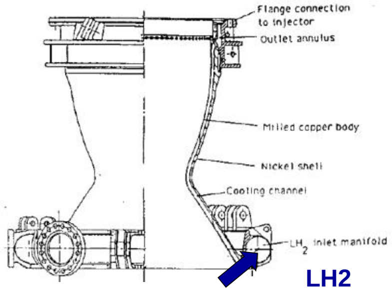
LRE TCA

Heat Transfer (Cooling)

Combustion Chamber Regenerative Cooling

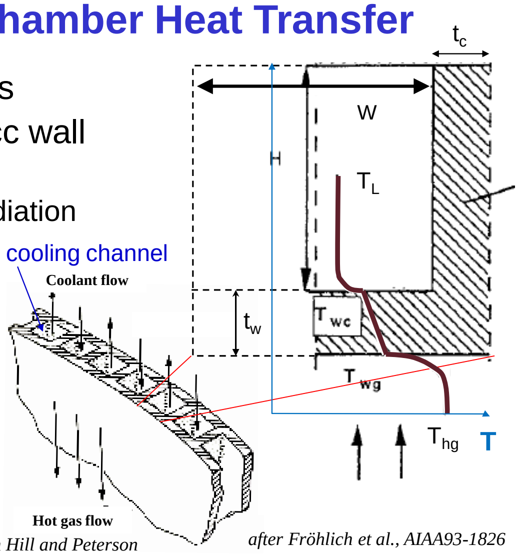


From Fröhlich et al., AIAA93-1826

- Vulcain (LH2/LOX) example
  - liquid H_2 used to cool walls
  - flowed through closely arranged small tubular cooling channels within combustion chamber wall

Combustion Chamber Heat Transfer

- Heat transfer occurs
  - from hot gas to cc wall
    - $T_{hg} \rightarrow T_{wg}$
    - convection + radiation
  - through cc wall
    - $T_{wg} \rightarrow T_{wc}$
    - conduction
  - from cc wall to low T coolant
    - $T_{wc} \rightarrow T_L$
    - convection

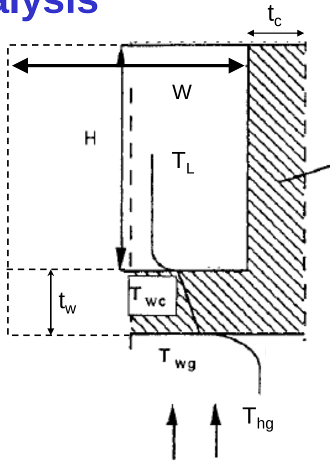


from Hill and Peterson

after Fröhlich et al., AIAA93-1826

Heat Transfer Analysis

- Convection + wall conduction ≡ **conjugate heat transfer**
- Advanced analysis methods, e.g., CFD and FEA
- Simplifying assumptions for 1st order analysis
  - 1-d
    - large H/t_w
    - $H/W > 1$
  - steady
    - "long" thrust time



Simplified Heat Transfer Analysis

- 1-d, **steady** analysis, no backside (rad.) cooling

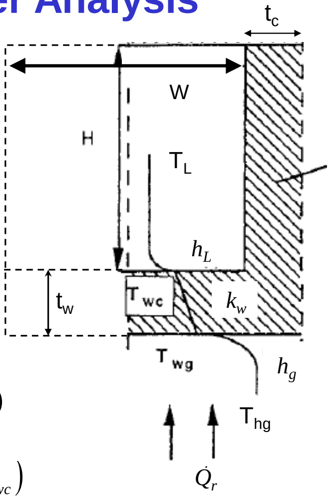
$$\dot{Q} = A_{wg}\left[h_g\left(T_{hg}-T_{wg}\right)+\dot{q}_r\right] = -k_w A_{wg}\frac{dT_w}{dx} = h_L A_{wc}\left(T_{wc}-T_L\right)$$

  - $h \equiv$ convective heat transfer coefficient
  - $k_w \equiv$ thermal conductivity of wall
- If side walls "thin", $t_c << W$

$$\dot{q} = h_g\left(T_{hg}-T_{wg}\right)+\dot{q}_r = -k_w\frac{dT_w}{dx} = h_L\left(T_{wc}-T_L\right)$$

  - $\dot{q} \equiv$ heat transfer flux (per unit area)
- For uniform (composition) walls and 1-d

$$-k_w\frac{dT_w}{dx} \cong k_w\frac{\left(T_{wg}-T_{wc}\right)}{t_w}$$



Simplified Heat Transfer Analysis

- Solution
  - heat flux from combustor

$$\dot{q} = \frac{T_{hg}-T_L+\dot{q}_r/h_g}{1/h_g + t_w/k_w + 1/h_L}$$

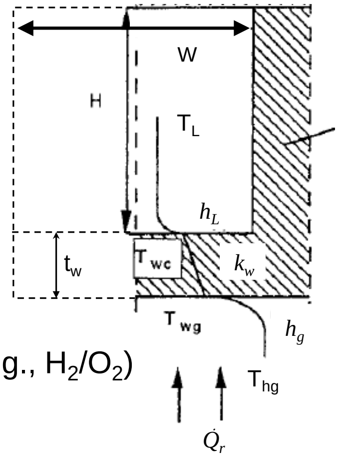
  - hot-side wall temperature

$$T_{wg} = T_{hg} - \left(\dot{q}-\dot{q}_r\right)/h_g$$

  - if radiation to wall is small
    - no particles (non-sooting, e.g., H_2/O_2)

$$T_{wg} = T_{hg} - \frac{T_{hg}-T_L}{1+t_w h_g/k_w + h_g/h_L}$$

Want this term to be large – cooler wall



Reducing Hot Wall Temperature

$$\frac{T_{hg}-T_L}{1+t_w h_g/k_w+h_g/h_L}$$

- Low temp. coolant (T_L)
    - in combustion chamber, $T_{hg} >> T_L$ (e.g., 3000K vs 100-200K), so small changes in T_L have minimal effect
- High conductivity wall material (k_w)

Material	k (Wm^{-1}K^{-1}) @ 1000°C	σ_y (MPa)
Nickel Super Alloy	20-30	>500 @ 600°C
Copper	350	70 @ 20°C

- Thin walls (t_w)
    - structural limits

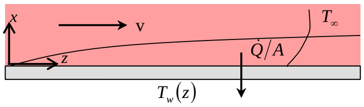
$$t_w \geq p_{o,cc}D_{cc}/2\sigma_{\max}$$

$\sigma_{\max}$ = maximum allowed wall stress ($<\sigma_y$)

- Low ratio h_g/h_L
    - reduce to hot wall, increase from cold wall

Convective Heat Transfer Coefficients

- Convective heat transfer
    - due to fluid moving over surface
    - thermal boundary layer develops, like momentum boundary layer



$$\tau_{shear}=\tau_{shear}(Re_z)$$

Reynolds number

- Heat transfer coefficient $h=h(Re_z, Pr)$

Prandtl number

$$Pr \equiv \nu/\alpha$$

Thermal diffusivity

$$\alpha \equiv k/\rho c_p$$

- so T_w varies downstream
- e.g., for laminar flow over flat plate

Stanton Number **Nusselt Number**

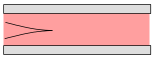
$$St=\frac{h}{\rho_\infty u_\infty c_{p_\infty}}=0.332Re_z^{-1/2}Pr^{-2/3}=\frac{Nu}{RePr}\Rightarrow Nu_z=\frac{hz}{k}=0.332Re_z^{1/2}Pr^{1/3}$$

Heat Transfer Coefficients: Coolant Side

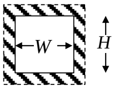
- For coolant side, can model as channel flow

except maybe near coolant entrance location

  - typically fully-developed turbulent flow



$$h_L(z) = C\frac{k_L}{D_H}\overbrace{\left(\frac{\overline{\rho_L u_L} D_H}{\mu_L}\right)^{0.8}}^{Re_D}\overbrace{\left(\frac{\mu_L c_{pL}}{k_L}\right)^{n}}^{Pr} \quad \begin{cases} C = 0.018,\ n = 0.37 \\ C = 0.023,\ n = 0.33 \end{cases}$$



  - D_H = hydraulic diam= 4×x-sect area/perimeter $= 2HW/(H + W)$
  - evaluate fluid properties (k, μ, c_p) at bulk average liquid temperature at location z

though can add correction for prop. variations through b.l

Recall want high h_L $\Rightarrow$ coolant with high k_L and c_{pL} ← *ability to increase energy without large T change*

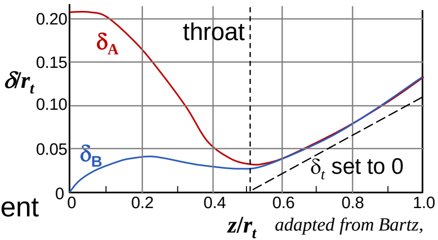
but usually limited to (one) propellant

  - Note scaling

$$\overline{\rho_L u_L} = \dot{m}_L / A \Rightarrow h_L(z) \propto \dot{m}_L^{0.8} / \underset{\substack{\downarrow \\ \text{actually } HW\big/\left(\frac{H+W}{2}\right)^{0.2}}}{D_H^{1.8}}$$

Heat Transfer Coefficients: Hot Side

- For hot side, boundary layer (b.l.) thickness (δ) not as simple to model
  - flow not fully-developed ($\delta << r$ at all z)
  - geometry complex
  - area is changing, and flow is accelerating
- Results from approx. calc., 15° conical nozzle with different inflow assumptions
  - δ minimum near throat (δ_t)
  - initial δ has little impact on δ_t, due to accelerating flow in converging section tending to thin b.l.
    - same holds for nozzle convergence angle
  - b.l. growth rate in diverging section weak function of δ_t



adapted from Bartz, Trans. ASME 77 (1955)

Heat Transfer Coefficients: Hot Side

- Correlations (semi-empirical) available, e.g. Bartz

D.R. Bartz in *Advances in Heat Transfer, Vol. 2,* Hartnett and Irvine Ed. (1965)

$$\frac{h_g D}{k_g} \propto Re^{0.8} Pr^{0.4} \quad \Rightarrow h_g(z) = \frac{0.026}{D^{0.2}} \left(\frac{D_t}{R_c}\right)^{0.1} \left(\frac{\dot m}{A}\right)^{0.8} k_g^{0.6} \left(\frac{c_{p_g}}{\mu_g}\right)^{0.4} \sigma$$

$R_c =$ throat radius of curvature

  - evaluate fluid prop's. (k_g, c_{pg}, μ_g) at freestream T_o
  - σ accounts for temperature dependence of properties across boundary layer

cpg

$$\sigma = \left(\frac{\rho(\bar T)}{\rho(T_{hg})}\right)^{0.8} \left(\frac{\mu(\bar T)}{\mu(T_o)}\right)^{0.2} \cong \left[\frac{1}{2}\frac{T_{wg}}{T_{hg}}\left(1+\frac{\gamma-1}{2}M^2\right)+\frac{1}{2}\right]^{0.2n-0.8} \left(1+\frac{\gamma-1}{2}M^2\right)^{-0.2n}$$

$$\bar T \equiv \tfrac{1}{2}\left(T_{hg}+T_{wg}\right)$$

with $\mu(T) \propto T^n$

***n*~0.8 for diatomics**

Heat Transfer Coefficients: Hot Side

$$h_g(z) = \frac{0.026}{D^{0.2}} \left(\frac{D_t}{R_c}\right)^{0.1} \left(\frac{\dot m}{A}\right)^{0.8} k_g^{0.6} \left(\frac{c_{p_g}}{\mu_g}\right)^{0.4} \sigma$$

- Many of the terms in this equation are constants
  - – only a few vary with axial position
  - – can scale local area to throat area

[ ] constant (post-burning)

$$h_g(z) = \left[\frac{0.026}{D_t^{0.2}} \left(\frac{D_t}{R_c}\right)^{0.1} \left(\frac{\dot m}{A_t}\right)^{0.8} k_g^{0.6} \left(\frac{c_{p_g}}{\mu_g}\right)^{0.4}\right] \left(\frac{A_t}{A(z)}\right)^{0.9} \sigma(z)$$

h_g high at min. area, i.e., throat

– or using characteristic velocity $c^* \equiv p_o A_t / \dot m$

$$h_g(z) = \left[\frac{0.026}{D_t^{0.2}} \left(\frac{D_t}{R_c}\right)^{0.1} \left(\frac{p_o}{c^*}\right)^{0.8} k_g^{0.6} \left(\frac{c_{p_g}}{\mu_g}\right)^{0.4}\right] \left(\frac{1}{\varepsilon(z)}\right)^{0.9} \sigma(z)$$

Solution at Given Axial Location

- Before we found (for our assumptions)

$$\dot{q} = \frac{T_{hg} - T_L}{1/h_g + t_w/k_w + 1/h_L} \qquad T_{wg} = T_{hg} - \frac{T_{hg} - T_L}{1 + t_w h_g/k_w + h_g/h_L}$$

  - but h_g (and possibly h_L) functions of wall temperature, e.g., $\sigma(z) = \sigma(T_{hg,} T_{wg}, T_o)$
  - can't write simple equation for T_{wg}, so generally iterative solution
    - for example, guess $T_{wg} \Rightarrow h_g \Rightarrow \dot{q} = h_g(T_{hg} - T_{wg})$
    - iterate until heat fluxes agree

$$T_{wc} = T_{wg} - \frac{t_w}{k_w}\dot{q}$$

$$h_L(T_{wc} - T_L) = \dot{q}$$

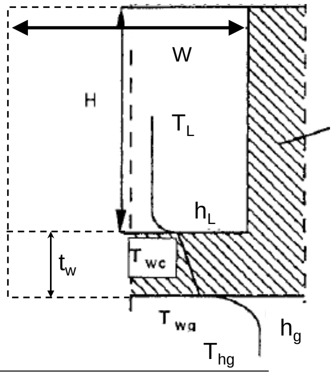
check

General Trends

$$\dot{q} = \frac{T_{hg} - T_L + \dot{q}_r/h_g}{1/h_g + t_w/k_w + 1/h_L}$$

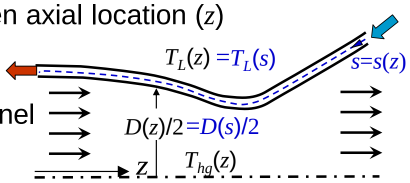
$$T_{wg} = T_{hg} - \frac{T_{hg} - T_L}{1 + t_w h_g/k_w + h_g/h_L}$$

- Noted previously, combustor wall typically made of thin material with high thermal conductivity
  - $T_{wg} \approx T_{wc}$
- Often $h_L/h_g >> 10^2$ $\qquad h_L \propto \dot{m}_L / D_H^{1.8}$
- Want high h_L to increase reduce T_w
  - high flowrate, small coolant passage + fins, wall roughness
  - tends to lower T_w without large change in Q rate (lower T_w-T_L)
- Want low h_g to decrease Q to wall, reduce T_w
  - smooth inner combustor, keep boundary layer thick
  - primary influence on Q



Axial Heat Transfer Solution

- Solved for heat transfer at given axial location (z)
- Need to find how conditions vary along z
- Calculate for one cooling channel
- First-order approach, combine previous results with
  - gas energy eqn. (1-d) neglecting heat loss
  - coolant energy and mom. eqns. (1-d)
    - with B.C. $T_L(s{=}0)$ known
  - numerical solution, e.g., finite differencing, iterative
  - if include heat loss impact on nozzle flow



$T_{hg}(z)$ from $h(A(z)/A_t)$

$W \cong D(z)/\#\,\text{channels}$

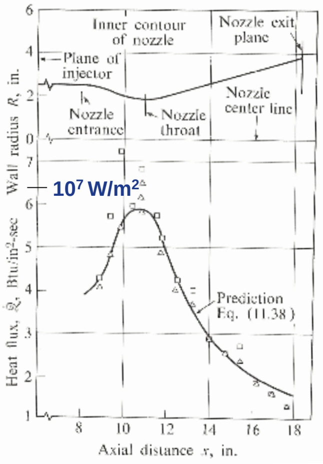
$$\dot{m}_L c_L(s)\,dT_L = \dot{q}(s)W(s)\,ds$$

$$dp_L(s) = -\frac{f_{Darcy}(s)}{2D_H(s)\rho_L(s)}\left(\frac{\dot{m}_L}{A_{chan}(s)}\right)^2 ds$$

$$dh_o(z) = \frac{\dot{q}(z)}{\dot{m}}\pi D(z)\,dz$$

Example: N_2O_4/N_2H_4 Rocket

- Correlation predictions vs. measured heat fluxes to wall
- Highest heat flux near throat
  - and largest prediction error
  - p_o dependence not completely captured?



Test conditions

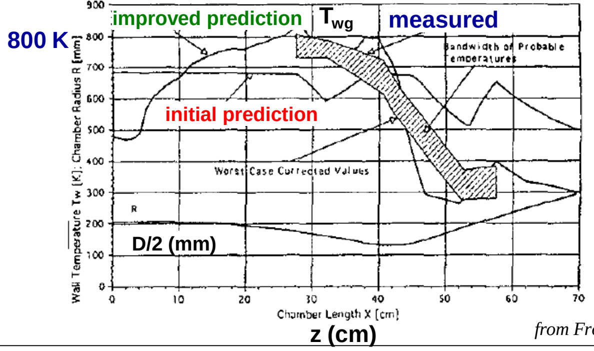
	p_0, psia	C^*, ft/sec	T_{wl}, °R
△	200	5421	860
□	199	5374	860

from Hill and Peterson

Vulcain Example

- Comparison of measured, predicted (hot) wall T's
  - improved prediction = turbulent (production) corrections for hot side

$$h_g(z)=\left[\frac{0.026}{D_t^{0.2}}\left(\frac{D_t}{R_c}\right)^{0.1}\left(\frac{\dot{m}}{A_t}\right)^{0.8}k_g^{0.6}\left(\frac{c_{p_g}}{\mu_g}\right)^{0.4}\right]\left(\frac{A_t}{A}\right)^{0.9}\sigma\;F_g(z)$$

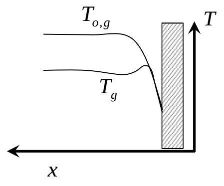


from Fröhlich et al., AIAA93-1826

Compressibility Effects on h_g

- For high Mach numbers (e.g., $M > 0.7$-0.8), static temperature of gas increases in velocity boundary layer as gas slows down
  - heat conduction through gas driven by static temperature gradient
  - so can impact heat transfer
  - can include in h

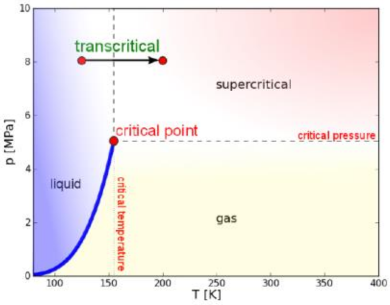
$$\dot{q}_{conv}=h_g\left(T_{hg}-T_{wg}\right)$$



- Shocks in nozzle can also disrupt boundary layer, impacting heat transfer to wall

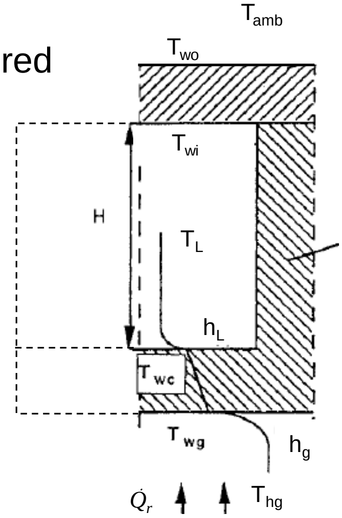
Overheating Coolants

- What happens of coolant gets “too hot”?
  - liquid can turn into gas
  - can “crack” multi-component liquids (RP-1)
  - potential to produce solid residues, “coking” (RP-1, CH_4)
  - autoignite (monopropellant like N_2H_4)
- If coolant below its critical point, can lead to boiling
  - nucleate boiling: gas bubbles form near wall, collapse as they reach cooler liquid
  - phase change increases heat transfer
  - but too much and gas film stays near surface, drastically reduces heat transfer
- If above critical point – no boiling
  - H_2 (p_c=13bar, T_c=33K)



External Wall Heat Transfer

- In previous analysis, we ignored back wall of coolant channel
  - heat conduction through wall
  - heat transfer to/from ambient
    - convection (in atmos.)
    - radiation (in space)
  - typically small effect
    - $T_{hg} - T_L >> T_L - T_{amb}$



Film Cooling

- Relatively cool fluid used to protect walls/surfaces from high temperatures
  - increase thermal boundary thickness
- To enhance regenerative cooling or other methods
  - can be used alone, but requires significant mass (propellant) $\Rightarrow I_{sp}$ loss
- Examples
  - introduce coolant (fuel?) as low velocity wall tangent jets through many small orifices around hot wall
  - introduce excess fuel through injectors located around periphery of injector plate
  - introduce “cold” turbine exhaust gases (e.g., gas generator) for downstream nozzle locations without regenerative cooling

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Low Thrust Rockets and Thrusters: Radiation Cooling

- Poor coolants, simplicity and heat loads small enough
  - radiation cooling may be viable
    - in-space, low thrust (thrusters)

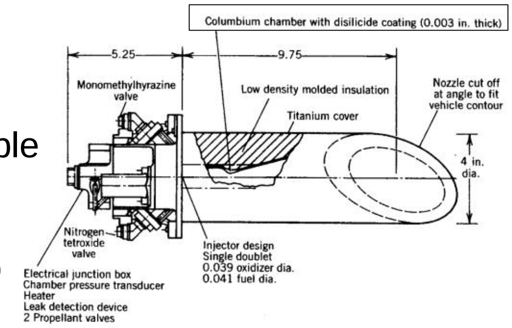


FIGURE 8–15. This radiation-cooled, insulated vernier thruster is one of several used on the Reaction Control System of the Space Shuttle vehicle for orbit stabilization and orientation, rendezvous or docking maneuvers, station keeping, deorbit, or entry. The

From Sutton

Radiation Cooled TCAs

- Radiation heat transfer from hot surface

$$\dot{q}_{rad} = \varepsilon\sigma\left(T^4 - T_\infty^4\right)$$

  - $\varepsilon \equiv$ emissivity (≤ 1)
  - $\sigma \equiv$ Stefan-Boltzmann constant

$\approx 5.670\times10^{-8}\ W\,m^{-2}\,K^{-4} \approx 1.191\times10^{-8}\ lb_f\,s^{-1}\,ft^{-1}\,R^{-4}$

- Example for $\varepsilon=1$, $T_\infty=0$

T (K)	500	1000	1500	2000
Q_{rad} (W/m^2)	3.5×10^3	5.7×10^4	2.9×10^5	9.1×10^5

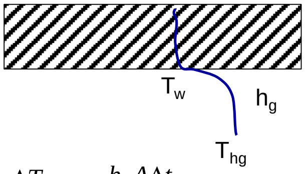
  - compare to conv. flux in N_2O_4/N_2H_4 rocket example: 10^6-10^7 W/m^2

High T Materials for Radiation Cooling

- Since radiation rates low except at high temperature, need for high T materials (>2000 K)
- Examples for in-space thrusters with oxidizer resistant coating on high temperature substrate
  - iridium/rhenium
  - oxide-iridium/rhenium
  - iridium/rhenium-carbon/carbon
  - oxide-iridium/rhenium-carbon/carbon
- Also high temperature ceramic matrix composites

Heat Sink Operation

- For pulsed operation, TCA walls never reach steady-state
  - can use thermal mass of wall to prevent reaching unacceptable temperatures
  - if assume high conductivity



$$\dot{Q} = h_g A\left(T_{hg} - T_w\right) = m_w c_w \frac{dT_w}{dt} \quad \Rightarrow \frac{\Delta T_w}{\left(T_{hg} - T_w\right)} = \frac{h_g A \Delta t}{m_w c_w}$$

- Low ΔT_w requires
  - sufficiently short thrust pulses
  - sufficient wall material (high thickness)
  - high heat capacity