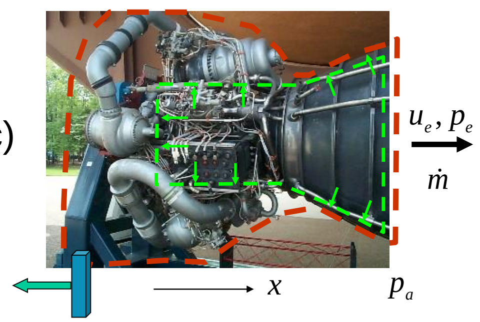
Rocket Propulsion Basics

Thrust and Impulse

Static Thrust

- Consider engine on ground test stand
  - – not moving (static)
  - – steady
  - – quasi 1-d flow



momentum conservation

$$\vec{F}_{\substack{solid\ body\ cross\\ CV\ on\ fluid}} - \int_{free} p\hat{n}dA + \int_{free}\vec{\sigma}_{shear}dA + \int_{CV}\rho\vec{f}dV = \frac{d}{dt}\int_{CV}\rho\vec{u}dV + \int_{CS}\rho\vec{u}\left(\vec{u}_{rel}\cdot\hat{n}\right)dA$$

$$-(-\tau)-(p_e-p_a)A_e+0+0=\frac{d}{dt}\int_{CV}\rho 0dV+\dot{m}u_e$$

$$\tau=\dot{m}u_e+(p_e-p_a)A_e$$

Can also get thrust from force balance (must know press. distribution around inside!! and outside of engine)

Georgia Tech College of Engineering

Equivalent Exhaust Velocity

- Definition

in Europe sometimes equated with I_{sp}

$$u_{eq} = u_e + \frac{(p_e - p_a)A_e}{\dot{m}}$$

sometimes ≡ c (instead of u_{eq})

- combines momentum change and pressure force terms
- written for convenience

$$\tau = \dot{m}u_e + (p_e - p_a)A_e \quad \Rightarrow \tau = \dot{m}u_{eq}$$

- $u_{eq} \leq u_e$ that would get for perfect expansion

Georgia Tech College of Engineering

Impulse

- **Impulse** definition

$$I \equiv \int F dt$$

- substituting equiv. velocity

$$I = \int F dt = \int \tau\, dt = \int \dot{m}u_{eq} dt$$

- assuming *steady* exit conditions

total mass of expelled propellant

$$I = \int \dot{m}u_{eq} dt = u_{eq} \int \dot{m} dt = m_p u_{eq}$$

Specific Impulse

- **Definition** $I_{sp} \equiv I/m_p$

  $= u_{eq}$ ← for steady-state $I = m_p u_{eq}$

  – gives performance of rocket per kg of propellant that rocket has to carry to achieve mission

  – higher I_{sp} means less propellant required
    - thus more payload that can be carried
    - or lighter, smaller rocket can be used

- **Normalization**

  – to get same specific impulse in all major unit systems, typically normalize I_{sp} by Earth's gravitational constant (gravity at Earth's surface)

units of time (seconds)

$$I_{sp} = \left(\frac{I}{m_p}\right)\Big/ g_e = u_{eq}/g_e$$

for steady-state

$g_e \approx 9.81 m/s^2$ **not necessarily *g* where rocket is**

Liquid Bipropellants - Examples

Oxidizer	BP/FP (°C)	Fuel	BP/FP (°C)	Combustor Temperature (K)	Bulk Avg. Density (g/cm^3)	C* (m/s)	Isp (s)
O_2	-183/-218	H_2	-253/-259	3010	0.3	2420	390
O_2		RP-1	~210/-50	3680	1.0	1810	300
O_2		UDMH	63/-58	3600	1.0	1860	310
O_2		NH_3	-33/-78	3080	0.9	1800	295
F_2	-188/-220	H_2		3960	0.5	2560	410*
F_2		Hydrazine	113/1.4	4680	1.3	2210	363*
N_2O_4	21/-12	MMH	86/-53	3390	1.2	1750	288*
N_2O_4		RP-1		3450	1.3	1650	275

Optimum performance; 1000psia (6.94MPa) combustor; p_e=p_a=14.7 psia (1 atm)

UDMH=Unsymmetrical dimethyl hydrazine $(CH_3)_2NNH_2$ **Hydrazine**=N_2H_4

MMH=Monomethyl hydrazine $CH_3NH\text{-}NH_2$ **NH_3**=Ammonia

*__Hypergolic Mixture__ (ignites on contact)

Solid Propellants - Examples

Propellant	Metal (wt %)	Combustion Temperature (K)	Density (g/cm^3)	Isp (s)
Double Base		2530	1.6	230
DB/AP	Al (20)	3870	1.8	265
Polyurethane-AP	Al (20)	3480	1.8	265
PBAN-AP	Al (16)	3480	1.8	263
HTPB-AP		3000	1.8	250
HTPB-AP	Al (17)	3480	1.9	265

Double Base=homogeneous mixture nitroglycerine-nitrocellulose
$C_3H_5(NO_2)_3$-$C_6H_7O_2(NO_2)_3$
AP=Ammonium Perchlorate **PBAN**=Polybutadiene-Acrylic Acid-Acrylonitrile Terpolymer
HTPB=Hydroxy-terminated Polybutadiene

Specific Impulse Limits

- **Chemical rockets**
  - liquid bipropellants typically higher I_{sp} than solids
  - typically 200-400 seconds at sea level operation
    - increases by ~17% for vacuum exhaust (40:1 nozzle expansion area ratio)

    $\Rightarrow I_{sp,max} \leq$ 480 seconds

    $\Rightarrow u_{eq,max} \leq$ 4700 m/s (15,400 ft/s)
  - limited by chemical energy stored in bonds
- **Electric propulsion (rockets)**
  - can have I_{sp} > 1000-3000 seconds

    $\Rightarrow u_{eq,max} \geq$ 10,000-30,000 m/s (33,000-98,000 ft/s)
  - limited by technology used to accelerate mass