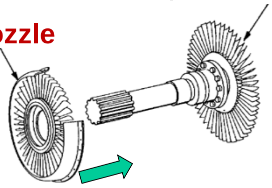
Turbomachinery for LRE

Turbines

Axial Turbine Analysis

- From Euler turbomachinery (conservation) equations need to understand change in tangential velocity to relate to forces on blades and power



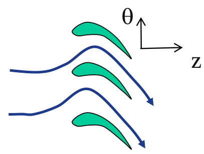
$$\mathrm{T} = \dot{m}\left[\left(rc_\theta\right)_e - \left(rc_\theta\right)_i\right]$$

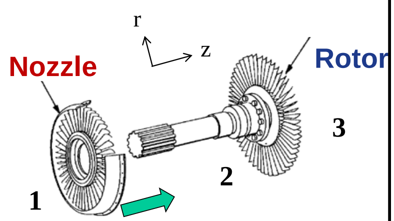
$$\dot{W} = \dot{m}\left[\left(uc_\theta\right)_e - \left(uc_\theta\right)_i\right]$$

- Analyze flow in a plane normal to rotational axis (cascade plane) to find c_θ

Cascade Analysis

- You may have previously analyzed flow over a “blade” (airfoil)
  - but in blade’s reference frame
- Here there are moving (e.g. rotor) and stationary blades
  - e.g., for turbine
    1→2 nozzle (stator)
    2→3 rotor
- Use velocity triangles to switch between frames





Mechanics and Thermodynamics of Propulsion, Hill and Peterson

Velocity Triangles

- Two reference frames to use for fluid velocity
  - engine’s $\vec{c}$
  - blade’s $\vec{w}$
- Difference due to blade motion

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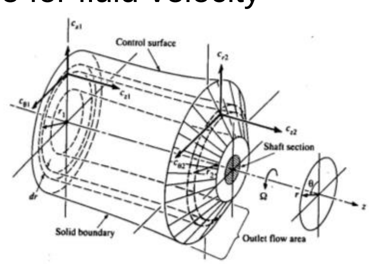
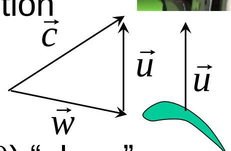
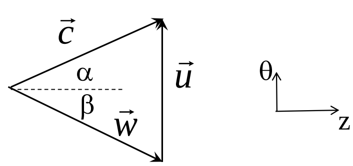


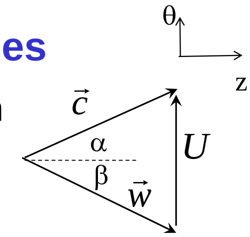
$$\vec{c} = \vec{w} + \vec{u}$$



- In 2-d (z,θ) “plane”
  - u is in θ direction
  - define angles (α,β) for each ref. frame



Rotor Velocity Triangles



- Blade moves in θ direction, and in (z,θ) plane, for fixed r, let $u_i=U$

$$\Rightarrow w_{z_i} = c_{z_i} \quad (1) \qquad w_{\theta_i} + c_{\theta_i} = U \quad (2)$$

- Also have general geometric relations

$$\vec{c} = \vec{w} + \vec{u}$$

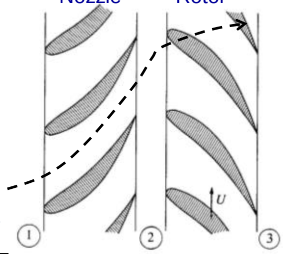
  – e.g., $c_{\theta_i} = c_i \sin\alpha_i = c_{z_i} \tan\alpha_i$

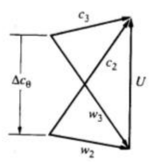
- Therefore

$$\boxed{c_{\theta_2} = c_{z_2} \tan\alpha_2}$$

$$(1,2) \Rightarrow c_{\theta_3} = U - w_{\theta_3}$$

$$\boxed{c_{\theta_3} = U - c_{z_3} \tan\beta_3}$$





Mechanics and Thermodynamics of Propulsion, Hill and Peterson

Single-Stage Characteristics (Axial Turbine)

- Goal - determine how turbine performance, e.g., Pr_T, affected by changes in operating conditions
- Start by analyzing single-stage turbine
- **Rotor** (2→3)
  – Euler $\qquad \dot{W}_R = \dot{m}\left(u_3 c_{\theta_3} - u_2 c_{\theta_2}\right) = \dot{m}\left(h_{o3} - h_{o2}\right)$

  – at fixed radial location $\qquad \dot{W}_R = \dot{m}U\left(c_{\theta_3} - c_{\theta_2}\right) = \dot{m}\left(h_{o3} - h_{o2}\right)$

$$U\Delta c_{\theta_{2,3}} = \Delta h_{o_{2,3}}$$

- **Nozzle** (1→2) $\qquad \dot{W}_S = 0 \Rightarrow h_{o2} = h_{o1}$ *while no work, there is still torque on stationary blades*
- So for stage $\quad \Delta h_{o_{1,3}} = \Delta h_{o_{2,3}} = U\Delta c_{\theta\,2,3}$

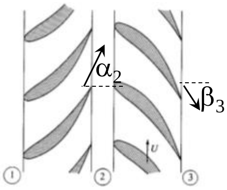
Axial Turbine Stage

- Combining results

$$\Delta h_{o_{1,3}} = \Delta h_{o_{2,3}} = U \Delta c_{\theta_{2,3}} \qquad c_{\theta_2} = c_{z_2} \tan\alpha_2 \qquad c_{\theta_3} = U - c_{z_3} \tan\beta_3$$

$$\Delta h_{o,stage} = U\left(U - c_{z_3} \tan\beta_3 - c_{z_2} \tan\alpha_2\right)$$

– assuming constant axial velocity



ψ, stage loading coeff. → $\dfrac{\Delta h_{o,stage}}{U^2} = 1 - \dfrac{c_z}{U}\left(\tan\alpha_2 + \tan\beta_3\right)$ ← **Φ, flow coefficient**

α_2 related to nozzle trailing edge angle

β_3 related to rotor trailing edge angle

IF flow attached (no separation)

$$\frac{\dot{W}_{T,produced}}{A_{inlet}} \approx \rho_{inlet} c_z U^2 \left[\frac{c_z}{U}\left(\tan\alpha_2 + \tan\beta_3\right) - 1\right]$$

***High output power*:**

1) *high flow* (c_z)
2) *high U* (*rpm, radius*)
3) *high* α_2 (*max* <90°)
4) *high* β_2 (*large rev. turn*)

Turbine Stage Pressure Ratio

- For adiabatic turbine with TPG/CPG

<0 for turbine

$$Pr_T = \frac{p_{o1}}{p_{03}} = \left[1 + \frac{1}{\eta_{st}}\left(\frac{T_{o3} - T_{o1}}{T_{o1}}\right)\right]^{\gamma/1-\gamma} = \left[1 + \frac{1}{\eta_{st}} \frac{(\gamma-1)U^2}{\gamma R T_{o1}} \frac{\Delta h_{o_{1,3}}}{U^2}\right]^{\gamma/1-\gamma}$$

>1 as written

$$\frac{\Delta h_{o_{1,3}}}{U^2} = \frac{\Delta c_{\theta_{2,3}}}{U}$$

$$\frac{p_{o1}}{p_{o3}} = \left[1 + \frac{\gamma-1}{\eta_{st}} \underbrace{\frac{U^2}{\gamma R T_{o1}}}_{M^2_{blade}} \underbrace{\frac{\Delta c_{\theta_{2,3}}}{U}}_{\psi}\right]^{\gamma/1-\gamma}$$

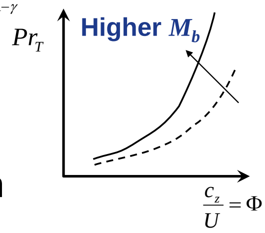
- Stage pressure ratio depends on
  1. ψ= f(**U= rΩ**, **$\Delta c_{\theta 2,3}$**)
  2. blade Mach number M_{blade}=f($r\Omega$, **T_{o1}**)
  3. **η_{st}**

Turbine Characteristics

- For given M_b, blade design, Ω, η_T

$$Pr_T = \left[1+\frac{\gamma-1}{\eta_{st}} M_b^2\left\{1-\frac{c_z}{U}\left(\tan\alpha_2+\tan\beta_3\right)\right\}\right]^{\gamma/1-\gamma}$$

$$Pr_T = \left[C_1 - C_2\frac{c_z}{U}\right]^{\gamma/1-\gamma}$$



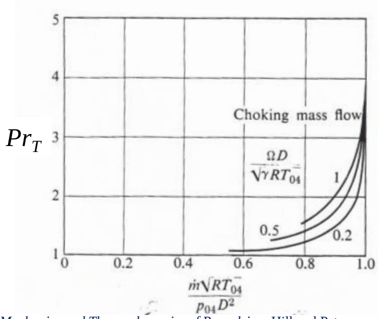
- As increase flowrate through turbine (at fixed rpm), larger pressure drop (more expansion) is produced
  - more work extracted per unit mass

Axial Turbine Maps

- Typically presented as separate curves for each rpm (M_b)
- x-axis - replace flow coefficient with corrected mass flow rate, recall

$$\dot{m} \propto A\, p_o \Big/ \sqrt{RT_o}$$

  - at high corrected mass flowrate, nozzle becomes choked
- Peak efficiency around design point

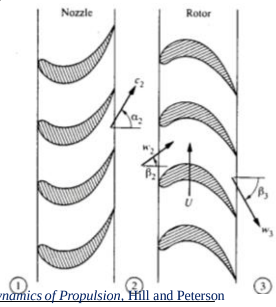


Mechanics and Thermodynamics of Propulsion, Hill and Peterson

Blade Design: Degree of Reaction

- We have TWO blade parameters to design $\frac{\Delta c_{\theta_{2,3}}}{U} = 1 - \frac{c_z}{U}\left(\tan\alpha_2 + \tan\beta_3\right)$
  - rotor trailing edge (match β_3)
  - nozzle trailing edge (match α_2)
- How to do this?
  1. Degree of reaction, ***R***
  2. Stage exit condition constraint (α_3)

$$c_{\theta_2} = c_z \tan\alpha_2 \qquad w_{\theta_2} = U - c_{\theta_2} = c_z \tan\beta_2$$
$$c_{\theta_3} = c_z \tan\alpha_3 \qquad w_{\theta_3} = U - c_{\theta_3} = c_z \tan\beta_3$$
$$\Delta c_{\theta_{2,3}} = -\Delta w_{\theta_{2,3}}$$



Mechanics and Thermodynamics of Propulsion, Hill and Peterson

Degree of Reaction

- Recall $R \equiv \Delta h_{rotor} / \Delta h_{stage}$
  - allows us to distribute load (static pressure change) between rotor and nozzle (or stator)
  - how to relate static enthalpy change to azimuthal velocity changes?
    - **ΔKE** !! $h_o = h + \mathrm{v}^2/2$
  - for stationary blade, no work done

$$\Delta h_o = 0 \Rightarrow \Delta h = -\Delta KE$$

- e.g., nozzle blade **if c_z constant, and negligible c_r**

$$h_2 - h_1 = \left(c_{z_1}^2 + c_{\theta_2}^2\right)/2 - \left(c_{z_2}^2 + c_{\theta_2}^2\right)/2 = \left(c_{\theta_1}^2 - c_{\theta_2}^2\right)/2$$

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Degree of Reaction (Turbine)

- Rotor blades??
  - are "stationary" in rotor's reference frame

$$h_3 - h_2 = \left(w_{\theta_2}^2 - w_{\theta_3}^2\right)/2$$

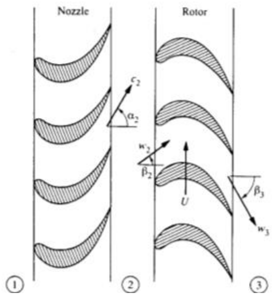
- Reaction

$$R = \frac{h_3 - h_2}{h_3 - h_1} = \frac{h_3 - h_2}{\left(h_{o3} - c_3^2\right) + \left(h_{o1} - c_1^2\right)}$$

if $c_1 \approx c_3$

$$\cong \frac{h_3 - h_2}{h_{o3} - h_{o1}} = \frac{w_{\theta_2}^2 - w_{\theta_3}^2}{2U\left(c_{\theta_3} - c_{\theta_2}\right)}$$

$$c_{z_3} = c_{z_2} = c_{z_1}$$



$$\Delta c_{\theta_{2,3}} = U - c_z\left(\tan\beta_3 + \tan\alpha_2\right)$$

R relates design blade angles to azimuthal KE change

$$w_{\theta_2}^2 - w_{\theta_3}^2 = \left(w_{\theta_2} + w_{\theta_3}\right)\left(w_{\theta_2} - w_{\theta_3}\right) = \left(w_{\theta_2} + w_{\theta_3}\right)\Delta c_{\theta_{2,3}}$$

$$R = \frac{w_{\theta_2} + w_{\theta_3}}{2U}$$

$$\Delta c_{\theta_{2,3}} = -\Delta w_{\theta_{2,3}}$$

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Impulse Turbine

$$R = \frac{\Delta h_{23}}{\Delta h_{13}} = \frac{w_{\theta_2} + w_{\theta_3}}{2U}$$

- $R = 0$
  - all the pressure change occurs across the nozzle, or the nozzle creates high KE

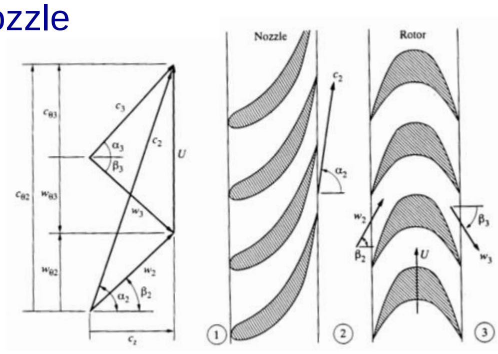
$$\Rightarrow w_{\theta_3} = -w_{\theta_2}$$

$$c_z \tan\beta_3 = -c_z \tan\beta_2$$

$$\Rightarrow \beta_3 = -\beta_2 \qquad \Delta w_{\theta_{2,3}} = -2w_{\theta_2}$$

$$\frac{\Delta c_{\theta_{2,3}}}{U} = \frac{-\Delta w_{\theta_{2,3}}}{U} = \frac{2\left(U - c_z \tan\alpha_2\right)}{U}$$

$$\frac{\Delta c_{\theta_{2,3}}}{U} = 2\left(1 - \frac{c_z}{U}\tan\alpha_2\right)$$



Mechanics and Thermodynamics of Propulsion, Hill and Peterson

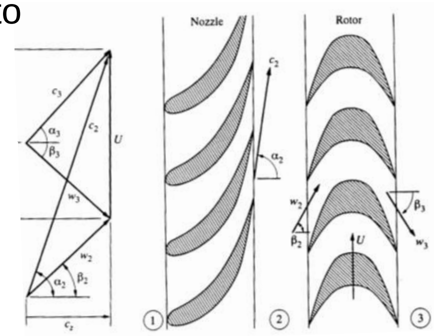
Impulse Turbine

- So for impulse turbine, blade loading coeff. $$\frac{\Delta h_{o\,stage}}{U^2} = \frac{\Delta c_{\theta_{23}}}{U} = 2\left(1 - \frac{c_z}{U}\tan\alpha_2\right)$$
- Relates blade loading to nozzle exit angle

$$\tan\alpha_2 = \frac{U}{c_z}\left(1 - \frac{\overbrace{\Delta h_{o\,stage}}^{<0}}{2U^2}\right)$$

- From previous & velocity triangles, rotor angles given by

$$\tan\beta_3 = -\tan\beta_2 = \tan\alpha_2 - \frac{U}{c_z}$$



Mechanics and Thermodynamics of Propulsion, Hill and Peterson

Impulse Turbine

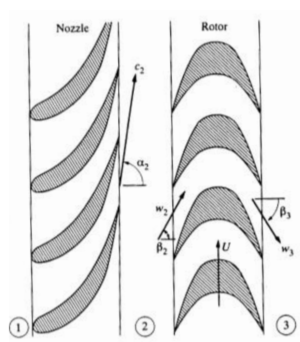
$$\frac{\Delta c_{\theta_{23}}}{U} = 2\left(1 - \overbrace{\frac{c_z}{U}\tan\alpha_2}^{=c_{\theta_2}/U}\right)$$

- To let largest power per unit mass flow rate ⇒ large α_2
  - tends to produce high velocities and p_o losses
  - practical limit, ~70-75°
- Further possible constraint
  - no exit swirl (⇒$c_{\theta 3}$=0)

$$\Delta c_{\theta_{2,3}} = c_{\theta_3} - c_{\theta_2} \;\Rightarrow \Delta c_{\theta_{2,3}} = -c_{\theta_2}$$

$$\Rightarrow \frac{\Delta c_{\theta_{23}}}{U} = 2\left(1 + \frac{\Delta c_{\theta_{23}}}{U}\right) \Rightarrow \boxed{\frac{\Delta h_{o_{stage}}}{U^2} = -2}$$

$$\Rightarrow \tan\alpha_2 = 2\left(U/c_z\right),\;\; \tan\beta_3 = U/c_z$$



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50% Reaction Turbine

- $R = 0.5$
  - balanced p drop across stage

less convergence in nozzle vs impulse turbine $R = \frac{w_{\theta_2} + w_{\theta_3}}{2U}$

$$\frac{\Delta c_{\theta_{2,3}}}{U} = 1 - \frac{c_z}{U}\left(\tan\beta_3 + \tan\alpha_2\right)$$

$$U = w_{\theta_2} + w_{\theta_3}$$
$$= \left(U - c_z \tan\alpha_2\right) + c_z \tan\beta_3$$

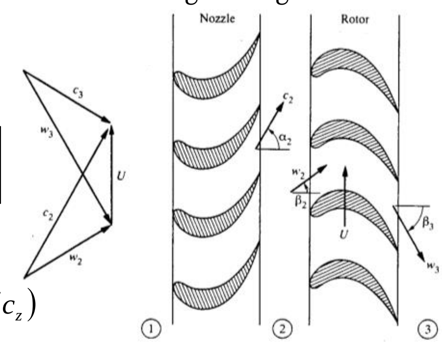
$$\Rightarrow \tan\beta_3 = \tan\alpha_2$$

$$\therefore \frac{\Delta h_{o\,stage}}{U^2} = \frac{\Delta c_{\theta_{2,3}}}{U} = \left(1 - 2\frac{c_z}{U}\tan\alpha_2\right)$$

- if no exit swirl

$$\Delta c_{\theta_{2,3}} = -c_{\theta_2}$$

$$\Rightarrow \frac{\Delta h_{o_{stage}}}{U^2} = -1 \quad \Rightarrow \alpha_2 = \beta_2 = \tan^{-1}\left(U/c_z\right)$$



half loading of impulse: less power/stage

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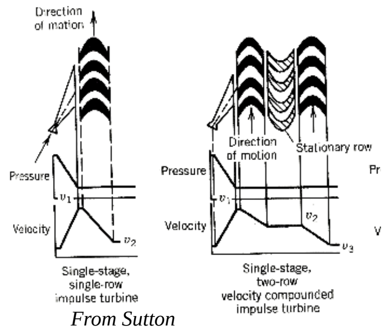
Rocket Turbines

- Can combine results for no exit swirl condition to show
  $$\Delta h_{o_{stage}} / U^2 = 2(1 - R)$$
  - as reaction decreases, power per stage increases
- To minimize size/weight, rocket turbopumps often employ impulse or low reaction turbines
  - but efficiencies typically lower (<70%) for impulse turbines compared to higher reaction turbines (~90%)
- Can improve efficiency by decreasing flow coefficient (c_z/U)
  - for given flowrate, requires higher blade speed, RPM
  - higher RPM = higher stresses = heavier, and larger gear ratio if geared to pump

Velocity-Compound Impulse Turbine

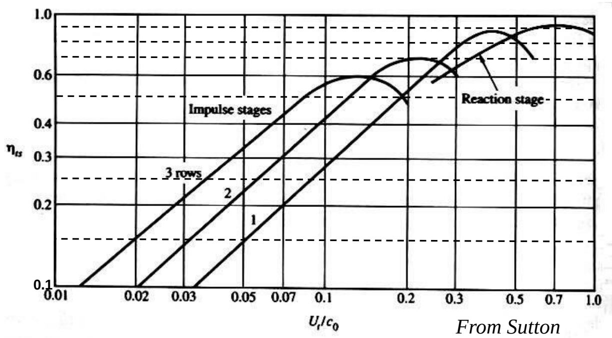
- Can increase stage power even more using velocity-compounding
  - multiple nozzle/rotors in series
- Example, two-row compounded impulse turbine
  - all Δp in 1st nozzle
  - 1st rotor exits with high swirl (so large α_2 allowed)
  - 2nd nozzle redirects flow without Δp
  - 2nd rotor extracts more work and reduces swirl
  - stage loading is 4x that of single-row impulse stage



From Sutton

Highly-Loaded Turbine Efficiencies

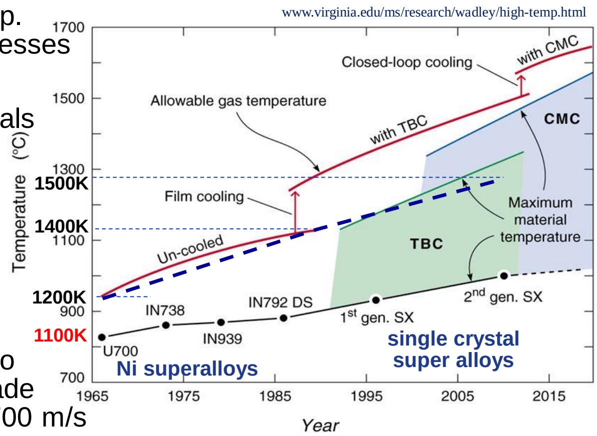
- Can provide lower or improved efficiency improvement over single row impulse stage
  - still lower than high reaction turbines



From Sutton

Turbine Inlet Temperature Limits

- Maximum inlet temp. limited by blade stresses
- Advances
  - higher T materials (superalloys)
  - coatings (TBC) and blade cooling, *not typical for rockets*
- Rocket turbine T_{max} historically limited to 900-1100K with blade tip speeds of 400-700 m/s
  - potential for increases to 1400-1500 K with better materials



Turbine Design Example

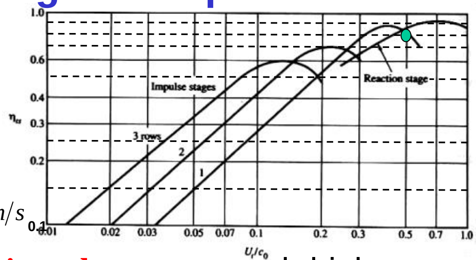
- Consider preliminary design requirements for gas-generator cycle LRE turbine
  - power/flow: 19.4 MW, 41.8 kg/s
  - gas properties: γ=1.15, MW=27.7
  - inlet: 1000K, 44 bar
  - outlet: 790K

$$T_{o,e} = T_{o,in} - \frac{\dot{W}}{\dot{m}c_p}$$

- Constraints
  - max tip speed 550 m/s *(would be more realistic to constrain blade-root stress)*
  - assume geared so rpm not fixed by pump rpm
  - assume zero swirl at exit, constant axial vel.

Power	19.4MW
Flowrate	41.8kg/s
γ	1.15
MW	27.7
$T_{o,in}$	1000K
$p_{o,in}$	44bar
$T_{o,exit}$	790K

Turbine Design Example



- Step 1: Turbine type
  - estimate U/c_o, c_o=theoretical gas spouting vel.

$$c_o = \sqrt{2(h_{oi} - h_{es})} \approx \sqrt{2\dot{W}/\dot{m}} \approx 1000\,m/s$$

suggests nozzle will be supersonic

  - $U/c_o \approx 0.5 \Rightarrow$ **single impulse stage**, much higher η than 2 row compounded, less stages than reaction turbine

$$\Rightarrow \eta_T \approx 0.75 \Rightarrow p_{o,e} \approx 3.8 bar, \rho_e \approx 1.6\frac{kg}{m^3} \qquad Pr_T = \left[1 + \frac{1}{\eta_{st}}\left(\frac{T_{oe} - T_{oi}}{T_{oi}}\right)\right]^{\gamma/1-\gamma}$$

$$\rho_e \cong p_{oe}/RT_{oe}$$

- Step 2: blade angles
  - use max α_2=70° $\Rightarrow \tan\beta_3 = \frac{1}{2}\tan\alpha_2 \Rightarrow \beta_3 = 53.9^\circ$

β_2=-53.9°

Power	19.4MW
Flowrate	41.8kg/s
γ	1.15
MW	27.7
$T_{o,in}$	1000K
$p_{o,in}$	44bar
$T_{o,exit}$	790K

Turbine Design Example

- Step 3: sizing

for zero-swirl impulse turbine

$$\alpha_2 = 70^\circ \Rightarrow \quad c_z/U_m = \frac{2}{\tan\alpha_2} = 0.728$$

flow coefficient (Φ) typical turbine values 0.5-1.5

$$U_m^2 = \frac{\Delta h_{o_{stage}}}{-2} \Rightarrow U_m = \sqrt{\frac{19MW/41.8\,kg/s}{2}} = 487\,m/s$$

$$\Rightarrow c_z = 354\,m/s \qquad M_e \sim 0.6\text{-}0.7$$

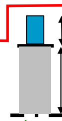
$$r_m^2 \equiv \frac{r_{tip}^2 + r_{root}^2}{2} \Rightarrow r_{root}^2 = 2r_m^2 - r_{tip}^2$$

$$A_e = \pi\left(r_{tip}^2 - r_{root}^2\right) = \pi\left(r_{tip}^2 - \left(2r_m^2 - r_{tip}^2\right)\right) = 2\pi r_m^2\left(r_{tip}^2/r_m^2 - 1\right)$$

$$r_m^2 = \frac{A_e}{2\pi\left(r_{tip}^2/r_m^2 - 1\right)}$$

$$A_e = \frac{\dot{m}}{\rho_e c_z} = \frac{41.8\,kg/s}{\left(1.58\,kg/m^3\right)354\,m/s} = 0.075m^2$$

$$r_m = \sqrt{\frac{A_e}{2\pi\left(1.13^2 - 1\right)}} = 0.207m$$



structure is mostly disk

root-tip ratio of 0.75

$$\frac{r_{tip}}{r_m} = \frac{U_{tip}}{U_m} = \frac{550}{487} = 1.13 \Rightarrow \frac{r_{root}}{r_m} = 0.85$$

$$\Omega = \frac{U}{r_m} = 2350\frac{rad}{s} \qquad N = 22{,}400 rpm$$