Hybrid Rockets

Design Approach and Example

Hybrid Design

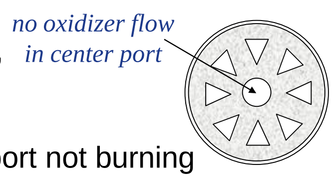
- Preliminary design approach similar to previous cases
- Assume given thrust and burn duration requirements, possibly maximum motor diameter or length constraints
  1. choose propellants: oxidizer and fuel
     - ⇒ based on performance and handling/storage
  2. try to find optimum I_{sp}, c^* (c_τ for nozzle)
     - ⇒ optimum O/F
     - ⇒ feed system (pressure) requirements (oxidizer)
  3. size system
     - ⇒ total propellant mass, oxid. storage, port/grain design (fuel)
  4. estimate initial conditions and performance variation over burn time
     - ⇒ ballistics (+ nozzle erosion)
  5. Iterate from 2 if necessary

Hybrid Rocket Prelim. Design Example

- Consider design from early point in the process
- **Application**: small hybrid rocket for in-space application
  - 1st stage of LEO to GEO (LEO to intermediate circular orbit)
- **Requirements**
  - needed velocity increment: Δu=**1893** m/s
  - payload mass: **6000** kg
  - assume additional **8.7% inert mass** (nozzle, casing, pressurization system…)
  - acceleration requirement: ≥1.5 thrust-to-weight

Hybrid Rocket Prelim. Design Example

- **Design choices**
  - HTPB/LOx propellants
    - well characterized, ρ_{HTPB} = 950. kg/m^3
  - initial chamber pressure $p_{o,init}$ =14.0 bar
    - in vacuum, thrust coefficient weak function of p_o
    - low pressure allows for reduced structural weight
  - nozzle ε=50.
    - higher thrust and I_{sp} for large ε, but also more mass and size
  - grain design
    - wagon wheel, 8 ports, center port not burning
    - choice of number of ports actually should be part of design process (try to improve vol. loading frac. ε_l)



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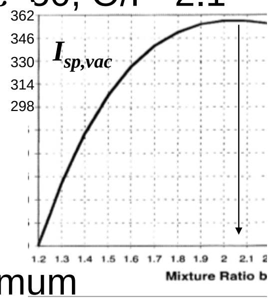
Preliminary Mass Estimates

- First estimate total propellant mass required

$$\Delta u = I_{sp} g_o \ln R$$

  - HTPB/LOx max $I_{sp,vac}$~355s for ε=50, O/F~2.1 reduce by ~10% for estimate



$$\Rightarrow m_p = m_i - m_f = m_f \left(e^{\Delta u / I_{sp} g_o} - 1\right)$$

$$= 1.087 \times 6000kg \left(e^{1893m/s / \left(320s \times 9.807 m/s^2\right)} - 1\right)$$

$$= \boxed{5400.kg} \longrightarrow m_{total} = 11{,}922kg$$

- Solid and liquid masses
  - assume **avg** O/F slightly > optimum

$$m_f = \frac{m_p}{1 + O/F} = \frac{5400kg}{1 + 2.2} = \boxed{1687.5kg} \quad \Rightarrow m_{ox} = \boxed{3712.5kg}$$

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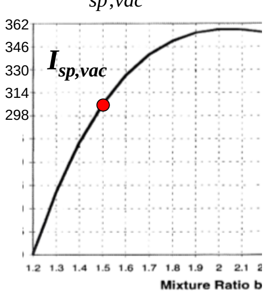
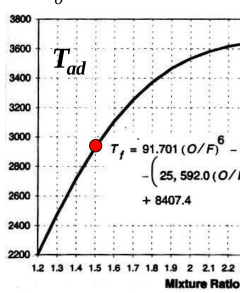
Initial Flow Rates

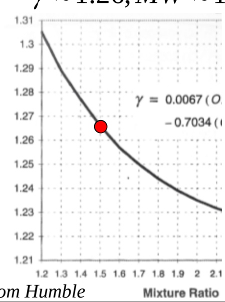
- Assume initial O/F lower than average O/F
  - O/F will increase during burn
  - guess initial O/F 1.4× below optimum,

    O/F_{init} = 2.1/1.4 = 1.5 (and p_o=14bar, ε=50)

$\Rightarrow I_{sp,vac} \approx 310s$ $\quad T_o \approx 3000K$ $\quad \gamma \approx 1.26, MW \approx 19.5$







from Humble

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$$\Rightarrow I_{sp,vac} \approx 310s \qquad T_o \approx 3000K \qquad \gamma \approx 1.26, MW \approx 19.5$$

- From required thrust-to-weight

$$\dot{m}_{p,i} = \frac{\tau}{I_{sp,i} g_o} = \frac{(\tau/w) m_{init}}{I_{sp}} = \frac{1.5(11{,}922kg)}{310s} = \boxed{57.7\,kg/s}$$

$$\dot{m}_{f,i} = \dot{m}_{p,i}/(1+1.5) = \boxed{23.1\,kg/s} \quad \Rightarrow \dot{m}_{ox,i} = \boxed{34.6\,kg/s}$$

$$\text{with } \gamma, MW, cpg \Rightarrow D_t = 30cm$$

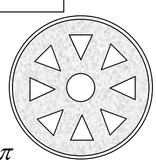
Initial Port Size Estimation

- For liquid oxidizers, try to limit flow through port to prevent extinguishing flame
  - suggested maximum flux, G_{ox}=350 kg/m^2s
- Initial port area

$$\Rightarrow A_{p,i}/A_t = 1.4 \quad ok$$

$$G_{ox,i} = \frac{\dot{m}_{ox,i}}{A_{p,i}} \Rightarrow A_{p,i} = \frac{\dot{m}_{ox,i}}{350\,kg/m^2s} = \boxed{0.099m^2}$$

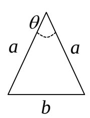
- With 8 ports, area of each port



$$\Rightarrow A_{p,single,i} = \boxed{124cm^2}$$

$$a = \sqrt{\frac{2A_{p,single,i}}{\sin\theta}} = 18.7cm$$

$$b = 14.3cm$$



$$\theta = \frac{2\pi}{N} = \frac{2\pi}{8}$$

$$A_{p,single} = \frac{ab}{2}\cos\frac{\theta}{2} \qquad \frac{b}{2a} = \sin\frac{\theta}{2}$$

Grain Length Estimate

- To determine grain length

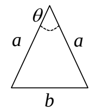
$$\dot{m}_{f,i} = \rho_{sf} r_{avg} S_{p,i} L \Rightarrow L = \frac{\dot{m}_{f,i}}{\rho_{sf} r_{avg} S_{p,i}} = \frac{G_{f,i} A_{p.i}}{\left(a' G_{ox}^{n} L^{-m}\right) \rho_{sf} S_{p,i}}$$

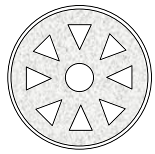
simplified regression rate law

$$\boxed{\Rightarrow L = \left(\frac{G_{ox,i}^{1-n} A_{p.i}}{a' (O/F) \rho_{sf} S_{p,i}} \right)^{\frac{1}{1-m}}}$$

  – need port perimeter

$$S_{p,i} = 2\sqrt{\frac{2A_{p,i}}{\sin\theta}} \left(1 + \sin\frac{\theta}{2}\right)$$





$$\boxed{L = 4.6m}$$

$a' = 2.0 \times 10^{-5}\ m/s$
(with G in $kg/m^2 s$, L in m)
$n = 0.75,\ m = 0.15$

$$a = \sqrt{\frac{2A_{p,single,i}}{\sin\theta}}$$

$$D_{p,i} = 4A_{p,i} / S_{p,i} \quad \Rightarrow L/D_p \approx 48$$

can characterize as long channels

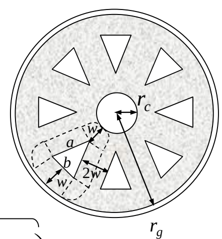
$$S_{p,i,single} = 2a + b = 2a\left(1 + \sin\frac{\theta}{2}\right)$$

Web Thickness and Radius Estimate

- For triangular ports $\dfrac{m_{f,i}/N}{\rho_{sf} L} \cong aw + aw + bw + \pi w^2$

$$w^2 + \frac{(2a+b)w}{\pi} - \frac{m_{f,i}/N}{\pi \rho_{sf} L} = 0$$

$$w = \frac{1}{2}\sqrt{\left(\frac{S_{p,single,i}}{\pi}\right)^2 + 4\frac{m_{f,i}/N}{\pi \rho_{sf} L}} - \frac{S_{p,single,i}}{2\pi}$$



$$\boxed{w = 6.7cm}$$

- Grain radius $r_g = \underbrace{2w + a\cos\frac{\theta}{2} + w\left(\sin^{-1}\frac{\theta}{2} - 1\right)}_{r_c}$

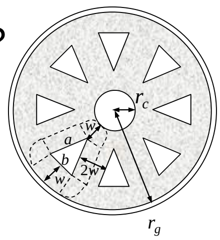
$$\boxed{r_g = 0.41m} \quad \Rightarrow \frac{L}{D_{grain}} = 5.5$$ reasonable

Fuel Check

- Initial estimate of fuel requirement was

$$m_f = 1687.5kg$$

- How much fuel will our design hold?
  - begin by examining volume loading fraction



high, but ignores slivers and no grain support structures

$$\varepsilon_l \equiv \frac{V_{propellant}}{V_{chamber}} = \frac{\pi\left(r_g^2 - r_c^2\right) - A_{p,i}}{\pi r_g^2} = 0.75$$

$$m_f = \varepsilon_l V_{chamber} \rho_{sf} = \varepsilon_l \pi r_g^2 L \rho_{sf} = 1753kg \checkmark$$

  - could try to make stored fuel closer to requirement, but…

Next Evaluation Step

- Perform ballistic analysis to see how O/F, mass flowrate, pressure, combustion product properties, thrust, I_{sp}, … vary with time
  - do we meet requirements and constraints (Δu, max p_o, thrust-to-weight,…)?
  - reduce excess fuel mass,…?
- Probably start by assuming constant oxidizer mass flow rate

and still need to design O_2 supply/pressurization system,…