Solid Rocket Motors

Two-Phase Flow in Nozzles

- Common for SRM to have droplets or solid particles (e.g., Al₂O₃ and soot) from propellant flowing through nozzle
  - how does 2-phase flow change T (energy), velocity (momentum) of nozzle flow
- Nozzle = expansion
- What can happen?
  - in accelerating flow, particle has inertia – lags flow velocity change
  - in cooling flow, particle not expanding – temperature drop lags due to thermal inertia
Two Phase Flow in Nozzles

- How do particles “catch up” to gas changes?
- Velocity - drag
  - \( u_p \uparrow, u_g \downarrow \)
- Temperature – heat transfer
  - \( T_p \downarrow, T_g \uparrow (\Rightarrow u_g \uparrow) \)
- Thrust effect
  \[ \tau \sim \dot{m}u_e = \dot{m}_g u_{e,g} + \dot{m}_p u_{e,p} \]
  - velocity
    - overall velocity lower due to particles
  - mass flow rate
    - overall mass flow rate increased by particles

Velocity Lag - Model Approach

- Drag
  \[ F_D = C_D A_f \frac{1}{2} \rho_g (\Delta u)^2 \]
  \[ = m_p \frac{d u_p}{d t} \]
  \[ \frac{d u_p}{d t} = \frac{3}{8} C_D \rho_g (\Delta u)^2 \frac{r_p}{\rho_p} \]
  \[ m_p = \rho_p V_p = \rho_p \left( \frac{4}{3} \pi r_p^3 \right) \]
  \[ A_f = \pi r_p^2 \]
- \( C_D = ? \)
  - small particles \( \Rightarrow \) laminar or Stokes flow
  - Stokes \( C_D \sim 24/\text{Re} \)
  - laminar \( C_D \sim \frac{24}{\text{Re}} \left( 1 + a \text{Re}^b \right) \)
Drag Model

- For Stokes flow, need Re<~2
  \[ \frac{du_p}{dt} = \frac{9 \mu_g \Delta u}{2 \rho_p r_p^2} \]
  - not dependent on gas density
  - acceleration
    - \( \propto 1 / \text{particle area} \)
    - \( \propto \text{velocity difference} \)
- For laminar flow
  - weak function of \( \rho_g \), 1/area\(^n\) (n<2), \( \Delta u^{1.2} \)

\[ \Delta u \sim 10 \text{ m/s} \]
\[ \rho_g \sim 1 \times 10^3 \text{ kg/m}^3 \]
\[ \mu_g \sim 5 \times 10^{-3} \text{ kg/m/s} \]
\[ \Rightarrow d_p < 2 - 20 \mu m \]

Particle Velocity Lag in a Nozzle

- For high temperature rocket nozzle flows
  - typical
    \[ \frac{du_g}{dt} \sim O(10^7 \text{ m/s}^2) \]
    \[ \frac{du_p}{dt} = \frac{9 \mu_g \Delta u}{2 \rho_p r_p^2} \]
  - Assuming Stokes flow and
    \[ \rho_p \sim O(10^3 \text{ kg/m}^3), \mu_g \sim O(10^{-4} \text{ kg/m/s}) \]
    - need \( r_p < O(0.1 \mu m) \) for no velocity lag
      - so no lag for “smoke” particles
      - large particles (e.g., agglomerates) can have significant velocity lag
Temperature Lag - Model Approach

• Heat transfer (negl. radiation)

\[ \dot{Q} = hA_s(T_p - T_g) \]

\[ h \approx k_g \left( 2 + a \text{Re}^{0.5-0.6} \text{Pr}^{0.3-0.4} \right) \]

\[ \frac{dT_p}{dt} = -\frac{3k_g}{c\rho_p} \frac{\Delta T}{r_p^2} \]

– particle temperature lag scales with \( r_p^2 \)

Particle Temperature Lag in a Nozzle

• For high temperature rocket nozzles
  – typical \( dT_g/dt \sim O(1000 \text{ K} / 1 \text{ ms}) \)

\[ \frac{dT_p}{dt} = -\frac{3k_g}{c\rho_p} \frac{\Delta T}{r_p^2} \]

• Assuming low Re flow and
  \( k_g \sim O(10^{-1} \text{ W/mK}), \rho_p \sim O(10^3 \text{ kg/m}^3), c \sim O(1 \text{ kJ/kgK}) \)

\[ \Rightarrow 3k_g/c\rho_p \sim (1-5)\times10^{-7} \text{ m}^2/\text{s} \]

– for small temperature lag
  • for \( r_p \sim 5 \mu\text{m}, \Delta T \sim \text{few hundred K} \)
  • for \( r_p \sim 0.3 \mu\text{m}, \Delta T \sim 1 \text{ K} \)
    – smoke at same temperature as gas
Specific Impulse

- Can solve flow through nozzle using separate energy and momentum equations for gas and particles
  - include drag and heat transfer exchanges
- Limiting case example (particle size independent)
  - gas: $T_o=2780$ K, $\gamma=1.2$, $MW=25$
  - particle loading: 10% (by mass), $c\approx 2$ kJ/kgK
  - nozzle: $p_o=50$ atm, $p_e=1$ atm

<table>
<thead>
<tr>
<th>No particles</th>
<th>No u lag</th>
<th>$\infty$ u lag</th>
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<tr>
<td></td>
<td>No T lag</td>
<td>$\infty$ T lag</td>
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<td>Isp (s)</td>
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<td>228</td>
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results from Hill and Peterson

| Particles can’t expand | No lags – still have “loss” (3%) | $T$ lag smaller effect | vel. lag has significant effect (9% “loss”) |