Georgia Tech College of Engineering

Rocket Propulsion Basics

Vehicle Acceleration/Rocket Equation

Thrust Coeff. & Characteristic Vel.

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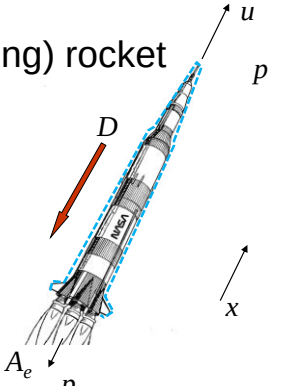
Thrust and Vehicle Acceleration

- Now look at moving (accelerating) rocket

$$\vec{F}_{\substack{solid\ body\ cross\\ CV\ on\ fluid}} + \int_{CS} \vec{\sigma}_{shear}\, dA - \int_{CS} p\hat{n} dA + \int_{CV} \rho \vec{f} dV = \frac{d}{dt}\int_{CV} \rho \vec{u} dV + \int_{CS} \rho \vec{u}\left(\vec{u}_{rel}\cdot\hat{n}\right) dA$$



drawn for u_e and rocket velocity (u) aligned

- Along x direction (rocket axis)

$$F_{\substack{x,solid\ cross\\ CV\ on\ fluid}} = 0$$ *no solid body crossing CV*

$$\int_{CS} \sigma_{x,shear}\, dA = -D_{viscous} - D_{wave} = -D$$ *Drag*

– other terms?

Includes effects of $p \neq p_a$ on solid surfaces

Thrust and Vehicle Acceleration

- Accelerating rocket

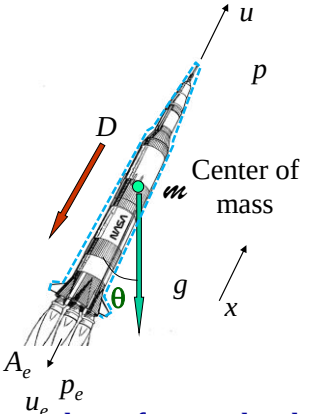
$$-\int_{CS} p\hat{n}dA = ?$$

Body force

$$\int_{CV} \rho f_x dV = ?$$

Unsteady term

$$\frac{d}{dt}\int_{CV} \rho \vec{u}_x dV = ?$$



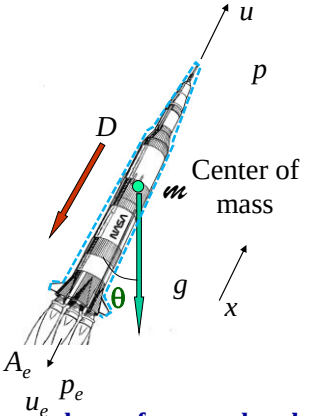
drawn for u_e and rocket velocity (u) aligned

Thrust and Vehicle Acceleration

- Accelerating rocket

Flux term

$$\int_{CS} \rho \vec{u}_x \left(\vec{u}_{rel} \cdot \hat{n}\right) dA = ?$$



drawn for u_e and rocket velocity (u) aligned

- Now combine all terms

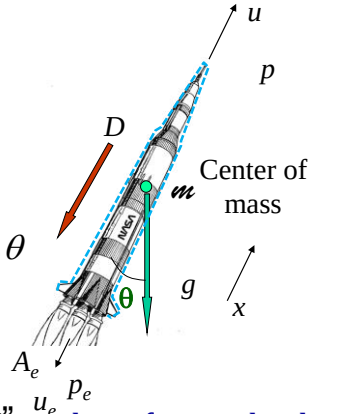
Thrust and Vehicle Acceleration

- Accelerating rocket

$$0 - D + (p_e - p_a)A_e - m\, g\cos\theta = -\dot{m}u + m\frac{du}{dt} + \dot{m}(u - u_e)$$

$$m\frac{du}{dt} = \dot{m}u_e + (p_e - p_a)A_e - D - m\, g\cos\theta$$

$$\boxed{m\frac{du}{dt} = \dot{m}u_{eq} - D - mg\cos\theta}$$



drawn for u_e and rocket velocity (u) aligned

- 1st term same as static "thrust"
- Other terms ??
  - thrust accel. "losses" (overcome drag, gravity, …)

Velocity Increment

- In orbital mechanics, important parameter for changing orbit in gravity field is the velocity increment
  - $\Delta u = u_{final} - u_{initial}$ (net incr. in vehicle speed)
  - e.g., launch to LEO requires ~9 km/s
- $\Delta u = \int_{\substack{accel\\period}} du = \int\left(\frac{\dot{m}u_{eq}}{m} - \frac{D}{m} - g\cos\theta\right)dt$ $\quad \dot{m} = -dm/dt$

$$= \int_{const.\ u_{eq}} u_{eq}\frac{-dm}{m} - \int\frac{D}{m}dt - \int(g\cos\theta)dt$$

$$\Delta u = u_{eq}\ln\frac{m_{initial}}{m_{final}} - \int\frac{D}{m}dt - \int(g\cos\theta)dt$$

Velocity Increment

$$\Delta u = u_{eq} \ln\left(\frac{m_{initial}}{m_{final}}\right) - \int \frac{D}{m}dt - \int (g\cos\theta)dt$$

$\equiv \mathcal{R}$, Mass Ratio

Rocket Equation

- 1st term $\equiv \Delta u_{thrust}$, Δu without losses
  - how to increase?
  -
- 2nd $\equiv \Delta u_{drag}$, Δu needed to overcome drag
  - $D = C_D \frac{1}{2} \rho u^2 A_{ref}$……how to decrease?
  -
- 3rd $\equiv \Delta u_{gravity}$, Δu needed to lift mass in gravity field
  - how to decrease?
  -

Thrust Coeff and Characteristic Velocity

- So important performance parameters: τ & Δu
  - depend on u_{eq} or I_{sp}, and $\dot{m}$
- For thermal rockets, typically parameterize these using
  - **Thrust coefficient**

    measure of ability of nozzle to convert thermal energy to KE

    $$c_\tau \equiv \frac{\tau}{p_o A_t}$$

    Operating (stagnation) pressure; Nozzle throat area
  - **Characteristic velocity**

    measure of thermal energy available; propellant property for chemical rockets

    $$c^* \equiv \frac{p_o A_t}{\dot{m}}$$

    vel. units

    $$c_\tau c^* = \frac{\tau}{\dot{m}} = u_{eq} = I_{sp} g_e$$
- Why A_t?