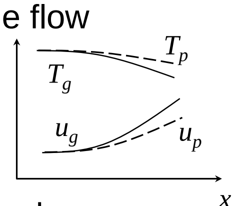
Solid Rocket Motors

Two-Phase Flow in Nozzles

Two Phase Flow in Nozzles

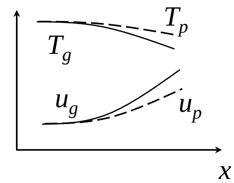
- Common for SRM to have droplets or solid particles (e.g., Al_2O_3 and soot) from propellant flowing through nozzle
  - how does 2-phase flow change T (energy), velocity (momentum) of nozzle flow
- Nozzle = expansion



- What can happen?
  - in accelerating flow, particle has inertia – lags flow velocity change
  - in cooling flow, particle not expanding – temperature drop lags due to thermal inertia

Two Phase Flow in Nozzles

- How do particles “catch up” to gas changes?
- Velocity - drag
  - $u_p \uparrow$, $u_g \downarrow$
- Temperature – heat transfer
  - $T_p \downarrow$, $T_g \uparrow$ $(\Rightarrow u_g \uparrow)$



- Thrust effect

$$\tau \sim \dot{m} u_e = \dot{m}_g u_{e,g} + \dot{m}_p u_{e,p}$$

  - velocity
    - overall velocity lower due to particles
  - mass flow rate
    - overall mass flow rate increased by particles

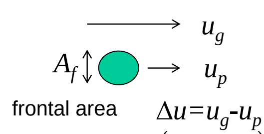
Velocity Lag - Model Approach

- Drag

$$F_D = C_D A_f \frac{1}{2} \rho_g (\Delta u)^2$$

$$= m_p \frac{du_p}{dt}$$

$$\frac{du_p}{dt} = \frac{3}{8} C_D \frac{\rho_g}{\rho_p} \frac{(\Delta u)^2}{r_p}$$



$$m_p = \rho_p V_p = \rho_p \left(4/3 \pi r_p^3\right)$$

spherical

$$A_f = \pi r_p^2$$

- $C_D = ?$
  - small particles $\Rightarrow$ laminar or Stokes flow
  - Stokes $C_D \sim 24/\mathrm{Re}$
  - laminar $C_D \sim \frac{24}{\mathrm{Re}}\left(1 + a\,\mathrm{Re}^b\right)$

Drag Model

- For Stokes flow, need Re<~2

$$\Delta u \sim 10\, m/s$$
$$\rho_g \sim 1-10\, kg/m^3$$
$$\mu_g \sim 5\times10^{-5}\, kg/m/s$$
$$\Rightarrow d_p < 2-20\,\mu m$$

$$\frac{du_p}{dt} = \frac{9}{2}\frac{\mu_g}{\rho_p}\frac{\Delta u}{r_p^2}$$

  – not dependent on gas density
  – acceleration
    - $\propto$ 1 / particle area
    - $\propto$ velocity difference
- For laminar flow
  – weak function of ρ_g, 1/area^n (n<2), $\Delta u^{1.x}$

Particle Velocity Lag in a Nozzle

- For high temperature rocket nozzle flows
  – typical

$$du_g/dt \sim O(10^7\ m/s^2)$$

$$\frac{du_p}{dt} = \frac{9}{2}\frac{\mu_g}{\rho_p}\frac{\Delta u}{r_p^2}$$

- Assuming Stokes flow and

$$\rho_p \sim O\left(10^3\, kg/m^3\right), \mu_g \sim O\left(10^{-4}\, kg/m/s\right)$$

  – need r_p < O(0.1 μm) for no velocity lag
    - so no lag for "smoke" particles
    - large particles (e.g., agglomerates) can have significant velocity lag

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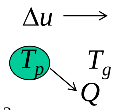
Temperature Lag - Model Approach

- Heat transfer (negl. radiation)



$$\dot{Q} = hA_s\left(T_p - T_g\right)$$

$$= -m_p c \frac{dT_p}{dt}$$

$A_s = 4\pi r_p^2$

$m_p = 4/3\pi r_p^3 \rho_p$

spheres

laminar flow

$$h \sim \frac{k_g}{d_p}\left(2 + a\,\mathrm{Re}^{0.5-0.6}\,\mathrm{Pr}^{0.3-0.4}\right)$$

low Re

$$h \sim k_g / r_p$$

$$\frac{dT_p}{dt} = \frac{-3k_g}{c\rho_p}\frac{\Delta T}{r_p^2}$$

  – particle temperature lag scales with r_p^2

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Particle Temperature Lag in a Nozzle

- For high temperature rocket nozzles
  – typical dT_g/dt ~ O(1000 K / 1 ms)

$$\frac{dT_p}{dt} = \frac{-3k_g}{c\rho_p}\frac{\Delta T}{r_p^2}$$

- Assuming low *Re* flow and

$$k_g \sim \mathrm{O}\left(10^{-1}\,W/mK\right), \rho_p \sim \mathrm{O}\left(10^3\,kg/m^3\right), c \sim \mathrm{O}\left(1\,kJ/kgK\right)$$

$$\Rightarrow 3k_g / c\rho_p \sim (1-5)\times 10^{-7}\,m^2/s$$

  – for small temperature lag
    - for r_p ~ 5 μm, ΔT ~ few hundred K
    - for r_p ~ 0.3 μm, ΔT ~ 1 K
      – *smoke at same temperature as gas*

Specific Impulse

- Can solve flow through nozzle using separate energy and momentum equations for gas and particles
  - include drag and heat transfer exchanges
- Limiting case example (particle size independent)
  - gas: T_o=2780 K, γ=1.2, MW=25
  - particle loading: 10% (by mass), c~2 kJ/kgK
  - nozzle: p_o=50 atm, p_e=1 atm

	No particles	No u lag		∞ u lag	
		No T lag	∞ T lag	No T lag	∞ T lag
Isp (s)	236	228	224	216	213
results from Hill and Peterson	*Particles can't expand*	*No lags – still have "loss" (3%)*	*T lag smaller effect*	*vel. lag has significant effect (9% "loss")*	