LRE Cooling and Solid Rocket Motors

- Homework solutions should be neat and logically presented, see format requirements at seitzman.gatech.edu/classes/ae6450/homeworkformat.html. Show ALL work in this format. If you use equations from the notes, class textbook or another book, please cite the reference.

- Remember the requirement to draw some IMPLICATIONS/CONCLUSIONS from the results of each problem.

- For these problems you may wish to use a numerical tool (spreadsheet, Matlab, etc.) to perform the analysis. If so, you still need to include in your write-up your analysis approach, the equations solved/used, etc. (this requirement cannot be met by inserting your code in the homework).

1. LRE Regenerative Cooling

In the previous assignment, you examined the design of a high thrust, methane-oxygen, staged-combustion rocket. The design called for using a portion of the liquid methane as a regenerative cooling fluid for the TCA. Now, we want to consider more details of the design of the regenerative cooling system.

It has been proposed to construct the combustion chamber walls from a high-strength copper-alloy with a thermal conductivity of 300 Wm⁻¹K⁻¹. Since the hot-side heat transfer coefficient is expected to be the highest at the nozzle throat, we will focus our analysis there.

For this analysis, we will make the cooling channels rectangular with a height-to-width ratio (H/W) of 1.2. Also, the nozzle is designed with throat diameter to local (near the throat) radius of curvature of 1.33. The Nusselt number for the cooling channels can be modeled as $\text{Nu} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.33}$ (which is one of the 2 sets of values given in the class notes), while the hot side heat transfer coefficient can be modeled by the expression provided from Bartz.

For this problem, use the following results from the overall rocket analysis.

### Operating and Nozzle Throat Conditions

<table>
<thead>
<tr>
<th>$\dot{m}_{gas}$ (kg/s)</th>
<th>$\dot{m}_{cool}$ (kg/s)</th>
<th>$T_o$ (K)</th>
<th>$p_o$ (bar)</th>
<th>$D_t$ (m)</th>
<th>$T_t$ (K)</th>
<th>$p_t$ (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>735</td>
<td>152</td>
<td>3660</td>
<td>162</td>
<td>0.33</td>
<td>3470</td>
<td>93</td>
</tr>
</tbody>
</table>

In addition, you may need the following fluid properties obtained from various sources.

**Combustion Product Stagnation Properties (from NASA CEA and GasEq)**

<table>
<thead>
<tr>
<th>MW</th>
<th>$c_p$ (kJ/kgK)</th>
<th>k (W/mK)</th>
<th>$\mu$ (kg/m/s)</th>
<th>$\mu(T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.6</td>
<td>2.36</td>
<td>0.40</td>
<td>$1.15 \times 10^{-4}$</td>
<td>$\propto T^{0.72}$</td>
</tr>
</tbody>
</table>
Methane Properties
(from https://webbook.nist.gov/cgi/cbook.cgi?ID=C74828&Mask=4)

<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>$p$ (bar)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$c_p$ (kJ/kgK)</th>
<th>$c_v$ (kJ/kgK)</th>
<th>$\mu$ (kg/m/s)</th>
<th>$k$ (W/m/K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>220</td>
<td>430</td>
<td>3.33</td>
<td>2.08</td>
<td>$1.25\times10^{-4}$</td>
<td>0.196</td>
</tr>
</tbody>
</table>

a) It is estimated that the maximum operating temperature of our alloy will be 900 K when the combustor wall thickness at the throat is 0.5 mm. Determine the channel width that can meet our maximum wall temperature constraint. In addition, estimate the number of cooling channels around the nozzle circumference.

b) Based on the results of part a), estimate the expected pressure drop per unit length for the coolant system in the region of the throat (your result should be in units of bar/m). You can use the following approximation for the Darcy friction factor valid for a smooth channel with $2\times10^4 < \text{Re}_D < 10^7$,

$$f = 0.00015 + 0.173 \text{Re}_D^{-0.195}$$

c) Repeat the analysis of parts a) and b) if the maximum wall temperature is reduced to 800 K.

d) Repeat the analysis of parts a) and b) if the maximum wall temperature is increased to 1000 K.

2. Solid Motor Design

You have been asked to help with the preliminary design of a solid rocket motor that is intended to produce three modes of operation during flight: 1) an initial, high thrust to produce a high acceleration (“boost” mode); 2) a lower, but longer duration, steady thrust (“sustain” mode); and 3) a high final thrust (“delivery” mode). One of the design requirements is to obtain a thrust of 1000 lb during the sustain mode when operating at an ambient pressure of 40.0 kPa.

Some of the design parameters have already been identified. The motor will be of a port-burning design, with a cylindrical casing, an inner diameter of the casing ($D_{\text{casing}}$) of 23 cm, and the propellant bonded to the casing. The nozzle has been chosen to have an expansion area ratio of 6.6 and a throat diameter of 4.5 cm.

Furthermore, the port-burning grain geometry has been chosen and will produce a burning perimeter ($P$) as a function of normalized regression distance as shown in the plot below. Here $x$ is the accumulated regression distance since ignition, $x^* = D_{\text{casing}}/3.5$, and $P_{\text{max}}$ is the perimeter of the burning propellant when the regressing surface has reached the casing.
For \( x/x^* < 0.2 \), this behavior can be accurately modeled with the expression

\[
\frac{P}{P_{\text{max}}} = 1 - 0.65 \left( \frac{x}{x^*} \right)^2 - 50 \left( \frac{x}{x^*} \right)^3.
\]

As seen in the graph, the burning surface perimeter remains constant for \( 0.2 \leq x/x^* \leq 0.8 \), and beyond 0.8, the profile is just the mirror image of the 0-0.2 range.

Finally, the motor will employ a composite propellant with the following properties:

<table>
<thead>
<tr>
<th>( \rho_{\text{solid}} ) (kg/m(^3))</th>
<th>( a ) (m/s/Pa(^n))</th>
<th>( n )</th>
<th>( MW_{\text{products}} )</th>
<th>( T_o ) (K)</th>
<th>( \gamma_{\text{products}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1780</td>
<td>6.5\times10^6</td>
<td>0.4</td>
<td>21</td>
<td>1700</td>
<td>1.20</td>
</tr>
</tbody>
</table>

where \( a \) and \( n \) are the parameters in St. Robert’s burning rate law.

As always, if you perform the analysis computationally, you need to show the expressions used to produce the results.

a) Determine the length of the motor necessary to meet the desired sustain thrust.

b) Determine and plot the pressure in the SRM as a function of time;

c) Determine and plot the thrust produced as a function of time, assuming the rocket operates at the same altitude during all portions of the flight;

d) Estimate the total mass of propellant stored in the motor and determine the volumetric loading fraction for this design; also compare it to typical values for SRM grain configurations;

e) Repeat items b) and c) for a new propellant having a pressure exponent \( n=0.65 \), and a new \( a \) coefficient - one that makes both propellants have the same regression rate at the sustain pressure for the original motor; keep the motor length, diameter and nozzle parameters the same as before;

f) Determine and compare the total impulse produced by both rockets.