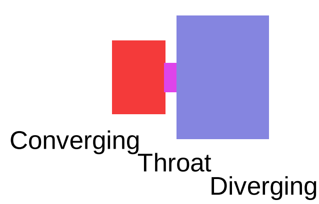
Nozzle Geometries

Nozzle Configurations

Nozzle Configurations

- So far considered 1-d ideal nozzles
  - primarily for over/underexpanded operation
- Described one real effect
  - flow separation
- Continue by looking at real nozzle configurations (nozzle geometry)
- Converging section
  - subsonic flow, favorable pressure gradient
  - can use almost any shape with minimal Δp_o loss
- Diverging section – design goals
  - high I_{sp}, low nozzle mass and length



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Major Nozzle Configurations

- Four major types (based on diverging section)
  1. **Conical**
     - oldest, simplest design, easy to manufacture

     small, inexpensive thrusters
  2. **Bell/contoured**
     - complex shape, high efficiency

     large rockets +
  3. **Plug/aerospike**
     - altitude compensating, annular or linear

     large static (X-33) and small flight test
  4. **Expansion-deflection**
     - altitude compensating, shorter than other enclosed nozzles

     static tested, primarily for upper stages

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Nozzle Configurations

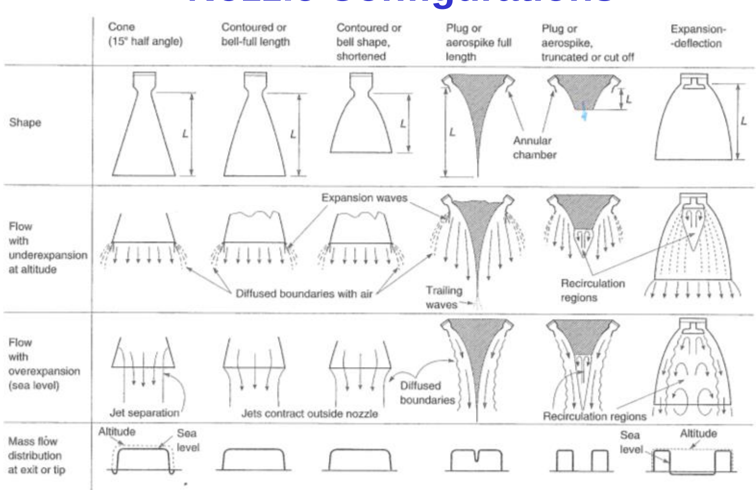

FIGURE 3–12. Simplified diagrams of several different nozzle configurations and their flow effects. *From Sutton*

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Linear Aerospike Nozzle

- From Boeing XRS-2200 test (X-33)

Photo Credit: Boeing

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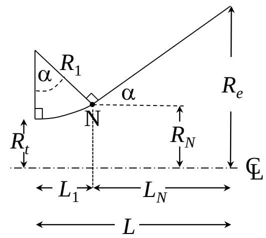
Nozzle Geometries

Conical Nozzles

Conical Nozzles

- Diverging section consists of 2 parts
  1. arc of sphere
     - begins at throat
     - radius R_1
  2. linear section
     - begins at transition point N
     - half angle α
- Design parameters
  – $R_t, R_1, \alpha, \varepsilon \Rightarrow L_1, L, R_e, \ldots$



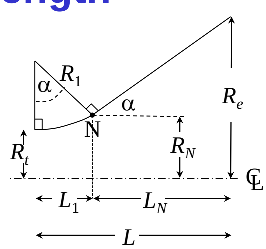
$$\varepsilon = A_e / A_t$$

$$\sqrt{\varepsilon} = R_e / R_t$$

$$\boxed{R_e = \sqrt{\varepsilon} R_t}$$

Conical Nozzle - Length

- Can write length in terms of design parameters



$$L = L_N + L_1$$

$$L_N = \frac{R_e - R_t + R_1(\cos\alpha - 1)}{\tan\alpha}$$

$$= \frac{R_t\left(\sqrt{\varepsilon} - 1\right) + R_1(\cos\alpha - 1)}{\tan\alpha}$$

$$L_1 = R_1 \sin\alpha$$

$$L_N \tan\alpha = R_e - R_N$$

$$R_N = R_t + R_1(1 - \cos\alpha)$$

$$L = \frac{R_t\left(\sqrt{\varepsilon} - 1\right) + R_1(\cos\alpha - 1)}{\tan\alpha} + R_1 \sin\alpha$$

$$= \frac{R_t\left(\sqrt{\varepsilon} - 1\right) + R_1\left(\cos\alpha - 1 + \sin^2\alpha / \cos\alpha\right)}{\tan\alpha}$$

$$R_e = \sqrt{\varepsilon} R_t$$

Conical Nozzle - Length

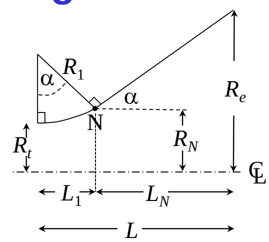
- Continuing

$$L=\frac{R_t\left(\sqrt{\varepsilon}-1\right)+R_1\left(\cos\alpha-1+\frac{\sin^2\alpha}{\cos\alpha}\right)}{\tan\alpha}$$

$$\frac{\cos^2\alpha}{\cos\alpha} \qquad \frac{1-\cos^2\alpha}{\cos\alpha}$$

$$L=\frac{R_t\left(\sqrt{\varepsilon}-1\right)+R_1\left(-1+\frac{1}{\cos\alpha}\right)}{\tan\alpha}$$

$$L=\frac{R_t}{\tan\alpha}\left[\sqrt{\varepsilon}-1+\frac{R_1}{R_t}\left(\frac{1}{\cos\alpha}-1\right)\right]$$



Typical value: $R_1/R_t \sim 1.5$

For ε= 50

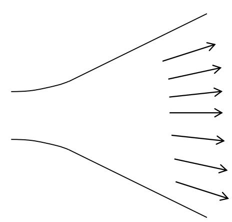
α	L/R_t
7	50
11	31
15	23
19	18
23	15
27	12

$\alpha\uparrow \Rightarrow L\downarrow$

Flow Divergence

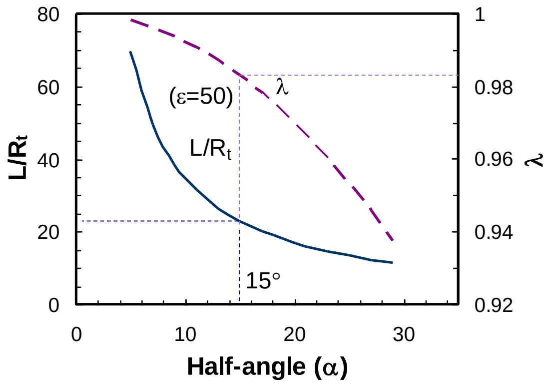
- Other effect of increasing nozzle angle
  - **flow divergence**
- Some of the momentum increase produced by nozzle is not aligned with nozzle axis
  - **thrust reduction/loss**
- For uniform $|u_e|$ can apply correction factor λ

$$\tau=\lambda\dot{m}u_e+\left(p_e-p_a\right)A_e$$



Conical Nozzles – Design Tradeoff

- **Shorter length** but **lower thrust** for higher cone-angle
  - tradeoff between size/mass and I_{sp}



$$\frac{L}{R_t} = \frac{\sqrt{\varepsilon} - 1 + \frac{R_1}{R_t}\left(\frac{1}{\cos\alpha} - 1\right)}{\tan\alpha}$$

$$\tau = \lambda \dot{m} u_e + (p_e - p_a) A_e$$

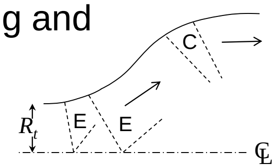
$$\lambda = \frac{1+\cos\alpha}{2}$$ *for spherical expansion*

Nozzle Geometries

Bell/Contoured Nozzles

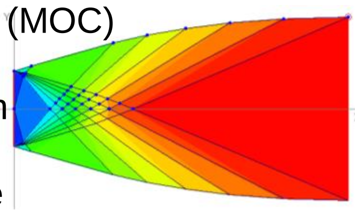
Bell/Contoured Nozzles

- Contoured to minimize turning and divergence losses
  - reducing divergence requires turning flow (more axial)
  - can result in compressions, could lead to shock losses
- Goal is to design nozzle contour such that all waves are isentropic and produce nearly axial flow at exit



Design Approaches

- **Method of characteristics** (MOC)
  - inviscid assumption (can use hybrid approach for wall boundary layers)
  - must account for variable speed of sound



from Aerorocket.com

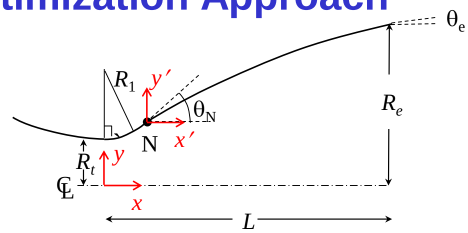
- **Approximate optimization method** of Rao
  - initial (near throat) section spherical
  - transition to parabola

Rao, *Jet Propulsion* **28**, pp. 377-382 (1958)

Rao, *J. Amer. Rocket Soc.* **31**, pp. 1488-1494 (1961)

Allman and Hoffman, *AIAA J.* **19**, pp. 750-751 (1981)

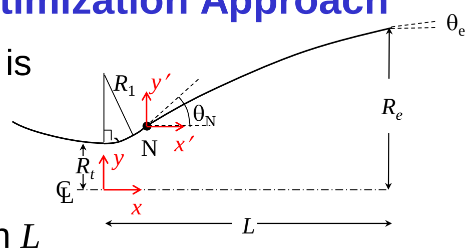
Approximate Optimization Approach



- Near throat region composed of two spherical sections
  - before throat: R_1/R_t=1.5
  - after throat and up to N: R_1/R_t=0.382
  - N given by $x_N = R_1 \sin\alpha \quad y_N = R_t + R_1(1-\cos\alpha)$
- Parabola (after N) with slope matched at N
  $$y' = Px' + Q + (Sx' + T)^{1/2}$$
  - 4 unknowns: P, Q, S, T

4 boundary conditions:
1) $x'_N = y'_N = 0$
2) $x'_e = L - x_N, \quad y'_e = \sqrt{\varepsilon} R_t - y_N$
3) θ_N = supplied (e.g., Rao)
4) θ_e = supplied (e.g., Rao)

Approximate Optimization Approach



- Output of approach is "optimal" contour given
  - ε
  - acceptable length L (shorter ⇒ larger divergence θ_e)
- Typically L is specified relative to length of conical nozzle with α=15°

$$L = f(\%) \times \frac{R_t}{\tan 15°}\left[\sqrt{\varepsilon} - 1 + 1.5\left(\frac{1}{\cos 15°} - 1\right)\right]$$

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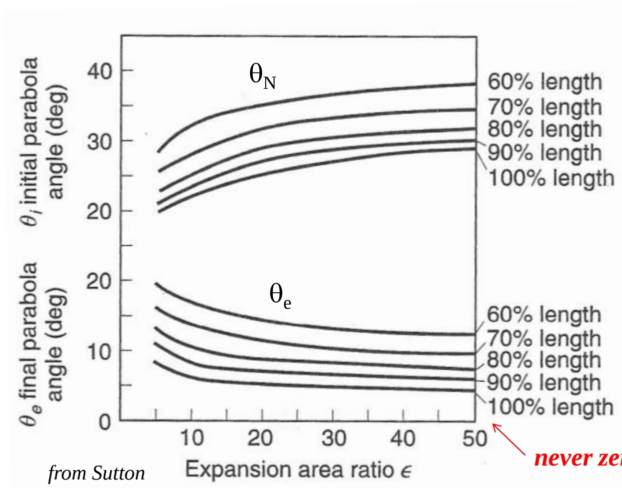
Approximate Optimal Design Angles



never zero angle

- Shorter nozzle
  - larger initial and final angles
- Larger ε
  - larger initial angle
  - smaller final angle

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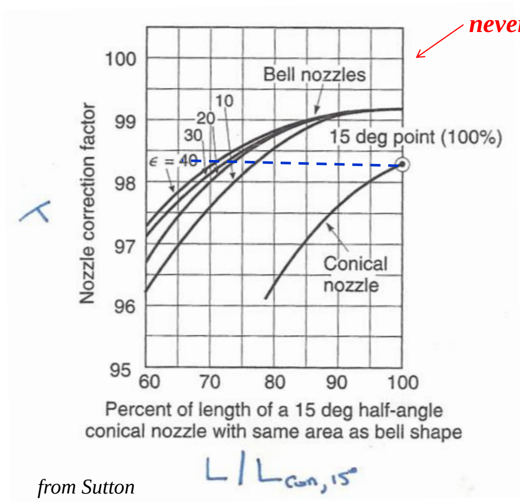
Approx. Optimal Design Performance

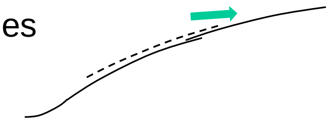
never 100% efficient



- Less divergence loss for full length bell vs. conical
- 70% bell has nearly same divergence loss but shorter and with less mass

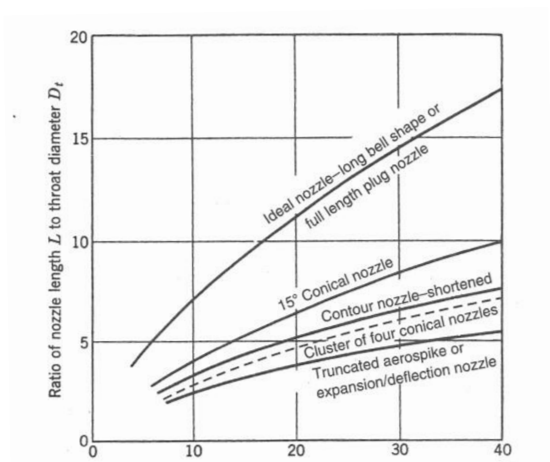
Altitude/Ambient Pressure Adjustment

- Can use variable expansion ratio nozzles
  - – extendable, two-step nozzles e.g., RL-10B-2 on Delta IV 2nd stage



- Plug/aerospike and ED nozzles
  - – requires full aerodynamic model to help determine nozzle boundaries
    - plug: outer boundary
    - ED: inner boundary
  - – full aerospike: high performance but cooling difficult
  - – truncated aerospike: can still get high λ with short L

Nozzle Length Comparison



from Sutton

- Ideal bell nozzle is longest for given ε
- Aerospike (and E/D) nozzles have potential for lowest weight