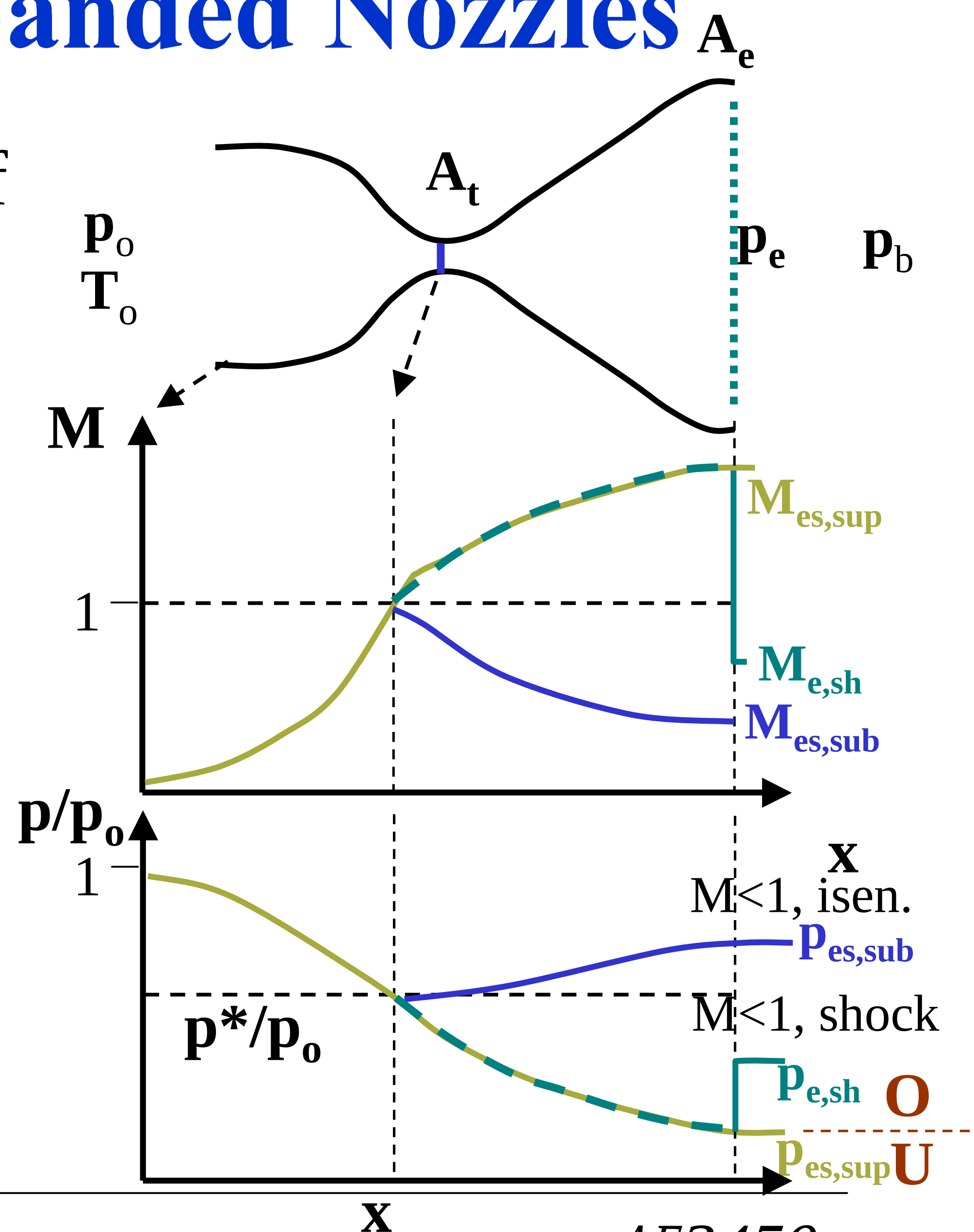
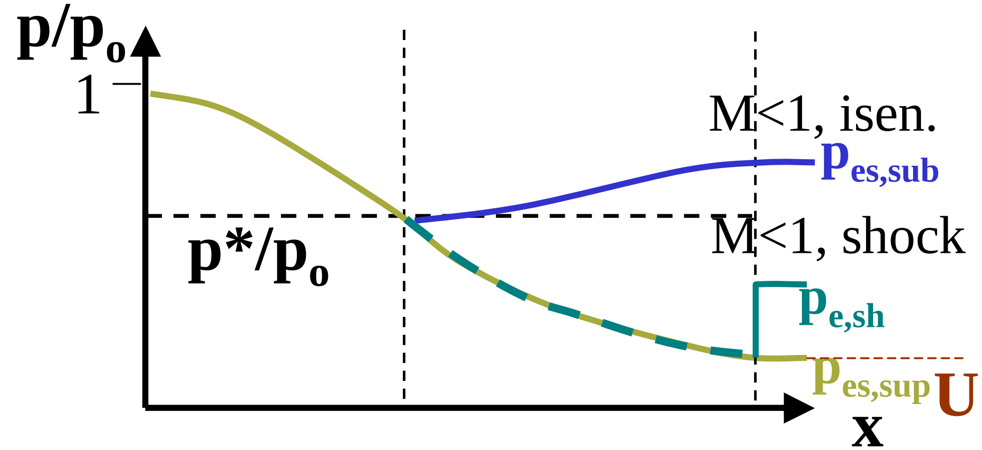
Under/Overexpanded Nozzles

- Recall previous analysis of CD nozzles, e.g., as back pressure is reduced
- Have looked at range of p_b that produce
  - *isentropic* solutions or
  - ***shocks in nozzle***
- What happens when
  - $p_b < p_{es,sup}$ **(U)**
  - $p_{es,sup} < p_b < p_{e,\,sh}$ **(O)**



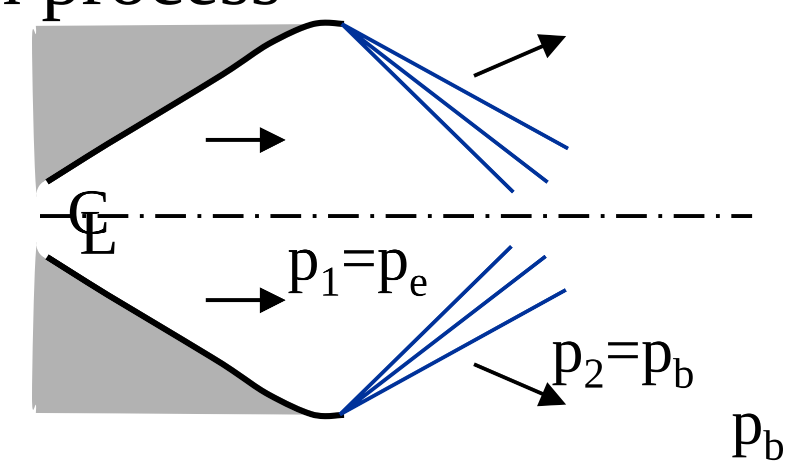
Underexpanded Nozzles

- Start with $p_b < p_{es,sup}$
  - **underexpanded case**
  - $p_e > p_b$ (not enough expansion has occurred)

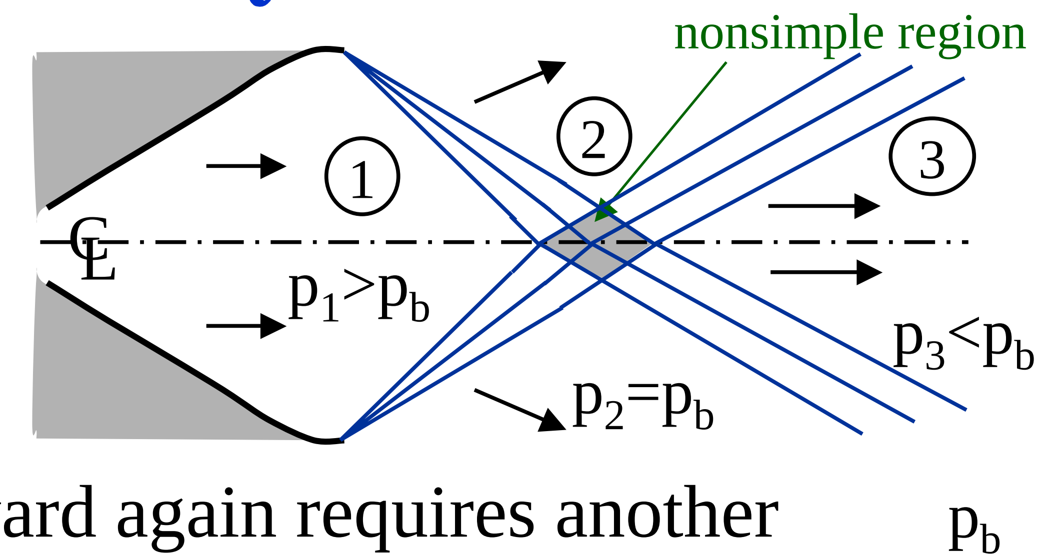


- So boundary condition at exit requires a supersonic expansion process

  ⇒ (2-d) Prandtl-Meyer flow
  - flow turns, pressure drops to back pressure
  - but flow must not cross centerline (symmetry line; like a wall)



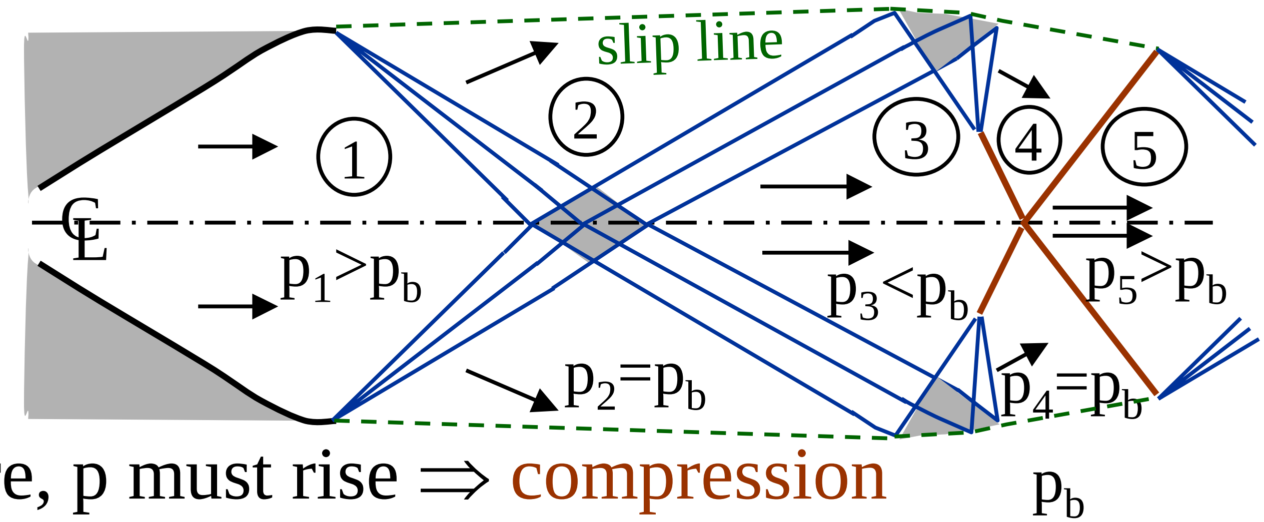
Centerline Boundary Condition



- Centerline
  - flow along centerline must be parallel to centerline
  - to get flow to turn inward again requires another PM expansion
  - but second expansion means pressure in region 3 drops below back pressure ($p_3<p_2=p_b$)
  - so now we will need to do something to match our pressure boundary condition at the edge of the jet

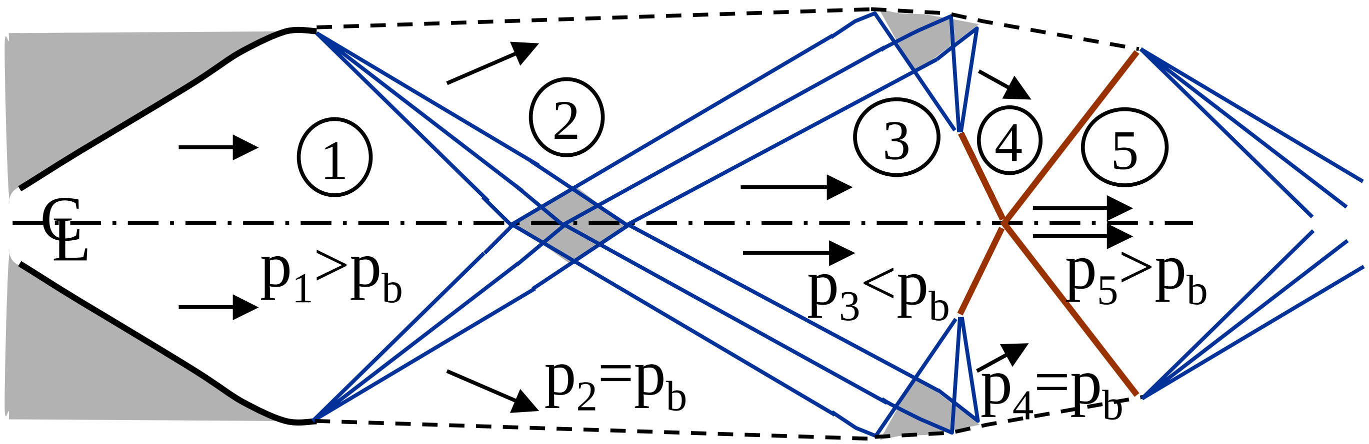
Jet Boundary Condition

- Jet boundary
  - along "edge" of jet must satisfy pressure b.c.



  - to match pressure, p must rise ⇒ compression
  - can get PM compression waves
  - combine to form oblique shocks, turn flow inward
  - centerline (velocity) boundary condition requires reflected compression waves
  - now pressure exceeds surrounding pressure ($p_5>p_b$), way we started, so process/cycle repeats itself

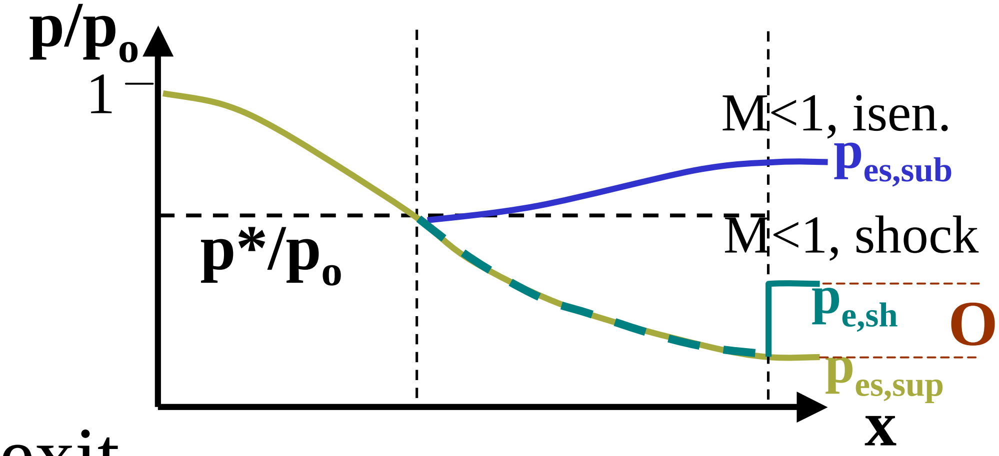
Underexpanded Jet



- For $p_e>p_b$, nozzle produces a jet flow consisting of regions of *decreasing* and *increasing* p (or density)
  - inviscid flow (ideal), process would continue endlessly
  - viscous case (real), viscous losses and turbulent mixing with surroundings causes wave pattern to decay after small number of cycles

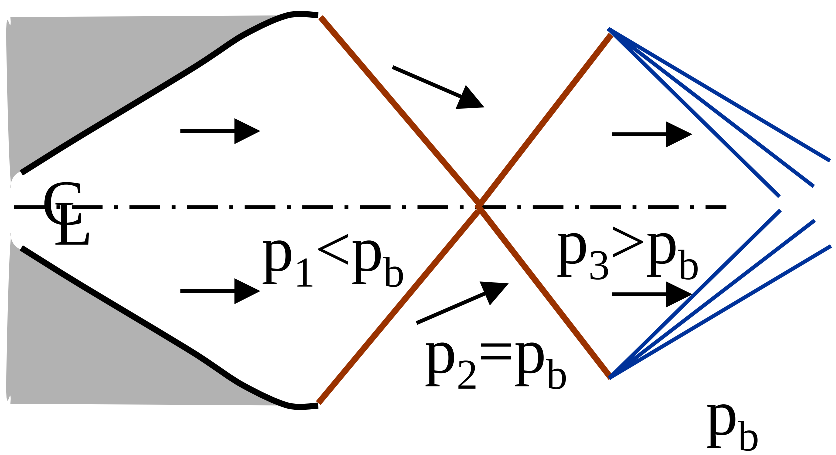
Overexpanded Nozzles

- $p_{es,sup} > p_b > p_{e,sh}$
  - **overexpanded case**
  - too much expansion has occurred

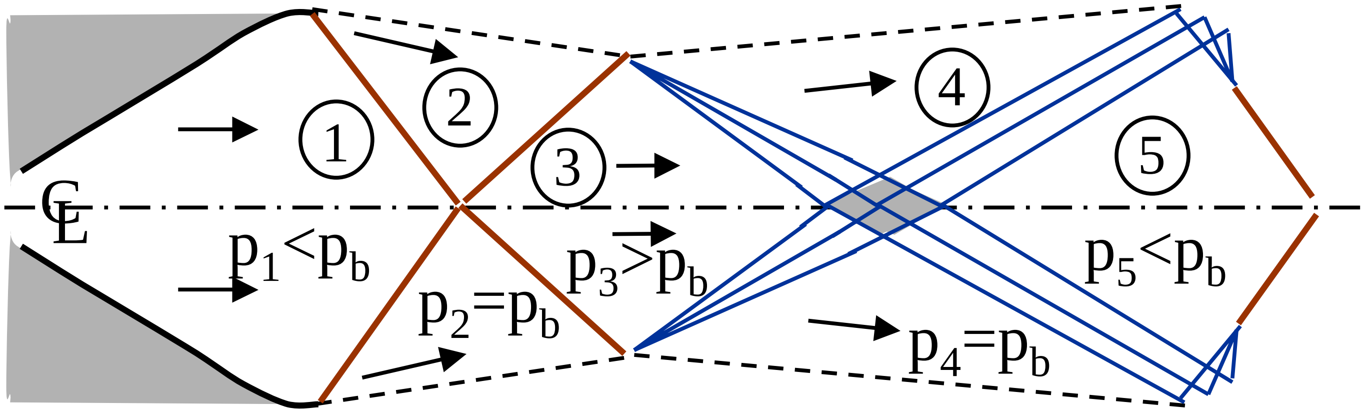


- So boundary condition at exit requires a supersonic compression process

  ⇒ oblique shock (2-d)
  - flow turns, pressure rises to back pressure
  - flow must not cross centerline, so get reflected shocks
  - then expansion to match p, etc.



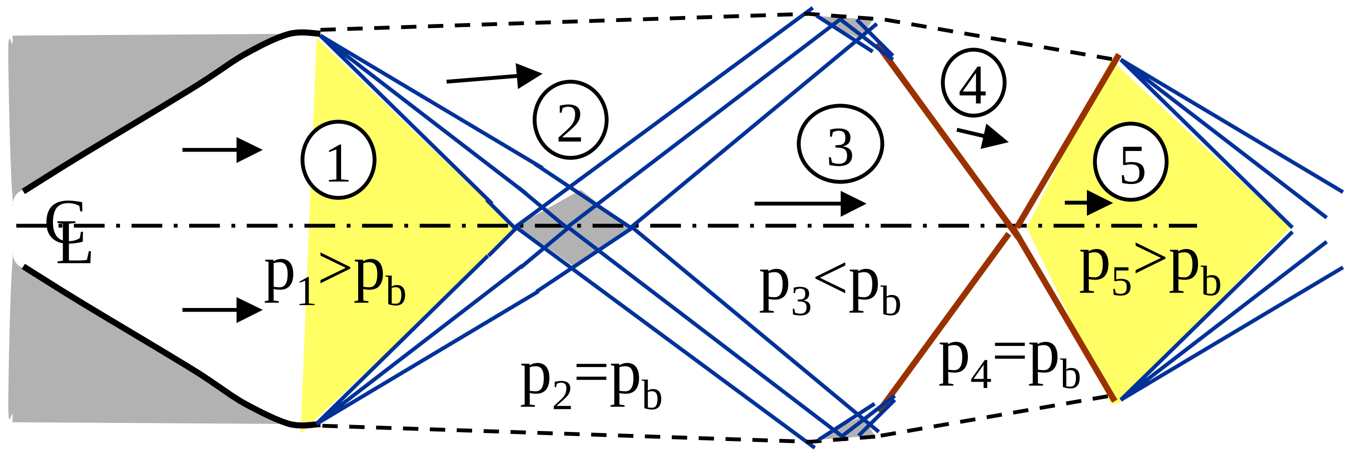
Overexpanded Jet



- For $p_e<p_b$, nozzle produces jet flow consisting of regions of *increasing* and *decreasing* pressure (or density)
  - same pattern as for underexpanded case, just "out-of-phase" (compressions first, then expansions for overexpanded vs. expansions then compressions for underexpanded)

Mach (Shock) Diamonds

- Regions of high pressure, also have high density (and temperature)
  - these hot, dense gases emit light (radiation)



SR-71 at takeoff