Premixed Flames: Flame Stretch and Flame Speed Measurements

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Methane Flame

Idealized Premixed Flame

- Simplest possible premixed flame configuration
  - 1-dimensional, planar
  - adiabatic
- \( T_b^o = T_{ad} \rightarrow \) Adiabatic Flame Temperature
- \( \rho_u u = \rho_u S_{L}^o = \rho_b u_b \rightarrow \) Mass Burning Flux
- What are the controlling parameters?
  - thermal and mass diffusivities
  - reaction rates
  - temperature of reactants
  - pressure
  - exothermicity of fuel/oxidizer
- \( S_{L}^o \rightarrow \) Fundamental property of fuel/oxidizer mixture
Non-Ideal Premixed Flames

- Non-adiabatic
- 3-dimensional
- Non-uniform and/or unsteady flow
- In general,
  - \( S_u \neq S_L^0 \)
  - Mass burning flux \( \neq \rho u S_L^0 \)

Rendering of 1200K isotherm from DNS of a turbulent \( \Phi = 0.37 \) H2 flame colored by local H2 consumption rate.


Stretched Flames: Basics

- Flames subject to aerodynamic non-uniformity and/or unsteady effects are called Stretched Flames
- Flames can be distorted, wrinkled or displaced by non-uniform or unsteady flow fields
  - called Hydrodynamic Stretch
- Stretched flames respond to non-uniform or unsteady flow fields they encounter by modification of the temperature and/or species profiles in the diffusive zone, changing flame propagation rate
  - called Flame Stretch
- Hydrodynamic stretch and flame stretch are inherently coupled concepts!
Stretched Flames: Physical Perspective

- Stretch causes a misalignment between convective and diffusive fluxes near the flame
- Stretched flames are not infinitely thin (otherwise kinematic restoration would dominate)
- Stretch effects help to explain a wide variety of interesting and important physics not observed in premixed 1D, planar, adiabatic laminar flames:
  - Flame speed
  - Flame shape
  - Flame stability

\[ S_L \neq S_L^0 \]
\[ \dot{m}^{\infty}_{D,\text{planar,adiabatic}} \neq \dot{m}^* \]

Tips of Bunsen Flames

- Propane (heavier than air) and methane (lighter)

Propane \((C_3H_8)\)  Methane \((CH_4)\)

\[ d=10\text{mm} \]
\[ \phi = 1.38 \quad 0.53 \quad 1.52 \quad 0.58 \]

Tip Opening

- Propane flames of increasing richness

\[ \phi = 1.07 \quad 1.17 \quad 1.32 \]

\[ 1.41 \quad 1.43 \quad 1.47 \]


Bunsen Tip Flame Temperature Data

- Curvature/stretch can cause enhancement or reduction in flame temperature
  - comparison of measured \( T_{\text{tip}} \) to \( T_{\text{ad}} \)
    - \( \text{CH}_4 \) vs. \( \text{C}_2\text{H}_4 \) vs. \( \text{C}_3\text{H}_8 \)

ref: C.K. Law, Combustion Physics
Stretched Flames: Geometry

- Stretch effects exist because of coupling between flame surfaces and flow fields.
- Real flames are 3-D unsteady deflagration waves.
- Assuming the flame is thin, smooth and continuous we can treat the flame as a 3-D surface.

\[ \vec{n} \equiv \vec{n}(\vec{x}, t) \]

### Flame Stretch Rate Definition

- Consider simple case of propagating flame.
- Flame stretch rate can be defined as normalized rate of change of flame surface area element.

\[ \kappa \equiv \frac{1}{A} \frac{dA}{dt} \quad \text{Williams (1975)} \]

- Lagrangian quantity.
- Units of flame stretch are 1/s, i.e. an inverse time scale.
Influence of Flame Stretch

- Two basic cases
  - positive stretch
    (flame in tension)
  - negative stretch
    (flame in compression)

- thermal diffusion ($\alpha$) “focuses” into reactants, enhances $S_L$ (same for mass diffusion of radicals)
- mass diffusion of reactants away from centerline, can reduce [reactants], reaction rate and thus $S_L$
- differential diffusion can lead to less stoichiometric mixture, greater diffusional loss of deficient reactant, reduces $S_L$
- $Le=1$, opposing effects tend to cancel

Stretched Flames: Geometry

$\bar{v}_F (\bar{x}, t) =$ local propagation velocity of the flame surface
$\bar{u}_u (\bar{x}, t) =$ unburned flow velocity
$\bar{u}_b (\bar{x}, t) =$ burned flow velocity
$\bar{n} (\bar{x}, t) =$ flame surface normal vector
$\bar{t}_1 =$ flame surface tangent vector
$\bar{t}_2 =$ second flame surface tangent vector

Flame surface moves, deforms, and accelerates/decelerates, as a function of space and time
**Sign Conventions**

- $\kappa \rightarrow$ Normalized time rate of change of flame surface area
- Given previous sign convention for stretch rate can interpret based on direction from which heat flux vectors cross the sidewalls of a differential control volume immediately adjacent to flame sheet
  - Heat flux into the sides of the control volume $\rightarrow$ negative stretch rate
  - Heat flux out of the sides of the control volume $\rightarrow$ positive stretch rate

\[
\kappa = \frac{1}{A} \frac{dA}{dt}
\]

**Flame Surface Propagation – G Eqn.**

- Level set approach arises very naturally for the purpose of defining the flame surface in thin, smooth, and continuous premixed flames
- Describe the surface of the flame as,

\[
G(\tilde{x}, t) = 0 \quad \tilde{n} = -\frac{\nabla G}{|\nabla G|}
\]

- Set $G > 0$ in products and $G < 0$ in reactants
- At flame

\[
\frac{dG}{dt} = \frac{\partial G}{\partial t} + \vec{v}_F \cdot \nabla G = 0
\]
Stretched Flame Analysis

• Full expression in terms of flame surface variables,

\[
\kappa = \frac{\partial u}{\partial t_1} + \frac{\partial u}{\partial t_2} + (\vec{v}_F \cdot \vec{n})(\nabla \cdot \vec{n}) = \nabla \cdot \mathbf{u} + (\vec{v}_F \cdot \vec{n})(\nabla \cdot \vec{n})
\]

\[
\kappa_a (\mathbf{u}_F + \nabla \cdot \vec{n}) = \kappa_b (\mathbf{u}_F)
\]

• \(\kappa_a\) - stretch due to tangential velocity gradients at the flame surface

• \(\kappa_b\) - stretch due to unsteady flame curvature

• Alternative expression,

\[
\vec{n} = \vec{n}(\vec{x},t)
\]

\[
S = \frac{1}{2}(\nabla \cdot \vec{u} + \nabla \cdot \vec{u}^*)
\]

\[
\kappa = -\vec{n} \cdot \nabla \vec{u} + \nabla \cdot \vec{u} - S \nabla \cdot \vec{n} = -\vec{n} \cdot S + \vec{n} + \nabla \cdot \vec{u} - S (\nabla \cdot \vec{n})
\]

• \(\kappa_s\) - stretch due to flow non-uniformities

• \(\kappa_{curv}\) - stretch due to a curved flame in a uniform approach flow

Stretched Flame Analysis

• If we consider stationary flames,

\[
\vec{v}_F = 0 \rightarrow \kappa = \nabla \cdot \mathbf{u} = -\vec{n} \times \nabla \cdot (\vec{n} \times \vec{u})
\]

since \(\mathbf{u} = \vec{n} \times (\vec{u} \times \vec{n})\)
Le Effects on Stretched Flames

- Each reactant has different $D_{ij}$ (multi-component mixture)
  - “most deficient” reactant and/or reactant with largest gradient is generally the controlling parameter
    - fuel if lean, oxygen if rich
- Three diffusivities of interest:
  - thermal diffusivity, $\alpha$
  - deficient reactant diffusivity, $D_i$
  - abundant reactant diffusivity, $D_j$
- Two effects arise because of differences in diffusivities
  - non-unity Le: imbalance between thermal and mass diffusivity
  - differential diffusion: imbalance between deficient and abundant species diffusivities

\[
Le_{\text{Non-unity}} = \frac{\alpha}{D}
\]

Non-Equidiffusive Effects

- Flame response to stretch effects is strongest in flames with Lewis numbers much different from one
- Flame response in stretched flames depends on the sign of the stretch rate and the Lewis number
  - for fixed premixed mixture, flame response will change entirely when stretch rate changes sign from positive to negative
  - for fixed stretch rate $\kappa$, flame response will change entirely when $Le$ transitions from greater than to less than a critical value ($Le_{\text{crit}} \approx 1$ for flame temperature)
- Theory predicts and experiments confirm that equidiffusive stretched flames are insensitive to the magnitude of stretch
**Stretch Effects in Bunsen Flames**

- Stationary, axisymmetric flame
- Curved flame with negative stretch
- Assume uniform velocity profile exiting the tube
- Diffusive fluxes do not align with convective fluxes
- For negative stretch case
  - if thermal conduction outweighs mass diffusion then flame speed enhanced ($\alpha$ vs. $D$)
    - so $S_L \uparrow$ for $Le > 1$
    - similarly $S_L \downarrow$ for $Le < 1$

**Non-unity Le and Negative Stretch**

- Consider Bunsen flames
  - tip of flame=negative stretch
- **Fuel lighter than air** (e.g., CH$_4$)
  - $MW_{fuel} < MW_{mix}$
  - for $\phi<1$, $Le_{deficient}=\alpha_{mix}/D_{fuel}$
    - drops as $\phi \downarrow$
    - $S_L \downarrow$ for leaner mixtures
  - tip weak (open) for sufficiently lean flames
  - enhanced for rich flames

Rearranging the equation for $Le_{deficient}$:

$$Le_{deficient} = \alpha_{mix}/D_{fuel}$$

For $Le_{deficient} < 1$, the flame speed decreases with decreasing $\phi$. For $Le_{deficient} > 1$, the flame speed increases with decreasing $\phi$. For $Le_{deficient} = 1$, the flame speed is constant with respect to $\phi$. This is consistent with the observation in the slide that $S_L \uparrow$ for $Le > 1$ and $S_L \downarrow$ for $Le < 1$.
Non-unity $Le$ and Flame Stretch

- For fuel heavier than air (e.g., $C_3H_8$)
  - $MW_{fuel} > MW_{mix}$
  - $Le_{deficient} \uparrow$ as $\phi \downarrow$
    $\Rightarrow S_L \uparrow$ for lean mixtures
  - tip enhanced for sufficiently lean flames
  - tip weak (open) for rich flames

Propane($C_3H_8$) $1.38$ $0.53$

Tip Opening

- Propane flames, richer=more stretch sensitivity

$\phi = 1.07$ $1.17$ $1.32$
$1.41$ $1.43$ $1.47$

**Stretch Effects: Opposed Flow Flames**

- Flow induced stretch
  - example: **stagnation flame** (positively stretched)
  - diffusion not aligned with flow
- Examples
  - premixed reactants impinging on a flat wall
  - premixed reactant impinging on hot products

![Diagram of Flame Stretch](image1)

**Stretch Effects: Unsteady Spherical Flames**

- Stretch rate changes as a function of radius
- Curvature is present but flow velocities align with flame surface normal
- Stationary spherical flame would be stretchless

\[
\kappa = \frac{\partial u_{jl}}{\partial t_1} + \frac{\partial u_{lr}}{\partial t_2} + (\vec{v}_F \cdot \vec{n})(\nabla \cdot \vec{n}) = \nabla \cdot \vec{u} + (\vec{v}_F \cdot \vec{n})(\nabla \cdot \vec{n})
\]

\[
\kappa_\lambda = \frac{\kappa}{\kappa_\nu}
\]

*Combustion Physics* by C.K. Law
(Cambridge University Press, 2006)
Flame Speed Corrections

- So for stretched flames, need to modify 1d flame speed to account for curvature, flow divergence
- *For small perturbations* (low amounts of stretch), asymptotic analysis (Markstein) leads to the following for the apparent flame speed

\[ S_L = S_L^o - \ell \kappa = S_L^o - Ma \delta_f \kappa \]

Markstein Length  Markstein number

\[ Ma \equiv \ell / \delta_f \]

- Sometimes expressed in terms of Karlovitz number

\[ Ka \equiv \frac{\text{residence time for crossing unstretched flame}}{\text{characteristic time for flame stretching}} = \frac{\delta_f / S_L^o}{1 / \kappa} = \frac{\delta_f \kappa}{S_L} \]

\[ S_L = S_L^o - Ma Ka S_L \quad \Rightarrow \quad S_L^o / S_L = 1 + Ma Ka \]
**Karlovitz Dependence**

\[ \frac{S_L^o}{S_L} = 1 + MaKa \]

- Flame speeds for propane-air mixtures for different \( Ka \)
- \( Ma = Ma(\phi) \)

*after Tseng et al., Combust. and Flame 95, 410 (1993)*

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**Markstein Number**

- Saw before \( Ma = Ma(\phi) \)
- Try simple linear correlation \( Ma = S(\phi - \phi_n) \)

<table>
<thead>
<tr>
<th>Air/Fuel</th>
<th>( S )</th>
<th>( \phi_n )</th>
<th>( \phi )</th>
<th>( Ka_{max} )</th>
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</thead>
<tbody>
<tr>
<td>( C_3H_8 )</td>
<td>-8.8</td>
<td>1.44</td>
<td>0.8-1.8</td>
<td>0.25</td>
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<tr>
<td>( C_2H_6 )</td>
<td>-4</td>
<td>1.68</td>
<td>0.8-1.6</td>
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<tr>
<td>( C_2H_4 )</td>
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<td>1.95</td>
<td>0.8-1.8</td>
<td>0.24</td>
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<tr>
<td>( H_2 )</td>
<td>3.9</td>
<td>1.5</td>
<td>1.4-8</td>
<td>0.21</td>
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<tr>
<td>( CH_4 )</td>
<td>10.2</td>
<td>0.74</td>
<td>0.6-1.4</td>
<td>0.30</td>
</tr>
</tbody>
</table>

*after Tseng et al., Combust. and Flame 95, 410 (1993)*
Flame Speed Measurements

- Measurements of "$S_L$" are difficult
  - hard to achieve adiabatic and 1-d conditions
  - usually compromise and try to correct
- Various methods
  - Bunsen-type burners ("simplest")
  - traveling tube
  - spherical "bombs"
  - soap bubbles
  - flat flame burners
  - stagnation flames

Bunsen (Tube) Burner Method

- Premixed fuel/air in cylindrical tube
  - laminar
  - various velocity profiles
  - stationary flame when normal component of local approach velocity = $S_L$

ref: Sébastien Ducruix

$CH_4/air \quad \phi = 1.05$
Bunsen (Tube) Burner Method

- Approaches
  - measure $\alpha, u$ (or $u_n$)
    - $u$ not necessarily same as $u_{exit}$
    - $u, \alpha$ may not be constant (depends on exit profile)
  - measure flame area
    - $m = \rho u A \Rightarrow S_L = \frac{m_{exit}}{\rho u A_{flame}}$
- how to identify flame surface
  - schlieren, shadow, luminosity, etc.


Bunsen Method: Flame Surface

- Flame surface measurement
  - luminous, schlieren, shadow give different $A_{flame}$
- Shadow closest to unburned region
  - but not 1-d flame
  - stretch effects
    - $\Rightarrow S_L \neq S_{\phi}$
- Luminosity corresponds to high $T_{rxn}$ zone

ref: Ibarreta and Sung, 3rd Joint US Section Meeting Comb. Instit.

Fig. 6. Comparison of flame luminosity contours and temperature gradient contours (percentage of maximum). Red indicates the maximum positive value, while blue indicates values below 25% of maximum. Temperature gradient values correspond to the data shown in Fig. 3.
Bunsen Method: Burned Surface Area

- Measure reaction zone area ($A_b$)
  - from flame luminosity
- Unstretched laminar flame speed ($S_L^\infty$)
  \[
  S_L^\infty = \frac{\rho_b S_b^\infty}{\rho_u} \approx \frac{\rho_b S_b}{\rho_u} = \frac{\dot{m}/A_b}{\rho_u} = \frac{\dot{Q}}{A_b}
  \]
  - flame curvature effect is small for $S_b$
  - flame strain primarily restricted to tip
    - use tall flames

Propagating Methods

- **Transparent Tube**
  - fill tube with premix gases, ignite, watch flame propagate (make sure it is sustained propagation)
  - gas moves ahead of flame front (compressed)
    - $u_f > S_L$, and gases may be slightly preheated
    - non-1d (boundary layer), buoyancy
- **Spherical Bomb**
  - fill closed spherical chamber, watch flame propagate
  - unburned velocity ($p, T_u$) continuously increasing
  - not 1-d (planar), affected by curvature and strain, must extrapolate back to zero stretch

Chemiluminescence image

ref: Koch, Seitzman

AE/ME 6766 Combustion
**Flat Flame Burner**

- Construct uniform velocity profile using porous plate, sintered metals, or small flow passages
- Burner is either water cooled or naturally cooled
- Flame can be essentially flat except at edges of burner
- Measurement of flame velocity easy ($u_{exit}$)
  - Nonadiabatic
    - measure cooling, extrapolate to $Q=0$
    - or $T$ profile and compare with computation; okay for low pressures
  
  ref: Glassman

**Stagnation Flames**

- Opposed jets or jet and stagnation plate
- Nearly flat flames (no curvature), but with aerodynamic “strain” due to deceleration caused by stagnation plane
  - extrapolate to zero-strain to obtain $S_L^o$
  
  ref: J. Natarajan PhD thesis