Overview

- Goals of this section are:
  1. examine influence of non-1D behavior on flames
  2. define flame stretch and stretched flames
  3. study dependence of flame properties, especially flame speed on flame stretch, and the influence of Lewis number(s)
  4. describe various methods used to experimentally measure flame speed
Idealized Premixed Flame

- So far we have focused on the simplest possible premixed flame configuration
  - 1-dimensional, planar, adiabatic (including no radiation), steady, no buoyancy
  - product temperature \( T_b = T_{ad} \)
  - mass burning flux
    \[ \rho_u u_u \equiv \rho_b S_L^{u} = \rho_b u_b \]
  - Controlling processes
    - diffusion: thermal and mass
    - reaction rates and exothermicity
- So solution, e.g., \( S_L \), is fundamental property of fuel/oxidizer mixture for given initial state
  - reactant composition
  - reactant temperature and pressure

Non-Ideal Premixed Flames

- Real flames are typically
  - non-adiabatic
  - non-uniform and 3-dimensional
  - unsteady
  - occur within gravity field
- In general,
  - \( u_u = S_L \neq S_L^{o} \)
  - equivalently, mass burning flux \( \neq \rho_b S_L^{o} \)
  - \( S_L \) can depend on velocity field and flame geometry
- Following focus is non-1d (unsteady?) flames

rendering of 1200K isotherm from DNS of a turbulent \( \phi=0.37 \) H\(_2\) flame colored by local H\(_2\) consumption rate.
**Stretched Flames: Basics**

- Flames subject to aerodynamic non-uniformity and/or unsteady effects are called **Stretched Flames**
- Flames can be distorted, wrinkled or displaced by non-uniform or unsteady flow fields - called **Hydrodynamic Stretch**
- Stretched flames respond to non-uniform or unsteady flow fields they encounter by modification of the temperature and/or species profiles in the diffusive zone, which can change the flame propagation rate - called **Flame Stretch**
- **Hydrodynamic stretch and flame stretch are inherently coupled concepts!**

**Stretched Flames: Physical Perspective**

- Flame stretch associated with misalignment between convective and diffusive fluxes in flame
- Can impact many flame properties – propagation speed, shape, stability and can explain interesting phenomena observed in non-ideal flames

*Note: stretch effects would not be observed in flames that are infinitely thin – no chance for diffusion to alter streamtube*
Example: Bunsen Flames Tips

- Non-ideal behavior observed for luminosity at tips of Bunsen flames for different fuel-air mixtures (fuel type and equivalence ratio)

<table>
<thead>
<tr>
<th>Fuel Type</th>
<th>Equivalence Ratio</th>
<th>Tip Appearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propane(C₃H₈)</td>
<td>1.38</td>
<td>Reduced</td>
</tr>
<tr>
<td></td>
<td>0.53</td>
<td>Enhanced</td>
</tr>
<tr>
<td>Methane(CH₄)</td>
<td>1.52</td>
<td>Enhanced</td>
</tr>
<tr>
<td></td>
<td>0.58</td>
<td>Reduced</td>
</tr>
</tbody>
</table>


Tip Opening

- Example: propane flames with increasing rich $\phi$, tip weakens until flame locally extinguished (“opens”)

<table>
<thead>
<tr>
<th>Equivalence Ratio</th>
<th>Tip Appearance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td>1.32</td>
<td></td>
</tr>
<tr>
<td>1.41</td>
<td>Tip Open</td>
</tr>
<tr>
<td>1.43</td>
<td>Tip Open</td>
</tr>
<tr>
<td>1.47</td>
<td>Tip Open</td>
</tr>
</tbody>
</table>

Bunsen Tip Flame Temperature Data

- Stretch can cause enhancement or reduction in flame temperature
  - compares measured $T_{\text{tip}}$ to calculated $T_{\text{ad}}$
  - CH$_4$ vs. C$_2$H$_4$ vs. C$_3$H$_8$
    - "Leaner" reduced, reduced, enhanced
    - "Richer" enhanced, reduced, reduced
    - $\phi(T_{\text{max}})$ shifts rich, no shift, shifts lean

Trend? ref: C.K. Law, Combustion Physics

Stretched Flames: Geometry

- Stretch effects exist because of coupling between flame surfaces and flow fields
- Assuming the flame is thin, smooth and continuous we can treat the flame as a 3-D surface
  - flame surface moves, deforms, and accelerates/decelerates
    - really a family of surfaces
    - $\vec{n} = \vec{n}(\vec{x}, t)$
Flame Stretch Rate Definition

- Consider simple case of propagating flame
- Flame stretch rate can be defined as normalized rate of change of flame surface area element

\[
\kappa \equiv \frac{1}{A} \frac{dA}{dt}
\]

- a Lagrangian quantity
- units of flame stretch are \( s^{-1} \), i.e., an inverse time scale or rate

\( \text{Flame Stretch Speed Measure} \)

Influence of Flame Stretch

- Two basic cases
  - positive stretch (flame in tension)
  - negative stretch (flame in compression)
- Diffusion for neg. stretch
  - thermal diffusion (\( \alpha \)) focuses heat into reactants, enhances reaction rate and thus \( S_L \)
  - mass diffusion of reactants away from (and products to) center streamline, can reduce [reactants], and thus reaction rate and \( S_L \)

\( \text{AE/ME 6766 Combustion} \)
**Stretch Rate (κ) Sign Convention**

- Given previous sign convention for \( \kappa = \frac{1}{A} \frac{dA}{dt} \)
  - can relate sign of \( \kappa \) to direction from which heat flux vectors cross the sidewalls of a differential flame tube (control volume) immediately adjacent to flame sheet
  - heat flux into sides of control volume \( \Rightarrow \) negative stretch rate
  - heat flux out of sides of control volume \( \Rightarrow \) positive stretch rate

**Le Effects on Stretched Flames**

- So we have the following diffusivities of interest
  - thermal diffusivity \( (\alpha) \) of mixture
  - mass diffusivity \( (D) \) in general, but recall each species has different \( D_{ij} \) (multi-component)
    - so for reactants
      - \( D_fj \) (or \( D_{f,M} \)) of fuel
      - \( D_{ox,j} \) (or \( D_{ox,M} \)) of oxygen
  - Two effects arise because of non-equidiffusivities
    - non-unity \( Le \)
      - imbalance between thermal and mass diffusivity
      - differential diffusion
      - imbalance between species diffusivities, e.g., fuel and oxygen
    - If all \( Le_i = 1 \), diffusive effects tend to counterbalance

\[
\frac{1}{A} \frac{dA}{dt} = \frac{\alpha}{D}
\]
Non-Equidiffusive Effects

- Flame response to stretch effects is strongest in flames with Lewis numbers much different from one
- Flame response in stretched flames depends on sign of stretch rate and Lewis number
  - for fixed (premixed) mixture, flame response (e.g., \( dS_L/\alpha \)) will change entirely when stretch rate changes sign (e.g., positive to negative)
  - for fixed stretch rate \( \kappa \), flame response will change entirely when \( Le \) transitions from greater than to less than some critical value
- Theory predicts and experiments confirm that equidiffusive \((Le=1)\) stretched flames are insensitive to the magnitude of stretch

Stretch Effects: Bunsen Flames

- Stationary, axisymmetric flame
- Stretch (negative) induced by flame curvature
  - assuming uniform velocity profile exiting the tube
- Diffusive fluxes do not align with convective fluxes
- Since negative stretch here
  - if thermal conduction outweighs mass diffusion then flame speed enhanced \((\alpha \text{ vs. } D)\) \( S_L \uparrow \text{ for } Le > 1 \)
  - similarly \( S_L \downarrow \text{ for } Le < 1 \)
  - but also must consider differential diffusion
Reexamine Bunsen Flame Tip Results

- Fuel lighter than air (e.g., CH$_4$)
  - in “preheat” zone
    \[ \tilde{W}_f < \tilde{W}_{\text{mix}} < \tilde{W}_{\text{ox}} \]
  - assuming molec. wt. dominates $D_{ij}$
    \[ D_{f,M} > \alpha_{\text{mix}} > D_{\text{ox,M}} \]
    \[ L_{f} < 1, \quad L_{\text{ox}} > 1 \]
  - so thermal vs “overall” mass diffusion will have some influence, but differential diffusion will tend to make mixture leaner near tip
    \[ \phi < \phi_{\text{crit}} \Rightarrow T_{\text{tip}} \text{ and local } S_{L} \downarrow \text{ flame weakened} \]
    \[ \phi > \phi_{\text{crit}} \Rightarrow T_{\text{tip}} \text{ and local } S_{L} \uparrow \text{ flame enhanced} \]

- Fuel heavier than air (e.g., C$_3$H$_8$)
  - in “preheat” zone
    \[ \tilde{W}_f > \tilde{W}_{\text{ox}} > \tilde{W}_{\text{mix}} \]
  - assuming molec. wt. dominates $D_{ij}$
    \[ D_{f,M} < D_{\text{ox,M}} \sim \alpha_{\text{mix}} \]
    \[ L_{f} > 1, \quad L_{\text{ox}} \sim 1 \]
  - now differential diffusion will tend to make reactant mixture richer near tip
    \[ \phi > \phi_{\text{crit}} \Rightarrow T_{\text{tip}} \text{ and local } S_{L} \downarrow \text{ flame weakened} \]
    \[ \phi < \phi_{\text{crit}} \Rightarrow T_{\text{tip}} \text{ and local } S_{L} \uparrow \text{ flame enhanced} \]
Tip Opening Propane Flames

- As we increase $\phi_{burner}$, diffusion effects at tip lead to conditions that no longer can sustain a flame, e.g., too rich
- Richer propane mixture has more stretch sensitivity

$\phi = 1.07$  \hspace{1cm} 1.17  \hspace{1cm} 1.32  \hspace{1cm} max$^{\text{stretch location at tip}}$

$\phi = 1.41$  \hspace{1cm} 1.43  \hspace{1cm} 1.47$

Stretch Effects: Stagnation Flames

- Example of stretch in flat flame
  - stagnation flame (positively stretched)
  - no flame curvature, BUT flow (velocity) field diverging
    - reacting flow is decelerating, strained
  - $\kappa = a$ (strain rate) near centerline
  - diffusion again not aligned with flow

- Ways to produce this flowfield
  - premixed reactants impinging on a (stagnation) body
  - premixed reactants impinging on flow moving in opposite direction (opposed jets)
Stagnation Flame Example Results

- \( T_u = 300 \text{ K} \)
  - light and heavy fuels have opposite sign slope (sensitivity) at \( \phi = 0.6 \)
  - ethene (MW=28) reduced sensitivity
  - opposite sensitivity for propane at \( \phi = 0.6 \) and 1.4

- \( T_u = 650 \text{ K} \)
  - sensitivity to stretch significantly reduced
  - sign change of sensitivity for propane \( \phi = 1.4 \)

S. Adusumilli, Effects of preheat temperature and vitiation on reaction kinetics of higher hydrocarbon fuels, 2019.

---

Stretch Effects: Unsteady Spherical Flames

- In general for “thin” flame, stretch is given by

\[
\kappa = \frac{\partial u}{\partial \alpha_1} + \frac{\partial u}{\partial \alpha_2} + \nabla_{\phi} \cdot \nabla \cdot (\nabla \cdot \nabla) = \nabla_{\phi} \cdot u_{\phi} + \nabla_{\phi} \cdot (\nabla \cdot \nabla)
\]

- \( v_F \) = local propagation velocity of flame surface
- \( \alpha \) = tangent to flame surface
- \( \kappa_a \) = stretch due to tangential velocity gradients at flame surface
- \( \kappa_b \) = stretch due to unsteady flame curvature

- For spherical flame
  - flame curved, but flow velocities align with flame surface normal; no tangential velocities
  - stationary spherical flame would not be stretched
  - for unsteady spherical flame, stretch rate changes as function of radius, \( r = r(t) \);
  - \( \kappa \) decreases with time for outwardly expanding flame
Flame Speed Corrections

- For stretched flames, need to modify 1-d flame speed to account for curvature, flow divergence effects
- **For small perturbations** (low stretch rates), asymptotic analysis (Markstein) leads to the following flame speed scaling

\[
S_L = S_L^0 - \ell \cdot \kappa = S_L^0 - Ma \cdot \delta_f \cdot \kappa
\]

(VII.16) \hspace{1cm} \text{Markstein Length} \hspace{1cm} \text{Markstein number}

- Can be expressed in terms of a non-dimensional stretch rate called a Karlovitz number* (or stretch factor)

\[
Ka \equiv \frac{\delta_f}{S_L^0} = \frac{1}{\kappa} \cdot \frac{\delta_f}{\ell} = \frac{1}{\ell} \cdot \frac{\delta_f}{\delta_f}
\]

(VII.17)

- residence time within 1d unstretched laminar flame
- characteristic time for flame stretching

\[
\tau_{1-d} = \frac{\delta_f}{S_L^0} = \frac{1}{\kappa} \cdot \frac{\delta_f}{S_L^0}
\]

(VII.18)

- and VII.16 becomes

\[
S_L = S_L^0 - Ma \cdot Ka \cdot S_L \Rightarrow \frac{S_L^0}{S_L} = 1 + Ma \cdot Ka
\]

(VII.16a)

*other Karlovitz number definitions used in turbulent combustion

Karlovitz Number Scaling

\[
\frac{S_L^0}{S_L} = 1 + Ma \cdot Ka
\]

- Flame speeds of propane-air mixtures for different \(Ka\)
- nearly linear scaling

\[
\Rightarrow Ma = Ma(\phi)
\]

after Tseng et al., *Combust. and Flame* 95, 410 (1993)
### Markstein Number $\phi$ Dependence

- What is $Ma = Ma(\phi)$ relation?
- Simplest is linear correlation
  \[ Ma = S \times (\phi - \phi_{crit}) \]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Air/Fuel} & S & \phi_{crit} & \phi & 0 & \text{max} \\
\hline
\text{C}_3\text{H}_8 & -8.8 & 1.44 & 0.8-1.8 & 0.25 \\
\text{C}_2\text{H}_6 & -4 & 1.68 & 0.8-1.6 & 0.25 \\
\text{C}_2\text{H}_4 & -2.9 & 1.95 & 0.8-1.8 & 0.24 \\
\text{H}_2 & 3.9 & 1.5 & 1-4.8 & 0.21 \\
\text{CH}_4 & 10.2 & 0.74 & 0.6-1.4 & 0.30 \\
\hline
\end{array}
\]

- Propane-Air, $1 \text{ atm}$

\[
\begin{align*}
\frac{S_L^0}{S_L} = 1 + MaK_a
\end{align*}
\]

- Correlation $\phi_{crit} = -8.8(\phi-1.44)$

- After Tseng et al., Combust. and Flame 95, 410 (1993)

### Flame Speed Measurements

- How does one experimentally measure flame speeds?
- Measurements of $S_L^0$ are difficult
  – hard to achieve adiabatic and 1-d conditions
  – usually compromise and try to correct
- Various methods
  – Bunsen-type burners (“simplest”)
  – traveling tube
  – spherically expanding flames
  – soap bubbles
  – flat flame burners
  – stagnation flames
Bunsen (Tube) Burner Method

- Premixed fuel/air in cylindrical tube
  - laminar
  - various velocity profiles
  - stationary flame when normal component of local approach velocity = $S_L$

- Approaches
  - measure $\alpha$, $u$ (or $u_n$)
    - $u$ not necessarily same as $u_{exit}$
    - $u$, $\alpha$ may not be constant (depends on exit profile)
  - measure flame area, reactant volumetric flowrate

\[ \dot{m} = \rho u A \Rightarrow S_L = \dot{m}_{exit} / \left( \rho u A_{flame} \right) \]

- how to identify flame surface?
  - schlieren, shadow, luminosity, …?

ref: Sébastien Ducruix

CH$_4$/air $\phi=1.05$

Bunsen Method: Flame Surface

- Flame surface measurement
  - luminous, schlieren, shadow give different \( A_{\text{flame}} \)
- Shadow closest to unburned region
  - but not 1-d flame
  - stretch effects \( \Rightarrow S_L \neq S_0 \)
- Luminosity corresponds to high \( T_{\text{rxn}} \) zone

Fig. 6. Comparison of flame luminosity contours and temperature gradient contours (percentage of maximum). Red indicates the maximum positive value, while blue indicates values below 25% of maximum. Temperature gradient values correspond to the data shown in Fig. 3.

ref: Ibarreta and Sung, 3rd Joint US Section Meeting Comb. Instit.

Bunsen Method: Burned Surface Area

- Measure reaction zone area (\( A_b \))
  - from flame luminosity
- Unstretched laminar flame speed (\( S_L^{o} \))

\[
S_L^{o} = \frac{\rho_b S_b^{o}}{\rho_a} = \frac{\dot{m}/A_{b}}{\rho_a} = \frac{\dot{m}}{\rho_a} A_b
\]

- flame curvature effect is small for \( S_b \)
- flame strain primarily restricted to tip
  - use tall flames

ref: Kochar, Seitzman

AE/ME 6766 Combustion
Propagating Methods

- **Transparent Tube**
  - fill tube with premix gases, ignite, watch flame propagate (make sure it is sustained propagation)
  - gas moves ahead of flame front (compressed)
    - $u_f > S_L$, and gases may be slightly preheated
    - non-1d (boundary layer), buoyancy
- **Spherical Flames**
  - fill closed (?) (spherical?) chamber, ignite and watch flame propagate
  - unburned velocity (and $p,T_u$?) continuously increasing
  - expanding (unsteady) flame experiences stretch ($\kappa(t) \downarrow$), extrapolate back to zero stretch

$$u_f = \frac{dR_f}{dt}$$

Flat Flame Burner

- Construct uniform velocity profile using porous plate, sintered metals, or small flow passages
- Burner is either water cooled or naturally cooled
- Flame can be essentially flat except at edges of burner
- Measurement of flame velocity easy ($u_{exit}$)
- **Nonadiabatic**
  - measure cooling, extrapolate to $Q=0$
  - or measure $T$ profile through flame and compare with computation (works for low pressures, thick flames)
Stagnation Flames

- Opposed jets or jet and stagnation plate

- Nearly flat flames (no curvature), but with aerodynamic “strain” due to deceleration caused by stagnation plane
  - extrapolate to zero-strain to obtain $S_L^o$

ref: J. Natarajan PhD thesis 9 mm

AE/ME 6766 Combustion