ \frac{p_o}{p} = 1 + \frac{\gamma-1}{2} M^2 + \frac{\gamma}{2} \left( \frac{M^2}{2} \right)^2 + \ldots = 1 + \frac{\rho v^2}{2} + \frac{\rho v^2 M^2}{8} + \ldots \]

  \[ \rho_o = \rho + \frac{1}{2} \rho v^2 + \frac{1}{2} \rho v^2 \frac{M^2}{4} + \ldots \]

  **Bernoulli** higher terms negligible for small \( M \) (<0.3) \[ 0.3^{0.5} \approx 0.0225 \]
### Stagnation Density and Tables

**Stagnation Density**
- from $T_o$, $p_o$ and ideal gas law ($\rho = p/RT$)
- $\rho_o$ constant for isentropic flow

\[
\frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{\gamma-1}
\]

(IV.8)

**Tables**
- $T_o/T$ and $p_o/p$ tabulated in text (John) as function of $M$
  - listed as $T_t/T$ and $p_t/p$ (t for total $T$ and total $p$)
- $\gamma=5/3$ (Table A.3): atoms (Ar, He, …) at “not too high” $T$
- $\gamma=1.4$ (Table A.1): diatomics (N$_2$, O$_2$, …) at “moderate” $T$
- $\gamma=1.3$ (Table A.2): more atoms or higher $T$
  - make your own?

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### Stagnation versus Static Properties

**Static Properties**
- represent the properties you would measure if you were moving with the flow (at the local flow velocity)
- always defined in the flow’s reference frame

**Stagnation Properties**
- always defined by conditions at a point
- represent the (static) properties you’d measure if you first brought the fluid at that point to a stop (isentropically) with respect to a chosen observer
- depends on observer’s reference frame
Stagnation Properties: Example

- **Supersonic** projectile (M=2) flying through still air
- **Static** conditions: $T_{\infty}=250\text{ K}$, $p_{\infty}=0.003\text{ atm}$

\[
T_{\infty}=250\text{K}
\]
\[
p_{\infty}=0.003\text{atm}
\]

- **Find:**
  1. $T_o$ at A ($T_{oA}$) relative to observer on projectile
  2. $T_{oD}$ (same observer) $<$, $>$, $=$ $T_{oA}$?
  3. $p_{oB}$ (same observer)
  4. $p_{oC}$ (same observer) $<$, $>$, $=$ $p_{oB}$?

Stagnation Properties: Example (con’t)
Stagnation Properties: Example 2

• Projectile flying through still air at 170 m/s
• Static conditions: $T_\infty = 288$ K, $p_\infty = 1$ atm
• Nose of projectile = point B
• Find:
  1. $p_{oA}$ (relative to observer on projectile)
  2. $p_B$
  3. $T_B$
• Hint, use $a = \sqrt{\gamma RT}$