Electric Propulsion for Rockets

Performance Considerations

Electric Propulsion System Elements

Propellant
Gas (Xe), liquid (N_2H_4), solid (teflon)

Storage

Feed System

Energy Source
chemical, nuclear, radiation (solar)

Storage

Conversion
to electricity at proper V, DC/AC(*freq*), current, steady/pulsed

Accel./Thruster
Electrothermal (nozzle)
Electrostatic
Electromagnetic

since power source separate from propellant
*⇒ **I_{sp} not limited by chemical bond energy***
(can exceed 10,000 sec)

Electric Propulsion - Accelerators

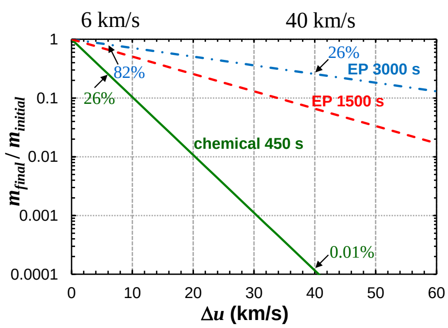
	Electrothermal	Electromagnetic	Electrostatic
Accel. Force	Pressure, ∇p Electrically heat propellant and use nozzle expansion	Lorentz, $\vec{F}_m$ Magnetic and elec. fields accelerate charged particles	Electrostatic, $\vec{F}_e$ Static electric field alone accelerates charged particles
I_{sp}(s)	300-1,500	1,000-10,000	2,000-100,000+
$\frac{\text{Thrust}}{\text{Weight}}$	$<10^{-1}$	$<10^{-4}$	$<10^{-4}$–10^{-6}

Advantages for In-Space Propulsion

- From rocket equation $m_{final}/m_{initial} = e^{-\Delta u/u_e} = e^{-\Delta u/I_{sp}g_e}$

In-space Propulsion Requirements

Mission	Typical Δu (km/s)
GEO stationkeeping (15 years)	1.0
LEO to GEO (< 1 day)	4.2
LEO to Mars (9 months)	5.7
LEO to GEO (8 months)	6.0
LEO to Jupiter (9 months)	50
LEO to Mars (1 month)	90
LEO to 1000 Aus (30 Yrs)	175



- **EP can provide greater (and reasonable) payload mass fractions**
  - especially for deep space missions, but even for GEO missions
  - lower launch costs, larger payloads,…

EP Power Requirements

- **Jet Power**

  minimum possible power required to operate EP device

$$P_j = \tfrac{1}{2}\dot{m}u_e^2 = \tfrac{1}{2}(\dot{m}u_e)u_e$$

$$\boxed{P_j = \tfrac{1}{2}\tau u_e} \quad \textbf{(IV.36a)}$$

if $u_{eq}=u_e$

$$\boxed{P_j = \tfrac{1}{2}\tau (I_{sp} g_e)} \quad \textbf{(IV.36b)}$$

- So increase in I_{sp} (or u_e) entails increase in power
  - $\propto u_e^2$ for constant propellant flowrate
  - $\propto u_e$ for constant thrust
- Comparison for rocket with moderate thrust of 4.5 kN (~1000 lb_f)
  - chemical rocket w/ I_{sp}= 350s $\Rightarrow$ P_j ~ 7.7*MW*
  - for elec. rocket w/ I_{sp}=3500s $\Rightarrow$ ***P_j ~ 77 MW***
- So **EP devices tend to be power-limited**
  - produce low thrust (and thus low acceleration)

Required Supply Power

- **Supply Power**

$$\boxed{P_s = \frac{P_j}{\eta_T}} \quad \textbf{(IV.37)}$$

η_T: total efficiency of energy conversion

- Overall conversion efficiency has 3 main components

$$\boxed{\eta_T = \eta_S \;\eta_{pp} \;\eta_{th}} \quad \textbf{(IV.38)}$$

η_S: energy conversion - of raw energy source to electricity
- ~100% for photovoltaics – direct conversion of photons (does NOT include fraction of solar radiation that can be converted to electricity by typical solar array, ~18-25%)
- 10-40% for nuclear thermal, thermodynamic cycle limits = lots of waste heat

η_{pp}: power preparation and conditioning electronics (losses in electronics)
- ~92% (electrostatic)
- ~98% (steady arc jets)

η_{th}: thruster efficiency - only part of delivered electrical energy converted to KE

30-75%

Mass Requirements

- What limits available power to EP systems?
  - typically it is mass of power plant and associated systems
- If power plant mass is significant fraction of propellant mass, then some advantages of higher specific impulse operation are lost
  - delivered "payload" may consist mostly of power plant/electronics for propulsion system
  - *not a problem if power plant part of desired payload*

Mass Requirements (con't)

- **Power source mass**

(IV.39) $$Mass_{power\ source} \cong \beta_s P_s$$

← nearly linear relationship

specific mass (e.g., kg/kW)

Note: sometimes α used for specific mass, but also used by others for specific power (kW/kg), i.e., $\alpha = 1/\beta$

- β_s~7-25 kg/kW$_{elec}$ for solar arrays (depends on cell design, substrate)
- β_s~2-4 kg/kW$_{thermal}$ for nuclear reactors (depends on shielding)
  - to reject waste heat, also require additional mass for radiators β_R~0.1-0.4 kg/kW$_{waste\ heat}$

Mass Requirements (con't)

- **Power preparation and conditioning**

$$Mass_{pp} \cong \beta_{pp} P_{pp} \quad \text{(IV.40)}$$

- Large variation with type of EP device (especially if need high voltage or power, and short pulse forming electronics/switches)
  - β_{pp}~0.2 kg/kW_{elec} for typical arcjets
  - β_{pp}~20 kg/kW_{elec} for PPT (pulsed plasma thrusters)

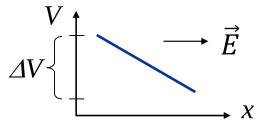
Electric Propulsion for Rockets

Physics Background

Electrical Definitions

- V – potential or voltage (sometimes ϕ) ($Volts$)
- E – electric field (V/m, N/C)
- q – charge (C) ($q_{e^-}=1.602\times10^{-19}\ C$)
- Force, $\vec{F} = \vec{E}q$
- Potential Energy $\int F dx = \int qE dx = q\Delta V$
- J – current (A, C/s)
- j – charge current density (A/m^2, C/sm^2) = J / A
- n_q – number density charged part. ($1/m^3$)
- u_q – velocity of charged particles (m/s)

$$\vec{E} = -\nabla V \text{ e.g.,} = -\frac{dV}{dx}$$



$$j = n_q q u_q$$

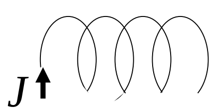
Electrical Heating

- Current passing through conductor heats it by amount proportional to its resistance

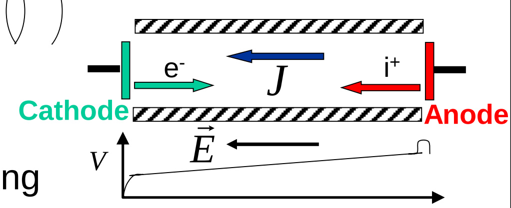
Current (A=C/s) Resistance (Ohms)

$$\dot{Q} = J^2 R$$

- Wire



- Gas (Plasma) Discharge
  - resistance heating due to collisions



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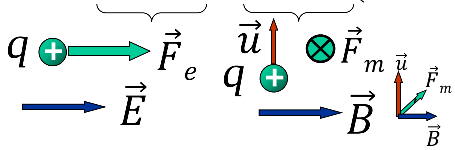
Forces on a Charged Particle

- To examine how to use electrical energy to accelerate a propellant, consider acceleration of a particle with mass ***m*** and charge ***q***

$$m\frac{d\vec{u}}{dt} = \overbrace{q\vec{E}}^{\text{Elec. Field}} + \overbrace{q\left(\vec{u}\times\vec{B}\right)}^{\text{Mag. Field}} + \vec{p}$$

Electrostatic Force — Lorentz Force — Collisional Force (Momentum Transfer)



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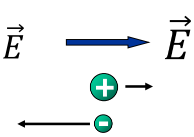
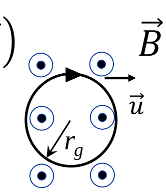
Motion of Charged Particle in E&B Fields

- How does charged particle move in electric and magnetic fields
- **Electric field only** $\quad \frac{d\vec{u}}{dt} = \frac{q}{m}\vec{E}$
  - electron lighter, higher accel.



- **Magnetic field only** $\quad \frac{d\vec{u}}{dt} = \frac{q}{m}\left(\vec{u}\times\vec{B}\right)$
  - particle gyrates (centripedal accel.)
  - radius of gyration $\quad r_g = \frac{m\left|\vec{u}\times\vec{B}\right|}{qB^2}$
    Larmor radius
  - frequency of gyration $\quad \omega_g = \frac{qB}{m}$
    cyclotron frequency
  - no work; B and F perpendicular



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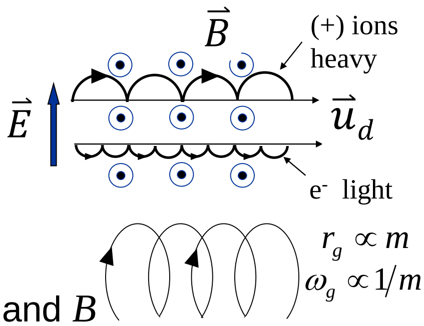
Motion of Charged Particle in E&B Fields

- **Crossed E and B fields** $m\frac{d\vec{u}}{dt} = q\vec{E} + q\left(\vec{u}\times\vec{B}\right)$
  - E field accelerates (+) particle upward
  - B field causes acceleration perpendicular to u
  - overall result is drift velocity normal to E and B



$$\vec{u}_d = \frac{\vec{E}\times\vec{B}}{B^2}$$

related to Hall effect

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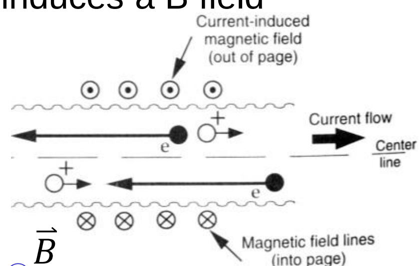
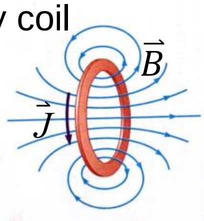
Induced Magnetic Fields

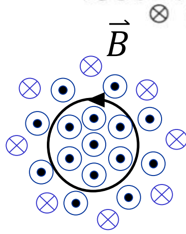
- In general, current flow induces a B field
  - B field induced by linear current



From *Space Propulsion Analysis and Design*, Humble, Henry and Larson, 1995

  - B field induced by coil





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Plasmas

- Gas composed of equal "amount" of negatively and positively charged particles
  - electrically neutral
  - negative particles usually e^-
  - positive particles are positive ions
  - most of gas molecules often remain neutral (weak plasma, or partially ionized gas)
- **Momentum equation for plasma**

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u}\right) = -\nabla p + \vec{j}\times\vec{B}$$

$\vec{j}$ ← Current Density (A/m^2)

$\nabla\vec{u} \equiv (\nabla u_1, \nabla u_2, \nabla u_3)$

Induced E Fields – Hall Effect

- **Electron acceleration**
  - lighter e^- accelerate more quickly and accommodate to flow field (most of j)
  - collisional coupling (momentum) of electrons and heavies (ions and neutrals) is weak

$$\Rightarrow \vec{E} = \eta\,\vec{j} - \vec{u}\times\vec{B} + \vec{j}\times\frac{\vec{B}}{n_e e} - \frac{\nabla p_e}{n_e e}$$

- η: plasma resistivity (Ωm), $\eta = m_e \nu_{ce} / n_e e^2$, where ν_{ce} is the collision freq. electrons with heavies
- $-\vec{u}\times\vec{B}$: Induced E field due to plasma motion
- $\vec{j}\times\vec{B}/n_e e$: **Hall Effect** accelerates heavy particles in charge neutral plasma
- $\nabla p_e / n_e e$: electron pressure term

Electric Propulsion for Rockets

Electrothermal Devices

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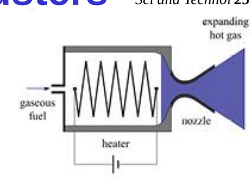
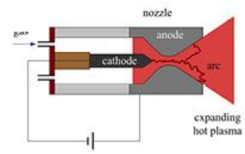
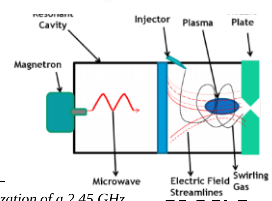
Electrothermal Thrusters

S. Mazouffre, Plasma Sources Sci and Technol ***25*** *(2016)*

- **Resistojets** (e.g., 10W to 10kW)
  - current through solid resistive heating element; heat transfer to flowing propellant
  - $T_{gas}<T_{solid}$, T_{solid} < 3000K
- **Arcjets** (e.g., kWs to MWs)
  - current flows through flowing propellant gas as arc discharge across electrodes
  - T_{max}>5000K
- **Microwave Heated Thrusters**
  - microwave transmitter produces and energizes plasma in propellant flow







from Yildiz et al., "Geometry Optimization of a 2.45 GHz Microwave Electrothermal Thruster Resonant Cavity" (2015)

Resistojets

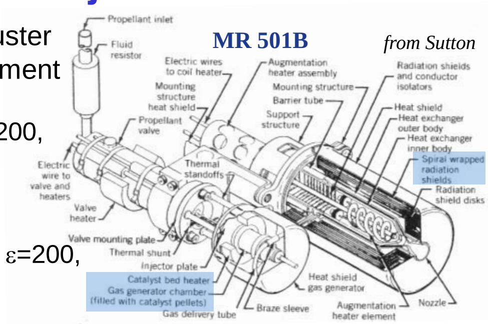
- Most used ET space thruster
- Early commercial deployment on Intelsat V, 1980
  - I_{sp}=295s, τ=450mN, ε=200, τ/w=0.13, P_{elec}~0.4kW
- Typical system Aerojet MR-501
  - 294-303s, 200-400mN, ε=200, τ/w~0.03, P_{elec}~0.5kW
- Propellants
  - most common N_2H_4 (hydrazine) decomposition products
    - I_{sp}=300 s, ~30% higher than using decomposition (chem.) alone
  - NH_3, H_2 (long term storage issues), ISS uses waste
- Power requirements
  - AC or DC

assuming no heat losses

$$\dot{Q} = V^2/R = \dot{m} c_p \Delta T \quad \textbf{(IV.41)}$$

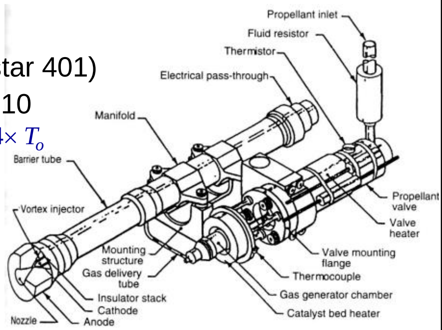
MR 501B

from Sutton



Arcjets

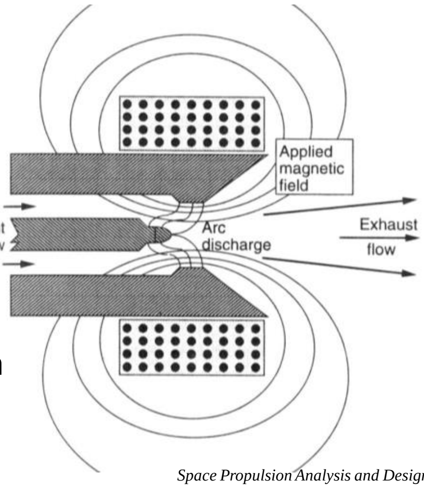
- 1st application 1994
  - satellite stationkeeping (Telstar 401)
- Upgraded version Aerojet MR-510
  - uses hydrazine $\quad 2\text{x } I_{sp} \Rightarrow 4\times\ T_o$
  - I_{sp}~600s, τ~250mN, τ/w~0.02, P_{elec}~2.2 kW
- Other propellants
  - NH_3 (800 s)
  - H_2 (1500-2000s)
- Challenges
  - excessive heating can limit life
  - higher frozen flow losses, so lower thrust efficiencies (e.g., ~35% of input power converted to useful thrust)
  - possible plume contamination



Space Propulsion Analysis and Design, Humble, Henry and Larson, 1995

Arcjet with Applied Magnetic Field

- High current density in arc tends to destroy electrodes
- Add soleniod B Field
  - adds azimuthal drift velocity to arc
  - improves azimuthal symmetry of gas temperature
  - reduces local electrode heating, reduces erosion
  - needed for high powers (≥100 kW)



Space Propulsion Analysis and Design, Humble, Henry and Larson, 1995

Electric Propulsion for Rockets

Electrostatic Devices

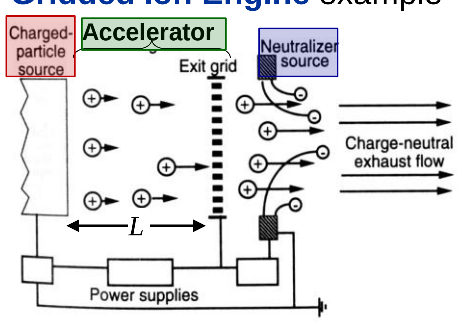
Electrostatic Thrusters

- Thrust provided by “static” electric field in the direction of the acceleration
- Propellant is often an ionized gas
  - so often denoted **ion thruster**
- Typically operate at low pressure (near vacuum)
- Handful of different technologies
  - **gridded ion thruster** or ion engine
    - original development at NASA, 1950’s-60’s
  - **Hall** effect **thruster** or ungridded ion thruster
    - original development in Russia (Stationary Plasma Thruster, SPT), 1950’s-60’s, higher thrust potential
  - electrospray (or colloidal) thruster
  - ….

Components

- **Ion sources**
  - usually **electron bombardment** plasma
  - **RF discharge**
  - **ion contact**: liquid metal (e.g., Cs) flowing through hot porous tungsten
  - **field emission**: charged droplets/ particles
- **Accelerator**
- **Neutralizer**
  - electrons added to make exhaust charge neutral
  - typically thermionic emitters or hollow cathodes

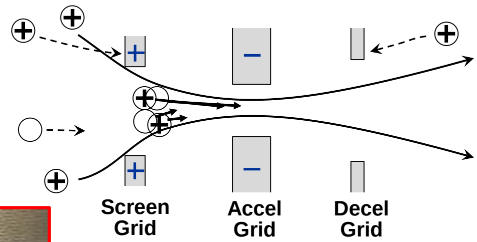
Gridded Ion Engine example



Space Propulsion Analysis and Design, Humble, Henry and Larson, 1995

Gridded Ion Thruster

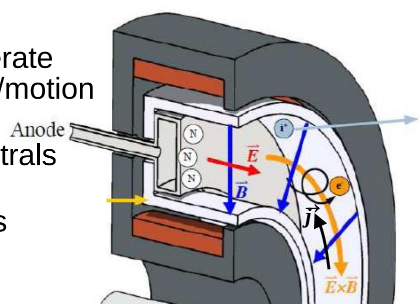
- **Acceleration** E field provided by electrodes
  - ions (and some neutrals) pass through many small cells
- Grid of multiple electrodes (forms ion optics)
  - 2-3 grids typical



- screen and decel grids help prevent high energy ions from impacting accel grid (sputtering)
- decel grid reduces sputtering from backflow of charge-exchange ions

Hall (Effect) Thruster

- **Ionization**
  - axial **E** and radial **B** generate azimuthal e^- acceleration/motion (Hall current)
  - high energy e^- ionize neutrals
  - heavy ions have larger radius of gyration, so less deflection by **B**
    - pick weak enough **B**
- **Acceleration**: two interpretations
  - electrons largely trapped by **B,** so negative plasma potential near exit accelerates ions
  - Hall effect from electron current, induced $\boldsymbol{E} \propto \boldsymbol{j}\times\boldsymbol{B}$



C. Mullins, *Non-invasive Hall Current Distribution Measurement System for Hall Effect Thrusters* (2015)

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Electrostatic Thruster Performance - u_e

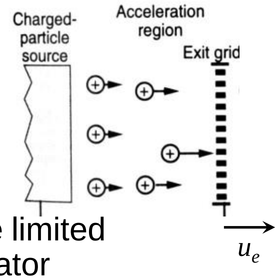
- To find specific impulse

  negligible pressure effect $I_{sp,ideal} = u_{e,ideal} / g_e$

- Find exhaust velocity from energy balance ***per particle with mass m***

  $$\overset{KE}{mu^2/2} = \overset{W \text{ by } E_{field}}{q\Delta V}$$

- So maximum (ideal) specific impulse limited by voltage difference across accelerator



(IV.42a) $$u_{e,ideal} = \sqrt{2\frac{q}{m}\left(V_{exit} - V_{inlet}\right)}$$

$1AMU = 1.6605\times10^{-27}\,kg$

for singly ionized $q = 1.6022\times10^{-19}\,C$

(IV.42b) $$u_{e,ideal} = 13{,}890\sqrt{\frac{\Delta V_{accel}(\text{volts})}{MW}}\ \frac{m}{s}$$

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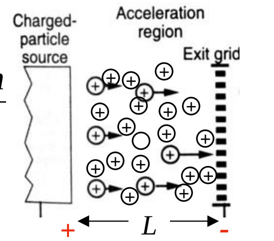
Electrostatic Thruster Performance - τ

- Mass flow rate related to current

  $j = nqu \qquad n = \rho/m \qquad \Rightarrow \dot{m}/A = j\dfrac{m}{q}$



accelerator electrode spacing

- So $\tau_{ideal} = \dot{m}u_{e,ideal} = j\,A\left(m/q\right)u_{e,ideal}$ **(IV.43)**
  - and maximum thrust limited by achievable current density
- For **gridded** ion engine, ion current limited by space-charge ***Child-Langmuir Law***
  - field from dense ions creates "shield" from applied E field

(IV.44a) $$j_{\max} = \frac{4\varepsilon_o}{9}\sqrt{\frac{2q}{m}}\frac{\Delta V_{accel}^{3/2}}{L^2}$$

permittivity of free space $\varepsilon_o = 8.854\times10^{-12}\,\dfrac{Farad}{meter}$

(IV.44b) $$j_{\max} = 5.467\times10^{-8}\frac{\Delta V_{accel}^{3/2}(\text{volts})}{\sqrt{MW}\,L^2(\text{m})}\ \frac{Amps}{m^2}$$

for singly ionized

Gridded Ion Thruster Performance - τ

- With space-charge limitation, maximum thrust

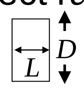
IV.42-44⇒ $\tau_{\max,ideal} = \frac{4\varepsilon_o}{9}\sqrt{\frac{2q}{m}}\frac{\Delta V_{accel}^{3/2}}{L^2} A \frac{m}{q}\sqrt{\frac{2q}{m}}\Delta V_{accel}$

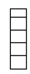
$$\boxed{\tau_{\max,ideal} = (8\varepsilon_o/9) A \Delta V_{accel}^2 / L^2}$$ **(IV.45a)**

for circular cross-section of diameter, D

$$\boxed{\begin{aligned}\tau_{\max,ideal} &= (2\pi/9)\varepsilon_o (D/L)^2 \Delta V_{accel}^2 \\ &= 6.18\times10^{-12} (D/L)^2 \Delta V_{accel}^2\end{aligned}}$$ **(IV.45b)** in *Newtons*

- High τ requires high ΔV and aspect ratio
  - space charge⇒$(D/L)_{max}$ ~1
  - use many small ion beams to get more thrust





Electrostatic Thruster - Propellant

- Thrust performance

from IV.43 $\frac{\tau}{A} = j\frac{m}{q}u_e = j\frac{m}{q}I_{sp}g_e$

- For fixed I_{sp}

$\frac{\tau}{A} \propto j\frac{m}{q}$

for gridded ion engine

$j_{\max} \propto \sqrt{\frac{q}{m}} \Rightarrow \frac{\tau_{\max}}{A} \propto \sqrt{\frac{m}{q}}$

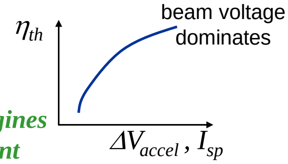
- **So choose propellant with high m/q**
  - heavy molecules
    - xenon (Xe) good choice (MW=131.3) and noble gas, so easy to store
    - Cs, Hg heavier, but storage issues
  - singly ionized ions preferable
  - also macro particles (colloidal thrusters)

Electrostatic Thruster Power

- **Jet power** from IV.36 $P_j = \frac{1}{2}\dot{m}_b u_e^2$ ← *Ion beam flowrate @ exit*
  - **propellant utilization efficiency** $\boxed{\eta_u \equiv \dot{m}_b / \dot{m}}$ **(IV.46)** *some propellant does not stay ionized*
  - **thrust correction** $\boxed{\gamma \equiv u_e / u_{e,ideal}}$ **(IV.47)** $= \dfrac{\tau / \dot{m}_b}{\tau_{ideal} / \dot{m}_b}$
    to account for beam divergence, multiple ionization, sputtering
- **Accelerator electrical power**
  $P_{elec} \overset{ideally}{=} P_{jet}$ $\boxed{P_{elec} = J\Delta V_{accel}}$ **(IV.48)**
  Recall $J = jA = (\dot{m}/m)q$
- **Power supplied to thruster**
  - other power requirements besides accelerator
    - ionization (make ions) $\boxed{P_{ion} = J\Delta V_{ion}}$ **(IV.49)**
    - neutralization (make e$^-$) $\boxed{P_{neut} = J\Delta V_{neut}}$ **(IV.50)**

Nonideal Performance and Typical Values

- Accounting for all these non-ideal effects
  $\boxed{I_{sp} = \gamma\eta_u I_{sp,ideal}}$ **(IV.51)** $\boxed{\tau = \dot{m}_b u_e = \eta_u \dot{m}\gamma I_{sp,ideal}\, g_e}$ **(IV.52)**
- **Thruster efficiency**
  (IV.53) $\boxed{\eta_{th} \equiv \dfrac{P_j}{P_{th}}}$
  $\boxed{P_{th} = J\Delta V_{th} = J(\Delta V_{ion} + \Delta V_{accel} + \Delta V_{neut})}$ **(IV.54)**
  $\boxed{\eta_{th} = \dfrac{\eta_u \gamma^2}{1 + (\Delta V_{ion} + \Delta V_{neut})/\Delta V_{accel}}}$ **(IV.55)**
- Typical values
  - ΔV_{ion} up to 100-300V (eV/ion)
  - ΔV_{neut} 10-20V
  - γ 0.8-0.95
  - η_u 0.8-0.95



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Electron Bombardment Xe+ Engine Example

- **Given:** Operating conditions
  - ΔV_{accel}=700 V, L=2.5 mm, 2200 holes (grids) each with D=2.0 mm, ΔV_{neut}=10V
  - MW(Xe)=131.3
    ΔV_{ion}=12.08V (min. = ionization potential)
- **Find:**
  - maximum τ, u_e and I_{sp}
  - $\dot{m}$
  - minimum thruster power required
- **Assume:**
  - ideal operation (to get maximum values and minimum power)

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Solution

using η_u=1, γ=1

- $\tau_{\max} = \tau_{\max,ideal} = 6.18\times10^{-12}(D/L)^2 \Delta V_{accel}^2$

  from IV.45b

  $= 6.18\times10^{-12}(2/2.5)^2 700^2 = 1.94\times10^{-6}\, N/grid$

- $\tau_{\max,total} = 2200\, grids(1.94\times10^{-6}\, N/grid) = \boxed{4.3mN}$ *device up to at least ~200mN*

- $u_e = u_{e,ideal} = 13{,}890\sqrt{\Delta V_{accel}/MW}\; m/s = 13{,}890\sqrt{700/131.3}\; m/s$

  from IV.42b

  $u_e = \boxed{32{,}070\, m/s} \Rightarrow \boxed{I_{sp} = 3270\, s}$

- $\dot{m} = \dot{m}_b = \tau/u_e = \boxed{1.34\times10^{-4}\, g/s}$

  ion mass

  $m = 1.66\times10^{-27} MW\;\; kg$
  $= 2.18\times10^{-25} kg$

- $P_{th} = P_{jet} + P_{ion} + P_{neut} = \dot{m}u_e^2/2 + (\Delta V_{ion} + \Delta V_{neut})q\,\dot{m}/m$

  from IV.36,43,49,50

  $= 68.9W + 2.17W$

  $q = 1.602\times10^{-19} C$ ***for singly ionized***

  $P = \boxed{71.1W}$

  maximum η_{th}=68.9/71.1 = 96.9%

kW engines flown

using η_u=0.9, γ=0.9, ΔV_{ion}=50V ⇒ η_{th}=67% ***more typical***

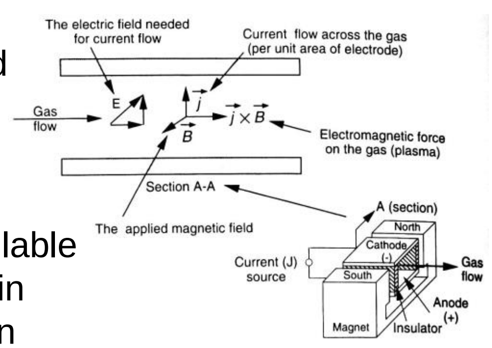
from IV.55

Electric Propulsion for Rockets

Electromagnetic Devices

Electromagnetic Propulsion Systems

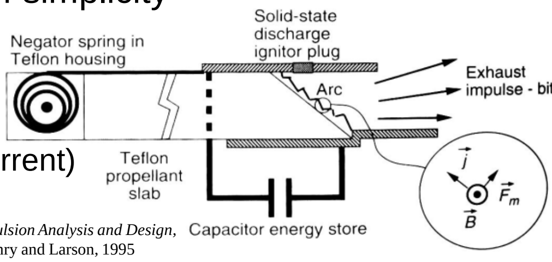
- Use applied or induced magnetic fields to produce acceleration of propellant
  - high currents/ powers required to produce significant induced fields
  - high power available only (normally) in pulsed operation



Space Propulsion Analysis and Design, Humble, Henry and Larson, 1995

Pulsed Plasma Thruster

- Propellant produced by vaporizing solid material with discharge
- B field induced by discharge also acts to accelerate vaporized propellant
- Advantage of simplicity
- Acceleration force $\sim j^2$ (discharge current)

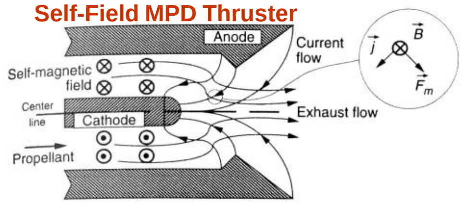


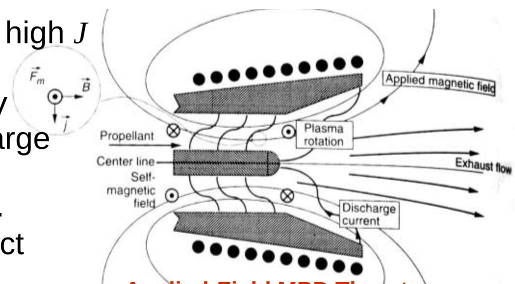
Space Propulsion Analysis and Design, Humble, Henry and Larson, 1995

Magnetoplasmadynamic (MPD) Thrusters

- Resemble arcjets
- Lower flow densities to attain higher exhaust velocity
- Diffuse discharge, low erosion
- Self field requires high J
- Applied B field
  - allows higher V at lower discharge currents
  - increase accel.
  - larger Hall effect

Self-Field MPD Thruster





Applied-Field MPD Thruster

Space Propulsion Analysis and Design, Humble, Henry and Larson, 1995

MPD Thrusters (con't)

- Most efforts focused on applications with exhaust velocities (Isp) greater than arc jets
- Typically require higher powers than currently available on in-space vehicles
- Exhaust speed $u_e \propto j^2 / \dot{m}$
  - limited by erosion and oscillations at high j