Solid Rocket Motors

Propellant Burning Rate and Pressure (Internal Ballistics)

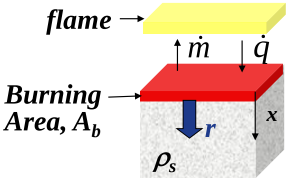
SRM Analysis

- Want to be able to find standard rocket performance parameters
  - $I_{sp}=c^*c_\tau / g_o\,,\ \tau = \dot{m}\, I_{sp}\, g_o$
- Does analysis change?
  - $c^*=c^*(T_o,\ \gamma,\ MW)$
  - $c_\tau = c_\tau(\varepsilon\ ,\ \gamma,\ p_o)$ …. and p_a
- Can still get ~fluid properties (T_o, γ, MW) from equilibrium code
- How to determine $\dot{m}$, p_o?
  - propellant flowrate not determined by pumps or pressurization system
  - controlled by how fast solid propellant burns

Mass “Production” Rate

- Propellant converted to gas at rate given by

(VI.1) $$\dot{m} = r\rho_s A_b$$



- (Surface) **Regression Rate *r***

$$r = dx/dt \quad \text{sometimes } \dot{r}_b$$

  - standard model (Burning Rate “Law” or St. Robert’s Law, also Vielle’s Law)

after M. de Saint-Robert

(VI.2) $$r = ap_o^n \quad \text{with } a = f(T, \dots)$$

  - also, $r = c + bp_o^n$ etc.

Solid Propellant Burning Rate

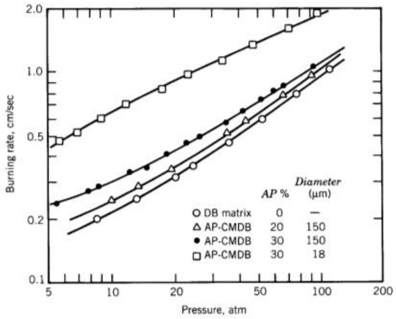


FIGURE 11–7. Measured burning rate characteristics of a double-base (DB) propellant and three composite-modified double-base (CMDB) propellants which contain an increasing percentage of small diameter (159 µm) particles of ammonium perchlorate (AP). When the size of the AP particles is reduced or the percentage of AP is increased, an increase in burning rate is observed. None of these data form straight lines.

$$r = ap^n \Rightarrow \boxed{\ln r \cong \ln a + n \ln p}$$

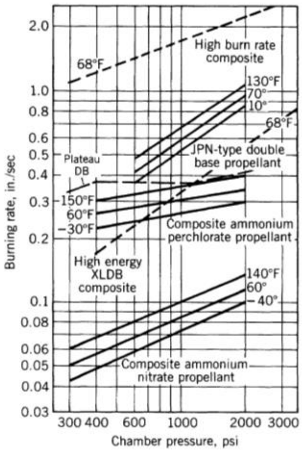
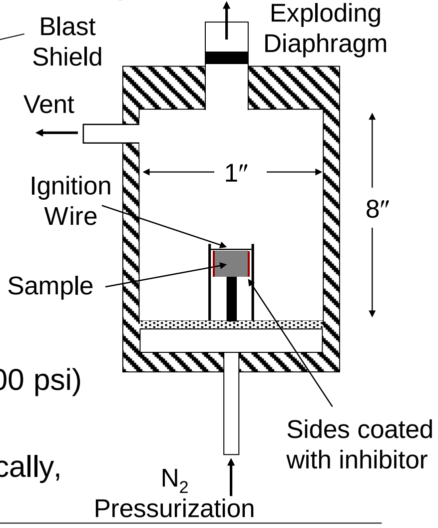


FIGURE 11–6. *From Sutton*

Measuring Burning Rate



- Pre-pressurize (e.g., up to 2500 psi)
- Ignite top of propellant
- Measure regression rate (optically, embedded wires, …)

Motor Internal Ballistics

- What governs motor internal conditions?
- Examine mass conservation

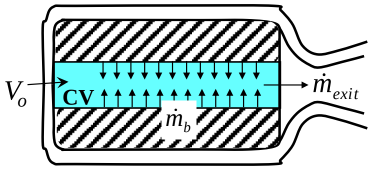
$$0=\frac{dm_{CV}}{dt}+\int\rho\left(\vec{u}\cdot\hat{n}\right)dA$$

$$0=\underbrace{\frac{d}{dt}\left(\rho_o V_o\right)}_{V_o\frac{d\rho_o}{dt}+\rho_o\frac{dV_o}{dt}}+\dot{m}_{exit}-\dot{m}_b$$

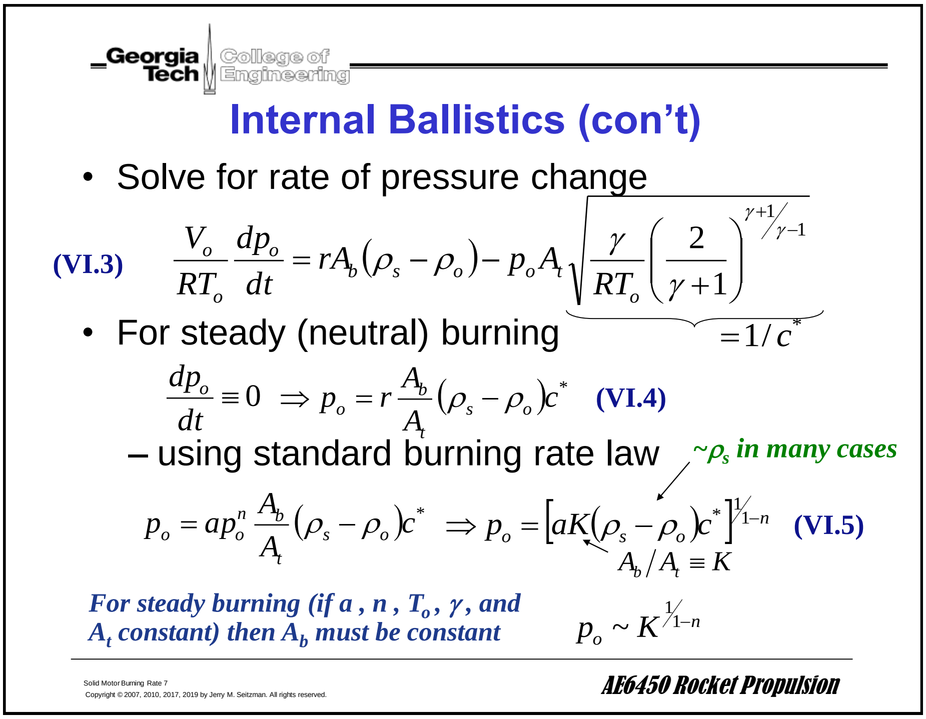
$$\dot{m}_b \rightarrow r\rho_s A_b$$

$$V_o\frac{d\rho_o}{dt} \rightarrow \frac{1}{RT_o}\frac{dp_o}{dt} \qquad \rho_o\frac{dV_o}{dt} \rightarrow \rho_o\left(A_b r\right)$$

$$\dot{m}_{exit} \rightarrow \frac{p_o}{\sqrt{RT_o}}\sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\gamma+1/2(\gamma-1)}A_t$$



Assuming:
1) *uniform gas prop's. in CV*
2) *TPG, CPG*
3) T_o*=constant (e.g.,* T_{ad}*)*
4) p_o*,* A_b*, r given at time t*



Internal Ballistics (con't)

- Solve for rate of pressure change

(VI.3) $$\frac{V_o}{RT_o}\frac{dp_o}{dt} = rA_b\left(\rho_s-\rho_o\right) - p_oA_t\underbrace{\sqrt{\frac{\gamma}{RT_o}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}}_{=1/c^*}$$

- For steady (neutral) burning

$$\frac{dp_o}{dt}\equiv 0 \;\Rightarrow\; p_o = r\frac{A_b}{A_t}\left(\rho_s-\rho_o\right)c^* \quad \textbf{(VI.4)}$$

  - using standard burning rate law

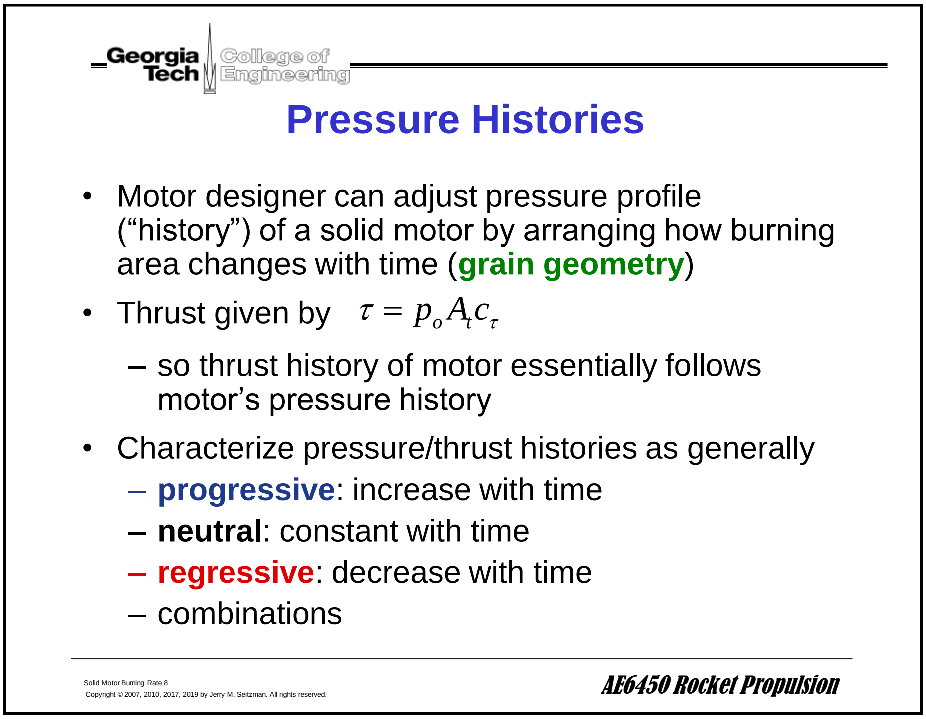
$\sim\rho_s$ in many cases

$$p_o = ap_o^n\frac{A_b}{A_t}\left(\rho_s-\rho_o\right)c^* \;\Rightarrow\; p_o = \left[aK\left(\rho_s-\rho_o\right)c^*\right]^{\frac{1}{1-n}} \quad \textbf{(VI.5)}$$

$$A_b/A_t \equiv K$$

For steady burning (if a , n , T_o, γ, and A_t constant) then A_b must be constant

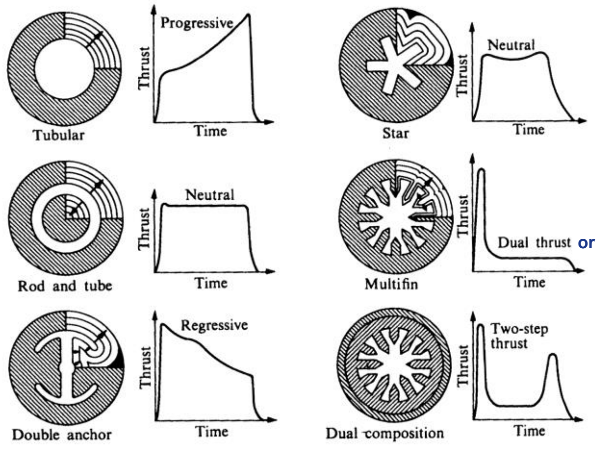
$$p_o \sim K^{\frac{1}{1-n}}$$



Pressure Histories

- Motor designer can adjust pressure profile ("history") of a solid motor by arranging how burning area changes with time (**grain geometry**)
- Thrust given by $\tau = p_oA_tc_\tau$
  - so thrust history of motor essentially follows motor's pressure history
- Characterize pressure/thrust histories as generally
  - **progressive**: increase with time
  - **neutral**: constant with time
  - **regressive**: decrease with time
  - combinations

Grain Geometries and Thrust History



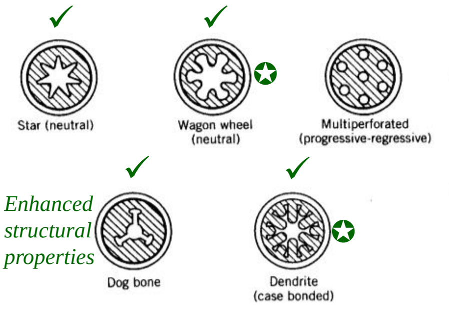
or "boost sustain"

FIGURE 12.17 Internal-burning charge designs with their thrust-time programs. (Courtesy Shafer [18].)

From Hill and Peterson

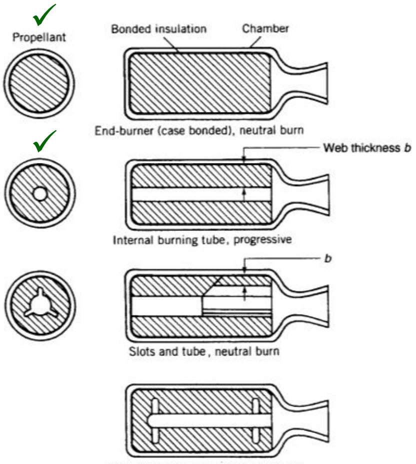
Many Solid Motor Grain Geometries

✪ *high thrust (A_b) web designs developed before high burn rate propellants (r>8 mm/s)*



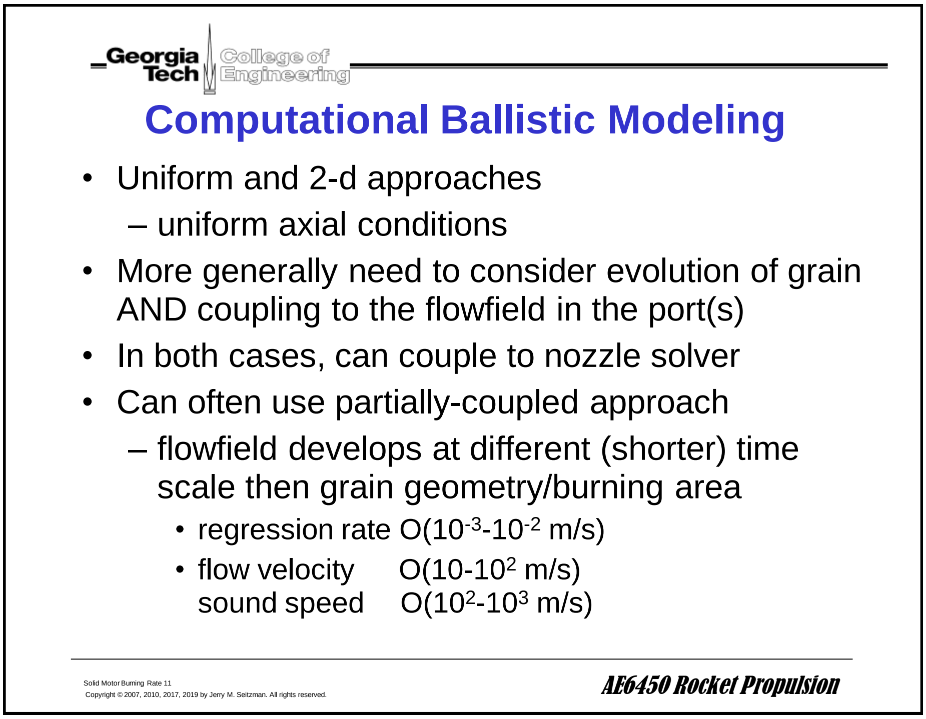
Enhanced structural properties

FIGURE 11–16. Simplified diagrams of several grain configu



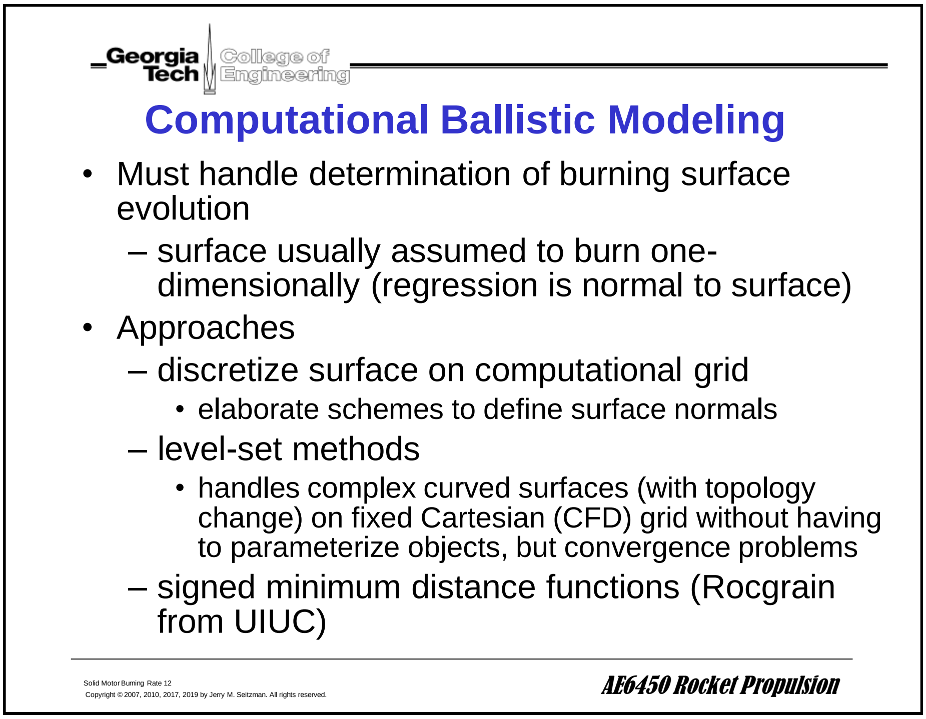
From Sutton

✓*analytic (2-d) expressions available for A_b*
Hartfield et al., AIAA 2003-4506



Computational Ballistic Modeling

- Uniform and 2-d approaches
  - – uniform axial conditions
- More generally need to consider evolution of grain AND coupling to the flowfield in the port(s)
- In both cases, can couple to nozzle solver
- Can often use partially-coupled approach
  - – flowfield develops at different (shorter) time scale then grain geometry/burning area
    - regression rate O(10^{-3}-10^{-2} m/s)
    - flow velocity O(10-10^{2} m/s)
      sound speed O(10^{2}-10^{3} m/s)

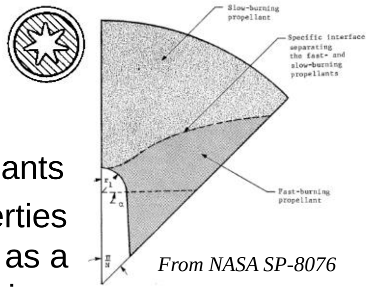


Computational Ballistic Modeling

- Must handle determination of burning surface evolution
  - – surface usually assumed to burn one-dimensionally (regression is normal to surface)
- Approaches
  - – discretize surface on computational grid
    - elaborate schemes to define surface normals
  - – level-set methods
    - handles complex curved surfaces (with topology change) on fixed Cartesian (CFD) grid without having to parameterize objects, but convergence problems
  - – signed minimum distance functions (Rocgrain from UIUC)

Other Approaches to Thrust "Control"

- Cast multiple propellants, e.g., bipropellant star configuration
- Functionally graded propellants
  - – propellants whose properties can be varied gradually as a function of depth into grain
    - e.g., 50% variation in r
  - – example: reduce burning rate with depth in otherwise progressive web design
  - – requires advanced manufacturing approaches



From NASA SP-8076

Figure 22.—Bipropellant star configuration (ref. 85).