Rocket Propulsion Basics

Thrust and Impulse

Static Thrust

- Consider engine on ground test stand
  - not moving (static)
  - steady
  - quasi 1-d flow

momentum conservation

\[
\tau = \dot{m}u_e + (p_e - p_a)A_e
\]

Can also get thrust from force balance (must know press. distribution around inside!) and outside of engine
**Equivalent Exhaust Velocity**

- **Definition**
  
  \[ u_{eq} = u_e + \frac{(p_e - p_a)A_e}{\dot{m}} \]

  - combines momentum change and pressure force terms
  - written for convenience
  
  \[ \tau = \dot{m}u_e + (p_e - p_a)A_e \quad \Rightarrow \quad \tau = \dot{m}u_{eq} \]

  - \( u_{eq} \leq u_e \) that would get for perfect expansion

**Impulse**

- **Impulse** definition
  
  \[ I = \int F dt \]

  - substituting equiv. velocity
    
    \[ I = \int F dt = \int \tau dt = \int \dot{m}u_{eq} dt \]

  - assuming *steady* exit conditions
    
    \[ I = \int \dot{m}u_{eq} dt = u_{eq} \int \dot{m} dt = \dot{m}' u_{eq} \]

  **total mass of expelled propellant**
Specific Impulse

• Definition

\[ I_{sp} = \frac{I}{m_p} \]

for steady-state

\[ I = m_p u_{eq} \]

– gives performance of rocket per kg of propellant that rocket has to carry to achieve mission
– higher \( I_{sp} \) means less propellant required
  • thus more payload that can be carried
  • or lighter, smaller rocket can be used

• Normalization

– to get same specific impulse in all major unit systems, typically normalize \( I_{sp} \) by Earth’s gravitational constant (gravity at Earth’s surface)

\[ I_{sp} = \frac{I_p}{m_p} g_e = \frac{u_{eq}}{g_e} \]

for steady-state

\[ g_e \approx 9.81 \text{ m/s}^2 \]

not necessarily \( g \) where rocket is

Liquid Bipropellants - Examples

<table>
<thead>
<tr>
<th>Oxidizer</th>
<th>BP/FP</th>
<th>Fuel</th>
<th>BP/FP</th>
<th>Combustor Temperature (K)</th>
<th>Bulk Avg. Density (g/cm^3)</th>
<th>C* (m/s)</th>
<th>Isp (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O_2</td>
<td>-183/-218</td>
<td>H_2</td>
<td>-253/-259</td>
<td>3010</td>
<td>0.3</td>
<td>2420</td>
<td>390</td>
</tr>
<tr>
<td>O_2</td>
<td>RP-1</td>
<td>UDMH</td>
<td>63/-58</td>
<td>3600</td>
<td>1.0</td>
<td>1860</td>
<td>310</td>
</tr>
<tr>
<td>O_2</td>
<td></td>
<td>NH_3</td>
<td>-33/-78</td>
<td>3080</td>
<td>0.9</td>
<td>1800</td>
<td>295</td>
</tr>
<tr>
<td>F_2</td>
<td>-188/-220</td>
<td>H_2</td>
<td>3960</td>
<td>0.5</td>
<td>2560</td>
<td>410*</td>
<td></td>
</tr>
<tr>
<td>F_2</td>
<td></td>
<td>Hydrazine</td>
<td>113/1.4</td>
<td>4680</td>
<td>1.3</td>
<td>2210</td>
<td>363*</td>
</tr>
<tr>
<td>N_2O_4</td>
<td>21/-12</td>
<td>MMH</td>
<td>86/-53</td>
<td>3390</td>
<td>1.2</td>
<td>1750</td>
<td>288*</td>
</tr>
<tr>
<td>N_2O_4</td>
<td></td>
<td>RP-1</td>
<td></td>
<td>3450</td>
<td>1.3</td>
<td>1650</td>
<td>275</td>
</tr>
</tbody>
</table>

Optimum performance; 1000psia (6.94MPa) combustor; \( p_e=p_a=14.7 \) psia (1 atm)

UDMH=Unsymmetrical dimethyl hydrazine (CH_3)NNH_2

Hydrazine=N_2H_4

MMH=Monomethyl hydrazine CH_3NH-NH_2

NH_3=Ammonia

*Hypergolic Mixture (ignites on contact)
Solid Propellants - Examples

<table>
<thead>
<tr>
<th>Propellant</th>
<th>Metal (wt %)</th>
<th>Combustion Temperature (K)</th>
<th>Density (g/cm³)</th>
<th>Isp (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Double Base</td>
<td></td>
<td>2530</td>
<td>1.6</td>
<td>230</td>
</tr>
<tr>
<td>DB/AP</td>
<td>Al (20)</td>
<td>3870</td>
<td>1.8</td>
<td>265</td>
</tr>
<tr>
<td>Polyurethane-AP</td>
<td>Al (20)</td>
<td>3480</td>
<td>1.8</td>
<td>265</td>
</tr>
<tr>
<td>PBAN-AP</td>
<td>Al (16)</td>
<td>3480</td>
<td>1.8</td>
<td>263</td>
</tr>
<tr>
<td>HTPB-AP</td>
<td></td>
<td>3000</td>
<td>1.8</td>
<td>250</td>
</tr>
<tr>
<td>HTPB-AP</td>
<td>Al (17)</td>
<td>3480</td>
<td>1.9</td>
<td>265</td>
</tr>
</tbody>
</table>

Double Base = homogeneous mixture nitroglycerine-nitrocellulose

C₃H₅(NO₂)₃-C₆H₇O₂(NO₂)₃

AP = Ammonium Perchlorate
PBAN = Polybutadiene-Acrylic Acid-Acrylonitrile Terpolymer

HTPB = Hydroxy-terminated Polybutadiene

Specific Impulse Limits

- **Chemical rockets**
  - liquid bipropellants typically higher $I_{sp}$ than solids
  - typically 200-400 seconds at sea level operation
    - increases by ~17% for vacuum exhaust
      - (40:1 nozzle expansion area ratio)
    - $I_{sp,max} \leq 480$ seconds
    - $u_{eq,max} \leq 4700$ m/s (15,400 ft/s)
  - limited by chemical energy stored in bonds

- **Electric propulsion (rockets)**
  - can have $I_{sp} > 1000$-3000 seconds
    - $u_{eq,max} \geq 10,000$-30,000 m/s (33,000-98,000 ft/s)
  - limited by technology used to accelerate mass