Translational Nonequilibrium and Radiation

• Homework solutions should be neat and logically presented, see format requirements (seitzman.gatech.edu/classes/ae6050/homeworkformat.html). Please note the requirement to draw some **implications** from the results of each problem. This could be implications for a practical device, a comparison of the results of different parts of a problem, a physical interpretation for an equation, etc.

• To receive credit, show all work in the format described above. If you use equations from the notes, the class textbook or another book, please cite the reference.

1. Translational Nonequilibrium Estimation

As we have seen previously, one way to estimate whether an energy mode will be able to maintain equilibrium is to compare its collisional relaxation time to a flow time, e.g., by evaluating a parameter $\beta \equiv \tau_{coll}/\tau_{flow}$; when β is sufficiently small we can assume the energy mode is able to maintain equilibrium.

This also holds for translational nonequilibrium, where we can estimate $\tau_{coll} = 1/\Theta$, where **Θ is the elastic collision frequency**. Similarly, we can estimate a flow time as $\tau_{flow} = L/c_{ref}$, where L is a characteristic flow length (e.g., a boundary or shear layer thickness), and c_{ref} is a characteristic velocity. For subsonic flows, c_{ref} is typically chosen to be the most probable random speed; for supersonic flows, c_{ref} is chosen to be the flow velocity.

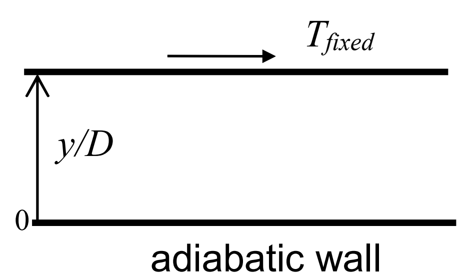
For the following, consider a flow of nitrogen (N_2) at 270 K and 0.10 bar.

a) Estimate the elastic collision frequency Θ using a hard sphere approximation of the total elastic collision cross-section, $\sigma^T=\pi d^2$, where d is the hard-sphere diameter of the molecule. A typical approach is to use the diameter parameter from a Lennard-Jones 12-6 potential.

b) Estimate values of β for subsonic nitrogen flows for two characteristic lengths: 1 cm and 100 μm.

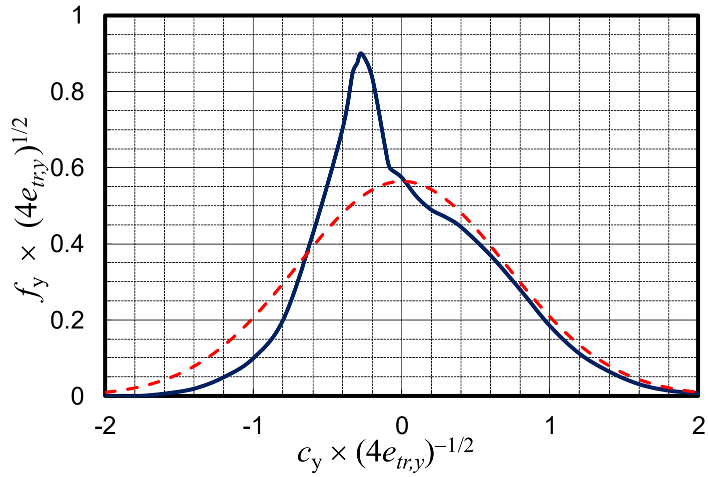
c) Estimate values of β for two supersonic nitrogen flows: M=3 with L=100 μm and M=10 with L=10 μm.

2. Couette Flow - Velocity Distributions



Consider a steady, planar Couette flow between two, infinitely long, parallel plates separated by a distance D. The top plate is moved at a **high** velocity, while the bottom plate is stationary. In addition, the top plate is maintained at a fixed temperature while the bottom plate is adiabatic. The vertical coordinate (y) is defined to be zero at the bottom wall and increasing upward as indicated in the schematic.

We can define separate distribution functions for each molecular velocity component, i.e., $f(c_i) = f_x(c_x)\, f_y(c_y)\, f_z(c_z)$. The graph[1] (solid curve) shows the normalized molecular velocity distribution function for the y-direction at y/D=0.5, where $e_{tr,y}$ is the mass-specific translational energy associated with just the y velocities of the molecules. For comparison, the dashed curve shows a normalized single-component Maxwellian (random velocity) distribution function.



a) Would you describe this flow as being close to translational equilibrium at y/D=0.5? Justify your answer.

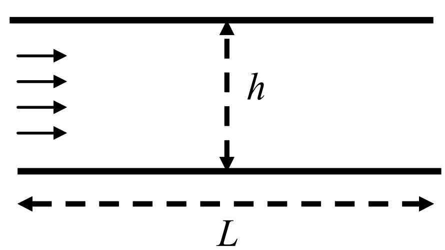
b) Based on $f_y(c_y)$, is there a non-zero heat flux in the y-direction (q_y) at y/D=0.5? If so, in which direction is energy being transported? Justify your answers.

c) Based on this data for $f_y(c_y)$, is there a non-zero shear stress τ_{xy} at y/D=0.5 (where x is the horizontal direction in the flow schematic)? Justify your answer.

d) On a single graph, draw qualitatively correct plots of the (un-normalized) $f_y(c_y)$ distribution functions at two locations, y/D=0.5 and y/D=0 (so just above the bottom plate). Include a justification.

3. Equation of Radiative Transfer

Consider a non-scattering, uniform, equilibrium gas flowing through a square duct of height h = 30. cm and length L = 3.0 m. The gas temperature is 1800 K, the pressure is 3.0 kPa and the molecular weight is 16. The absorption coefficient of this gas at a wavelength of 2.25 μm and at these conditions is 0.0004 cm^{-1}. Assume the walls of the duct are completely transparent and non-emitting.



a) What is the optical depth (or optical thickness) for radiation at 2.25 μm traversing in the vertical direction from the top to the bottom of the duct?

b) Estimate the spectral intensity (at 2.25 μm) of the radiation passing through the bottom wall of the duct that is also traveling normal to the bottom wall.

c) What is the radiant emitted spectral power emitted at 2.25 μm from the gas in the duct?

[1]The data is based on a computational solution of the Boltzmann equation for this flow using DSMC, assuming a hard-sphere molecular model.

EXTRA CREDIT: **Radiative Properties of Carbon Monoxide Transition**

Behind a Mach 8 normal shock passing through carbon dioxide, the gas has a temperature of 1920 K, a pressure of 3.8 atm and contains 0.59% (mole fraction) of CO.

a) What wavelength would correspond to a CO transition having an upper energy level with a vibrational quantum number of 1 and a rotational quantum number of 14, and a lower energy level with a vibrational quantum number of 0 and a rotational quantum number of 15? Base your answer on the information about the molecular constants for CO presented in the class review lecture https://seitzman.gatech.edu/classes/ae6050/StatisticalMechanicsReview.pdf.

b) Does this transition fall in the ultraviolet, visible, infrared, or microwave region of the electromagnetic spectrum?

c) If this transition has an Einstein coefficient for spontaneous emission of 12 s^{-1} and a peak line shape value $\varphi=9\times10^{-11}$ Hz^{-1}, calculate the absorption coefficient a_ν at the peak of this CO absorption transition. Be sure to include units in your answer.