Rocket Propulsion Basics

Ideal Thrust Coefficient and Characteristic Velocity

Ideal Thrust Coefficient

- Recall definition of thrust coeff.

$$c_\tau \equiv \frac{\tau}{A_t p_o}$$

- Also steady thrust

$$\tau = \dot{m} u_e + \left(p_e - p_a\right) A_e$$

- Combine with ideal nozzle results

$$u_e = \sqrt{\frac{2\gamma}{\gamma - 1} \frac{\bar{R} T_o}{MW} \left[1 - \left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]}$$

$$\dot{m}_{choked} = A_t \frac{p_o}{\sqrt{R T_o}} \sqrt{\gamma \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

Ideal Thrust Coefficient

$$c_\tau = \sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}\left[1-\left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]} + \left(\frac{p_e}{p_o}-\frac{p_a}{p_o}\right)\frac{A_e}{A_t}$$

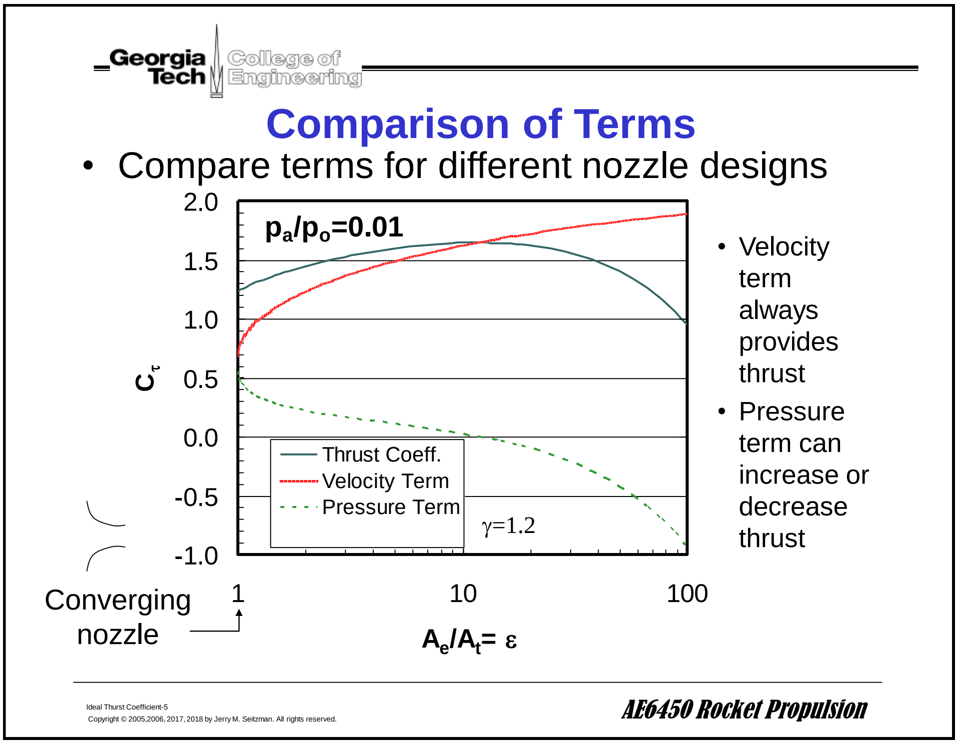
- Ideal thrust coefficient is only function of
  - – γ, ε ($=A_e/A_t$), p_a/p_o
  - – recall p_e/p_o = fn(ε)
- Note: $c_\tau \neq$ fn(T_o, MW)
- Thrust coeff. depends mostly on pressure distribution in thrust chamber
  - – from normalizing by p_oA_t

$$c_\tau \equiv \frac{\tau}{p_oA_t}$$

Ideal Thrust Coefficient

$$c_\tau = \underbrace{\sqrt{\frac{2\gamma^2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/\gamma-1}\left[1-\left(\frac{p_e}{p_o}\right)^{\gamma-1/\gamma}\right]}}_{\dot{m}u_e/p_oA_t} + \underbrace{\left(\frac{p_e}{p_o}-\frac{p_a}{p_o}\right)\frac{A_e}{A_t}}_{(p_e-p_a)A_e/p_oA_t}$$

- 1st term = contribution to thrust by **exit velocity**/momentum
- 2nd term = contribution to thrust by **exit pressure**



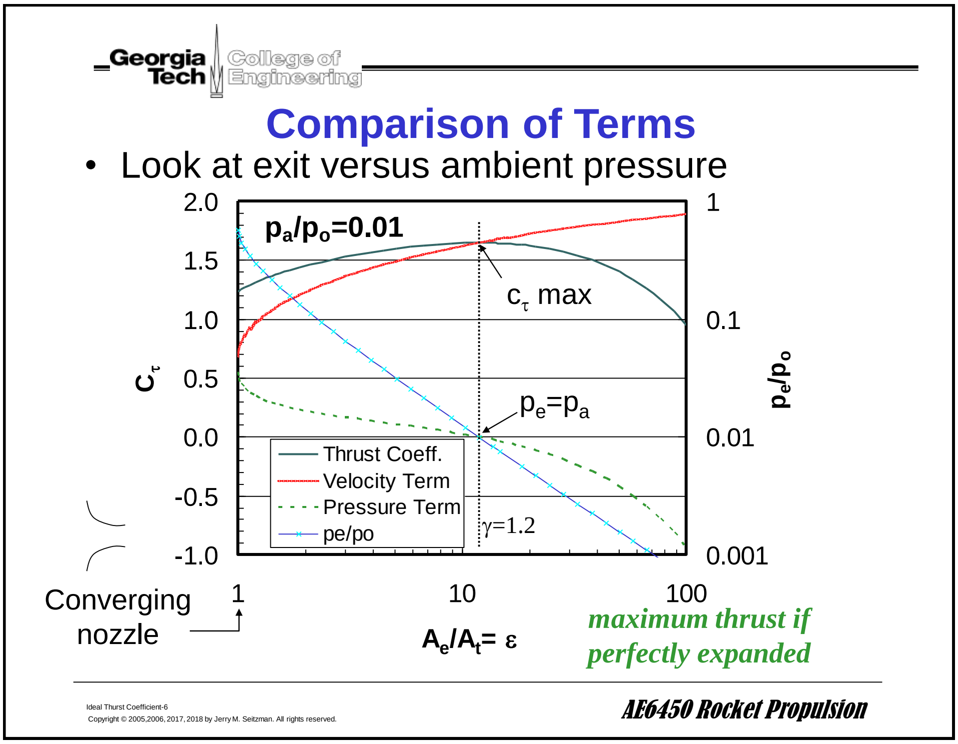
Comparison of Terms

- Compare terms for different nozzle designs

- Velocity term always provides thrust
- Pressure term can increase or decrease thrust

Ideal Thrust Coefficient-5

AE6450 Rocket Propulsion



Comparison of Terms

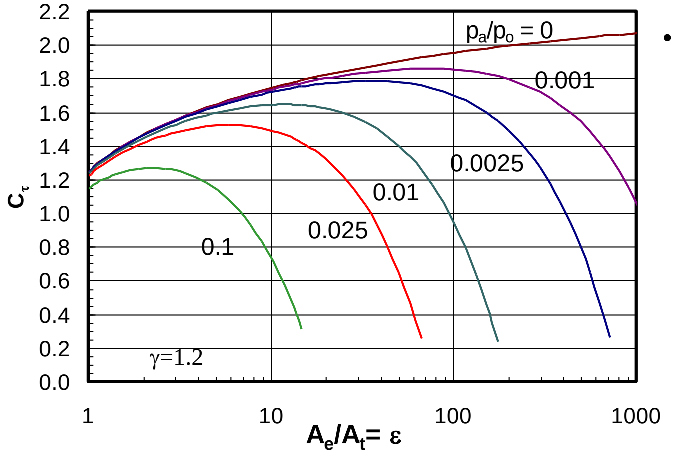
- Look at exit versus ambient pressure

maximum thrust if perfectly expanded

Ideal Thrust Coefficient-6

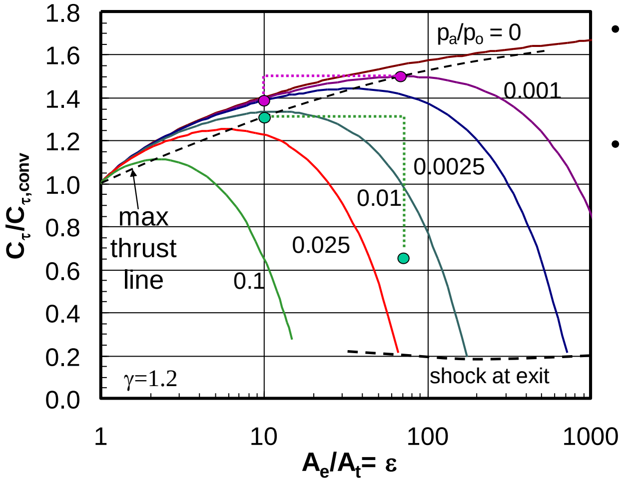
AE6450 Rocket Propulsion

Effect of Ambient Pressure on c_τ



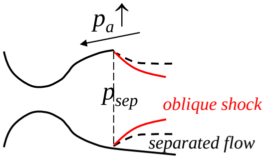
- Can get higher thrust coeffient by:
  - –reducing ambient pressure
  - –increasing rocket pressure

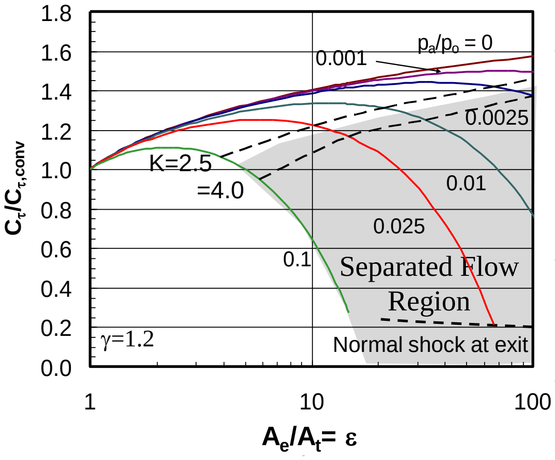
Normalize by Converging Nozzle c_τ



- Large ε needed for optimum c_τ for small p_a/p_o
- Optimum ε (for c_τ, I_{sp}) varies with altitude (p_a)
  - – for p_o=1000 psia
  - – $p_a/p_o \cong$
    - **0.015** **sea-level**
    - **0.001** **60,000 ft**

Flow Separation





- For p in nozzle enough below p_a, flow (b.l.) separates
  - occurs in over-expanded operation and before normal shock would enter
  - expansion essentially ends at separation (lower ε)
- Summerfield* found oblique shock enters nozzle for $K \equiv p_a/p_{e,sep} = 2.5\text{–}4$

$$\Rightarrow \quad p_e/p_o < 25\text{–}40\% \;\; p_a/p_o$$

- Kalt and Bendall** give empirical-based criteria (one of many)

$$p_{sep}/p_a = \tfrac{2}{3}\left(p_o/p_a\right)^{-0.2}$$

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*Summerfield et al., *Jet propulsion* **24** (1954)

***J. Spacecraft and Rockets* **2** (1965)

Ideal Characteristic Velocity

- Recall defn. of c* $\quad c^* \equiv A_t p_o / \dot{m}$
- Inserting expression for choked flow

$$c^* = \frac{A_t p_o}{A_t\left(p_o/\sqrt{RT_o}\right)\left[\gamma\, 2/(\gamma+1)\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

Ideal $$c^* = \frac{\sqrt{\gamma R T_o}}{\gamma\left[2/(\gamma+1)\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$\longrightarrow$ ***a_o*, sound speed**

$\longrightarrow$
0.71 for γ=1.2
0.81 for γ=1.4
0.94 for γ=1.67

Typical *c values**
1200-2400 m/s
(4000-8000 ft/s)

- So c^*=fn(γ, T_o, MW) ≅ 1.1-1.4×*sound speed* $\neq$ fn(p_o, p_a)

measure of thermal energy available → flow KE

measure of obtainable u_e for given KE

not dependent on nozzle performance