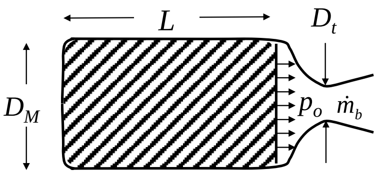
Solid Rocket Motors

Design Examples

Design of an End-Burning Motor



- Start with end burning motor
  - easiest to analyze
  - constant thrust
  - used in some motors and in gas generators
- **Requirements**
  - Δt_b=75 s, τ_{vac}=200 kN (45 klb$_f$) $\Rightarrow I_{tot}=15\ MN{\cdot}s$
- **Constraints** (already chosen)
  - p_o=4 MPa (assume uniform)
  - nozzle: c_τ=1.85 (ε~30-50)
  - propellant: ρ_s=1800 kg/m^3, γ=1.2, MW=24, c^*=1500 m/s
    r=0.40 [p_o(MPa)]$^{0.3}$ cm/s $\Rightarrow T_o=2730K$
- **Design Variables (**assuming axisymmetric/cylindrical geometry)
  - $\boldsymbol{D_t}$, motor diam. ($\boldsymbol{D_M}$), motor length ($\boldsymbol{L}$)

End-Burning Motor Example

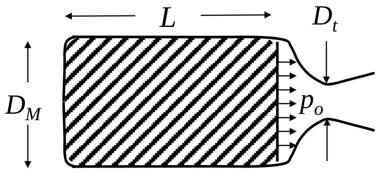
- Nozzle throat size, $\boldsymbol{D_t}$

$$A_t = \frac{\tau}{p_o c_\tau}$$

$$= \frac{2\times10^5\,N}{\left(4\times10^6\,N/m^2\right)1.85}$$

$$= 0.0270 m^2$$

$$\boxed{\Rightarrow D_t = 19cm\left(\sim 7.5in.\right)}$$



End-Burning Motor Example

- Motor length, L
  - for end burning, $L=\ell_{web}$

$$r = \frac{dx}{dt}$$

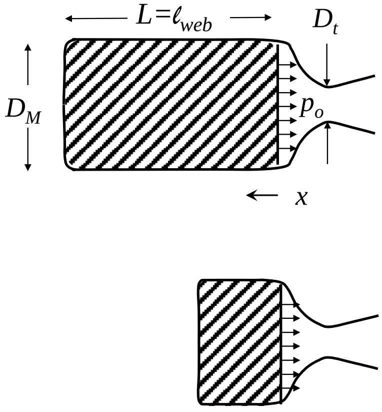
$$= \frac{\ell_{web}}{t_b} \quad \text{steady burning}$$

$$\ell_{web} = rt_b = 0.4(4)^{0.3}\,cm/s\,(75s)$$

$$= 0.61cm/s\,(75s)$$

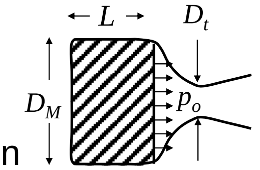
$$\boxed{\Rightarrow L = 46cm} \qquad \Rightarrow L/D_t \approx 2.5$$

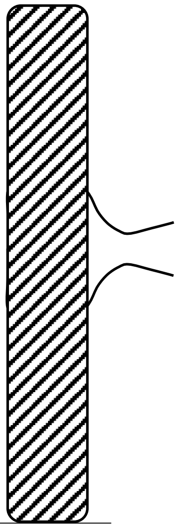
$$D_t = 19cm$$



End-Burning Motor Example

- Motor diameter, $\boldsymbol{D_M}$
  - for end burning, set by burn area
  - recall for steady-burn





(VI.5) $p_o = r\dfrac{A_b}{A_t}(\rho_s - \rho_o)c^*$

$$\frac{A_b}{A_t} \equiv K = \frac{p_o}{r(\rho_s - \rho_o)c^*} \overset{?}{\cong} \frac{p_o}{r\rho_s c^*} \qquad \rho_o = \frac{p_o}{RT_o} = 4.2\,kg/m^3$$

$$= \frac{4\times10^6\ N/m^2}{0.0061\,m/s\left(1800\,kg/m^3\right)1500\,m/s} = 244$$

$A = \pi D^2$

$$\boxed{D_M = \sqrt{K}D_t = \sqrt{244}(19cm) = 2.9m} \Rightarrow D_M/L \approx 6.4!!!!$$

Wide end-burning motors if high thrust reqr.

Scaling For Fixed Propellant

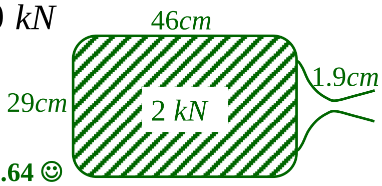
- What happens if we change p_o?
  - increase 4→10 MPa

$$\left.\begin{aligned} \Rightarrow D_t &: 19cm \to 12cm \\ L = \ell_{web} &: 46cm \to 60cm \\ K &: 244 \to 464 \\ D_M &: 2.9m \to 2.5m \end{aligned}\right\} \frac{D_M}{L} : 6.4 \to 4.2$$

$$D_t \propto \sqrt{\tau/p_o} \leftarrow \dot{m}$$
$$\ell_{web} \propto p_o^n t_b \leftarrow r$$
$$K \propto p_o^{1-n}$$
$$D_M \propto D_t\sqrt{K} \propto \sqrt{\tau/p_o^n}$$

- Instead what happens if we change *thrust*?
  - decrease 10 ×, 200→20 kN

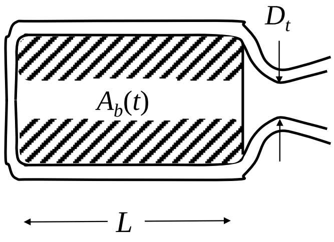
$$\left.\begin{aligned} \Rightarrow D_t &= 5.9cm \downarrow \\ L = \ell_{web} &= 46cm\ - \\ K &= 244\ - \\ D_M &= 0.92m \downarrow \end{aligned}\right\} \frac{D_M}{L} = 2$$



D_M/L ~0.64 ☺

So endburning primarily limited to low thrust motors (and gas generators, igniters, …)

Internal Burning Motor



- So end burning motors not desirable for most high thrust applications
  $\Rightarrow$ internal (port) burning grain designs
- Question
  - given initial grain geometry - how to calculate temporal profiles

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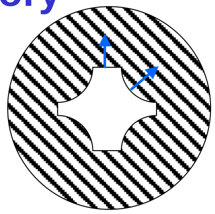
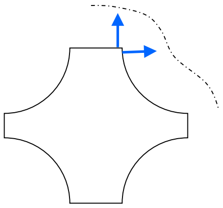
Port Regression History



- Want to know local dx/dt
  - $dx/dt=r=a(p_o)^n$
  - x normal to local surface
- Tends to remove sharp "corners" over time
- Simplest analysis
  - assume p_o uniform in port
  - quasi-steady burning, p_o ~constant over small amount of regression



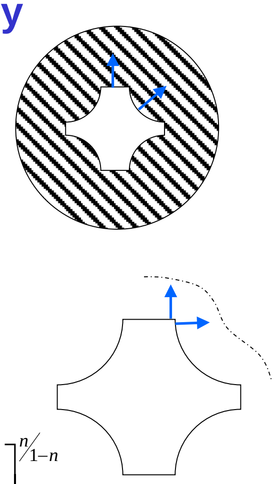
$$\frac{d}{dt}\left(\rho_o V_o\right)=V_o\frac{d\rho_o}{dt}+\rho_o\frac{dV_o}{dt}\approx\rho_o\frac{dV_o}{dt}$$

Regression History

- Then can use

$$p_o \cong \left[aK(\rho_s - \rho_o)c^*\right]^{1/(1-n)}$$

$$\frac{dx}{dt} = r = a\left[a\,\frac{A_b(t)}{A_t}\,\rho_s c^*\right]^{n/(1-n)}$$



  - note units, e.g., if

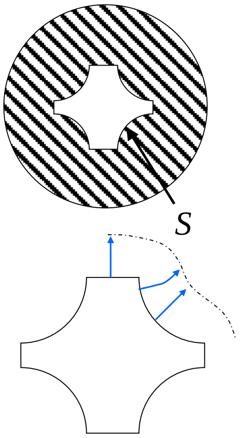
$$r = a\left[p_o(MPa)\right]^n \frac{cm}{s}$$

$$\frac{dx}{dt} = r = a\left(\frac{cm}{s}\right)\left[\frac{a}{100}\left(\frac{m}{s}\right)\frac{A_b(t)}{A_t}\,\rho_s c^*\,\frac{MPa}{10^6\,Pa}\right]^{n/(1-n)}$$

Regression History

$$\frac{dx}{dt} = r = a\left[a\,\frac{A_b(t)}{A_t}\,\rho_s c^*\right]^{n/(1-n)}$$

$$\frac{dx}{dt} = r = a\left[a\,\frac{S(t)L}{A_t}\,\rho_s c^*\right]^{n/(1-n)}$$



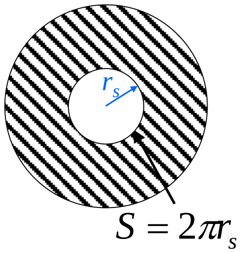
- Goal: find local x(t)
  - integrate along instantaneous normal

$$\int_{x_i}^{x} \frac{dx}{S^{\frac{n}{1-n}}} = \int_0^t a^{\frac{1}{1-n}}\left[\frac{L}{A_t}\,\rho_s c^*\right]^{n/(1-n)} dt$$

Example – Circular Port

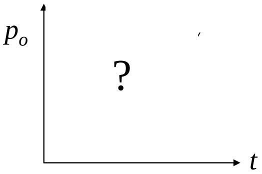
- Integral

$$\int_{r_s(0)}^{r_s(t)} \frac{dr'}{(2\pi r')^{\frac{n}{1-n}}} = \int_0^t \overbrace{a^{\frac{1}{1-n}}\left[\frac{L}{A_t}\rho_s c^*\right]^{n/1-n}}^{constant} dt$$

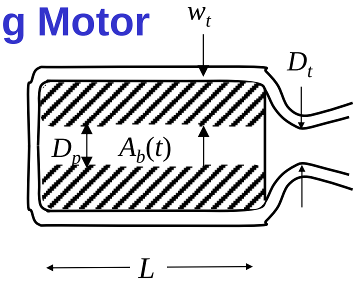


$$\int_{r_s(0)}^{r_s(t)} r'^{\frac{n}{n-1}} dr' = (2\pi)^{\frac{n}{1-n}} a^{\frac{1}{1-n}}\left[\frac{L}{A_t}\rho_s c^*\right]^{n/1-n} t$$

$$p_o \cong \left[a\frac{2\pi r L}{A_t}\rho_s c^*\right]^{1/1-n}$$



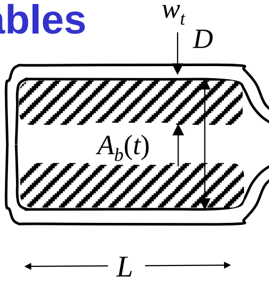
Internal Burning Motor



- **Typical Requirements**
  - $\tau(t)$, Δt_b (or I_{tot}) at given environmental conditions
  - Max. Expected Operating Pressure (MEOP)
- **Design Variables**
  - independent: propellant (r, c^*,…)
  - dependent: nozzle (ε, D_t), grain (L, w_t, D_p,…geometry) $\Rightarrow A_b(t)$
- **Unsteady (dep.) variables**
  - $p_o(t)$, $\dot{m}(t)$, …
- Other versions of design variables?
  - e.g., D_p/D_t (erosive burning)

w_t = web thickness
D_p = port (initial) diameter

Other Design Variables



- Web fraction

$$w_f \equiv \frac{\text{web thickness}}{\text{grain outer diameter}} \qquad w_f = \frac{2r\Delta t_b}{D}$$

- Volume loading fraction

$$V_l \text{ or } \varepsilon_l \equiv \frac{V_{propellant}}{V_{chamber}} \qquad \varepsilon_l = \frac{I_{tot}}{I_{sp} g_o \rho_s V_{chamber}}$$

not including insulation

$$V_{chamber,cyl} = L\,\pi D^2/4$$

- Length to diameter ratio, L/D
  - large value $\Rightarrow$
    - more likely erosive burning
    - tendency for combustion instability (lower freq.)
    - end effects less important
- Sliver fraction $\varepsilon_s \equiv V_{sliver}/V_{propellant}$

sliver≡propellant left over when burn ends

Other Constraints

- Propellant options
- Processing (manufacturing)
- Structural integrity