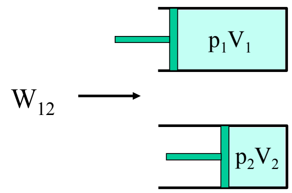
Example: Work as Path Function

- **Given:** Piston-cylinder containing compressible fluid at low pressure that is compressed by external force



- **Find:**
  1. **Work done by expanding fluid** from initial state (p_1,V_1) to final state (p_2,V_2) if **pV=constant**
  2. **Work done in two "step" process**:
     - first **V=constant** from p_1 to p_2,
     - then **p=constant** from V_1 to V_2
- **Assume:** Slow enough to be quasi-equilibrium processes

Solution: Work with pV=const

- **Analysis: Part 1**

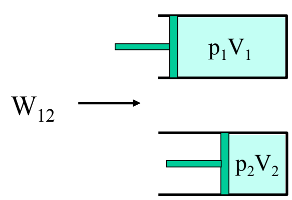
pdV work

$$W_{12} = \int_1^2 pdV = \int_1^2 \frac{const}{V} dV$$

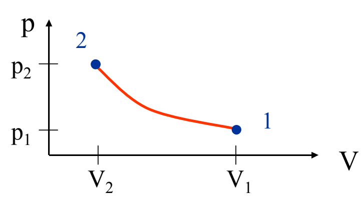
$$= p_2V_2 \int_{V_1}^{V_2} \frac{dV}{V}$$

$$= p_2V_2 \ln\frac{V_2}{V_1}$$

<0 ⇒PdV work negative if done TO system



p-V (State) Diagram



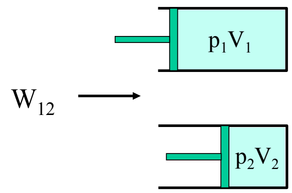
Solution: Work in 2 Steps

- **Analysis: Part 2**

pdV work

$$W_{12} = W_{1a} + W_{a2} = \int_1^a \cancel{p\,dV}^{\,0} + \int_a^2 p\,dV$$

$$= p_2 \int_{V_1}^{V_2} dV = p_2 \left(V_2 - V_1 \right)$$



$$W_{12,Path2} = p_2 V_2 \left(1 - \frac{V_1}{V_2} \right)$$

$$W_{12,Path1} = p_2 V_2 \ln \frac{V_2}{V_1}$$

$$\left| 1 - \frac{V_1}{V_2} \right| > \left| \ln \frac{V_2}{V_1} \right|$$

2nd path takes more work

p-V (State) Diagram