Solid Rocket Motors

Static Stability, Sensitivity and Axial Variations/Erosive Burning

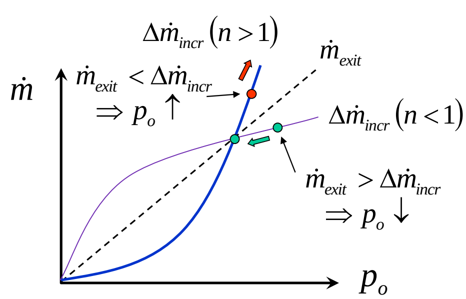
Motor Stability

- Recall mass conservation for steady operation (p_o=constant)

$$\dot{m}_{exit} = \dot{m}_b - \rho_o A_b r = A_b(\rho_s - \rho_o) r = \Delta\dot{m}_{incr}$$

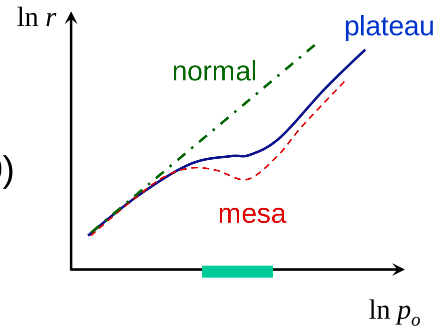
$$\dot{m}_{exit} \propto p_o \qquad \Delta\dot{m}_{incr} \propto p_o^n$$

- Is this condition (point) stable?
  - only if $n \le 1$
  - normally use $0.3<n<0.7$



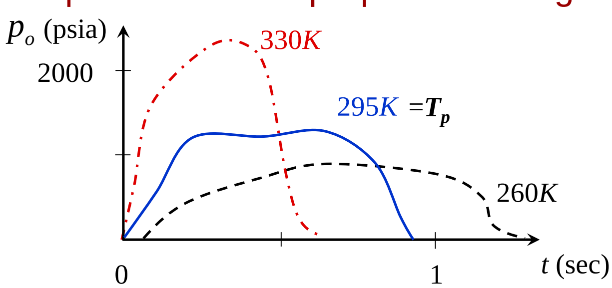
Plateau and Mesa Burning

- **Plateau burning**
  - region of weak p_o dependence ($n\rightarrow 0$) of burn rate
- **Mesa burning**
  - locally negative ($n<0$)
- **Payoff**
  - enhance p_o stability region
  - can keep p_o constant even with changes in K (A_b)



Temperature Sensitivity

- What happens if the initial (storage) temperature of the propellant changes?



- Burn rate and pressure increase
  - shorter duration burn, $\dot{m} \propto p_o$
  - similar total impulse

Temperature Sensitivity

- **Sensitivity of burning rate to T_p** *(at specific p_o)*
  - $\sigma_p \equiv \left.\frac{\partial \ln r}{\partial T_p}\right|_{p_o} = \frac{1}{r}\left.\frac{\partial r}{\partial T_p}\right|_{p_o}$
  - with St. Robert's Law $\sigma_p = \frac{1}{ap_o^n}\left.\frac{\partial a p_o^n}{\partial T_p}\right|_{p_o} = \frac{1}{a}\frac{\partial a}{\partial T_p}$
  - empirically (e.g., strand/ Crawford burner tests)

$$a \approx \frac{A}{T_1 - T_p} \Rightarrow \sigma_p \cong -\frac{1}{T_1 - T_p}$$

A, T_1 empirical constants

Also have r sensitivity to T at fixed K

$$\sigma_K \equiv \left.\frac{\partial \ln r}{\partial T_p}\right|_K$$

want high T_1 or get small range of usable T_p

typically $O(10^{-3}\text{-}10^{-2}\,K^{-1})$

ΔT_p=25K$\Rightarrow\Delta r/r$~few % to 25%

Temperature Sensitivity

- **Sensitivity of pressure to T_p** *(at specific K)*
  - $\pi_K \equiv \left.\frac{\partial \ln p_o}{\partial T_p}\right|_K = \frac{1}{p_o}\left.\frac{\partial p_o}{\partial T_p}\right|_K$
  - with St. Robert's Law

from (VI.5)

$$p_o = \left[aK(\rho_s - \rho_o)c^*\right]^{1/(1-n)}$$

$$\pi_K \cong \frac{1}{1-n}\frac{\partial a/\partial T_p}{a} = \frac{\sigma_p}{1-n}$$

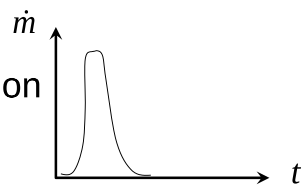
If $0.3 < n < 0.7 \Rightarrow \pi_K \cong 1.4 - 3.3 \times \sigma_p$

p_o more sensitive than r to T_p changes

  - another reason to keep n small (<0.8)
  - usually measure π_K in motor tests
  - additionally, $T_p\uparrow$ can $\Rightarrow$ flow/"slump"; $T_p\downarrow$ (+ cycling) $\Rightarrow$ cracking ($A_b\uparrow$, hot gas at insulation/casing);

Combustion Limits

- If n or p_o too low
  - do not get stable combustion
  - after ignition, propellant soon stops burning (r→0)



- At high p_o
  - possibility of erratic, unpredictable burning (usually > 5000 psi)

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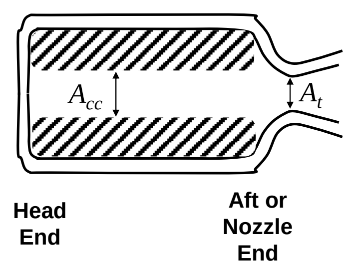
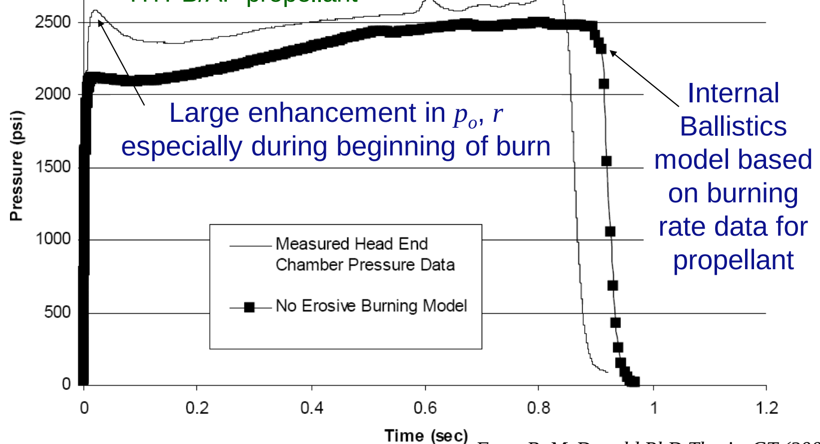
Axial Variations

- So far neglected variable gas properties along bore
- Okay if low Mach number (M) in port, $A_{cc} >> A_t$
- Initially, however, $A_{cc} \sim O(A_t) \Rightarrow M \rightarrow 1$ somewhere in port
- Leads to
  1) erosive burning
  2) axial pressure distribution



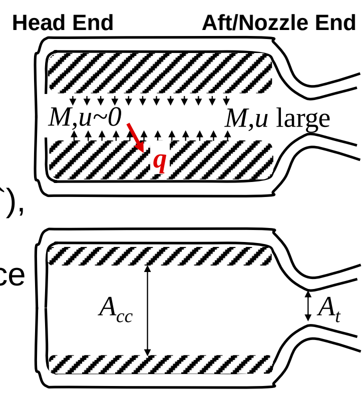
Erosive Burning

- Compact Kinetic Energy Missile
  - HTPB/AP propellant



From B. McDonald PhD Thesis, GT (2004)

Erosive Burning - Cause



Can get similar erosive burning if flow accelerated by vehicle acceleration or spin

- Variation in heat feedback to propellant surface along bore leads to variations in r
- Higher M_∞ (or u_∞)
  - ⇒ thinner boundary layers (∇T↑), and **higher turbulence**
  - ⇒ more heat transfer q to surface
  - – happens **more at aft end** (M, u increase downstream due to mass addition) and **during early times** (A_{cc}/A_t near 1)
  - – still most of q to surface is from heat release of near surface flame

Erosive Burning - Scaling

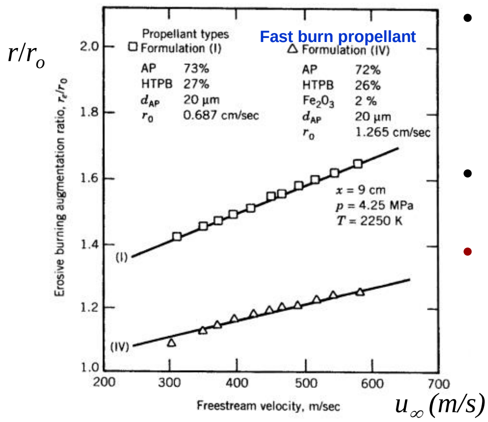


FIGURE 11–10. Effect of gas velocity in the perforation or grain cavity on the erosive burning augmentation factor, which is the burning rate with erosion r divided by the burning rate without erosion r_0. (Reproduced with permission of the AIAA from Chapter 10 of Ref. 11–3.)

- Can approximate effect with additional term to burning rate
  $$r = r_o + r_e$$
- Here 5-80% r increase with erosive burning
- **Reduced effect for higher r_o** (cataylzed) **propellant**
  – less dependent on additional heat xfer

From Sutton

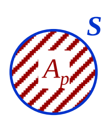
Erosive Burning: Empirical Models

$$r = r_o + r_e$$

- Lenoir-Robillard model (1957) $r_e = \dfrac{\alpha G^{0.8}}{D^{0.2}} e^{-\beta r \rho_s / G}$

$$G \equiv \text{mass flux} = \dot{m}/A = \rho u$$

$$D \equiv \text{port hydraulic diam.} = 4A_p/S$$



$$\alpha, \beta = \text{empirical constants}$$

$$\text{e.g., } \beta \sim 50-55 \qquad \alpha\left(\frac{m}{s}\right) \sim 0.0288 \frac{c_p}{c_s} \frac{\mu(kg/m \cdot s)^{0.2}}{\rho_s(kg/m^3)} Pr^{-2/3} \frac{T_o - T_s}{T_s - T_p}$$

- Green (1954)

$$r_e = r_o k_G \frac{G}{p_o \sqrt{\gamma / RT_o}} \qquad k_G = constant$$

Erosive Burning: Empirical Models (con't)

- Kriedler (1964)

$$r_e = r_o k_K \frac{G}{p_o^{0.485}} \qquad k_K = constant$$

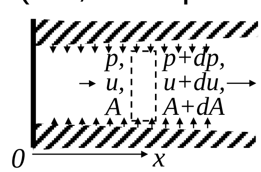
- Exponential "Law"

$$r = r_o p^{(k_e M)} \quad \Rightarrow r_e = r_o \left(1 - p^{(k_e M)}\right) \qquad k_e = constant$$

- These models quasi-predictive (constants may change with propellant and motor)
- Must know how G or M vary along port

Axial Variations

- Even without erosive burning, we can expect property variations along port (i.e., axial position)
- Consider effect of mass addition on $p_o(x)$
- Momentum equation (inviscid)



mass addition

$$-Adp = ud\dot{m} + \dot{m}du \longrightarrow -dp = \frac{d(\dot{m}u)}{A}$$

flow area

Integrate

$$p_{x=0} - p_x = \int_{x=0}^{x} \frac{d(\dot{m}u)}{A}$$

since mass flow rate and velocity both generally increase (RHS+)

pressure must drop downstream – but small variation usually

Constant Area Example

- Assume port area axially uniform

$$p_{x=0} - p_x = \int_{x=0}^{x} \frac{d(\dot{m}u)}{A} \longrightarrow p_x = p_{x=0} - \frac{(\dot{m}u)_x}{A}$$

$\dot{m}/\rho A$

$G = \dot{m}/A$

$$p_x = p_{x=0} - \left(\frac{G^2}{\rho}\right)_x$$

Recall, perimeter

$$G_x = \frac{\rho_s \int_0^x rSdx}{A} = \frac{\rho_s S}{A} a \int_0^x p_x^n dx$$

assuming V change takes longer than flow residence time

must link p(x) to $p_o(t)$ due to $A_b(t)$ and choked throttle constraint

$$\rho_x = \frac{p_x}{RT_o}$$

assuming little change in T except very close to surface