Georgia Tech College of Engineering
School of Aerospace Engineering

Laminar Nonpremixed Combustion: Laminar (Round) Jet Flames

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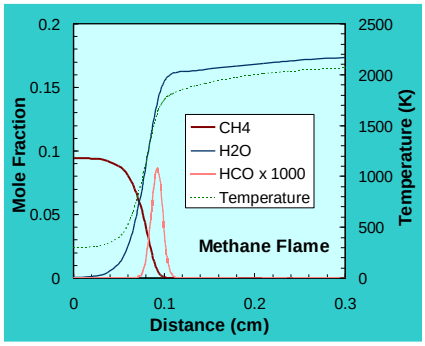

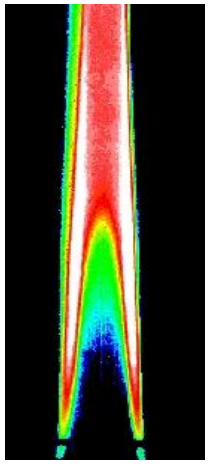

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Overview

- The specific goals of this section are:
  1. Extend description for laminar, nonreacting jets to **jet flames**
  2. Introduce concept of **conserved scalars**
  3. Describe buoyancy effects on laminar jet flames

Wang et al., Proc. Comb. Instit. ***37****(2), 2019*

CH_4

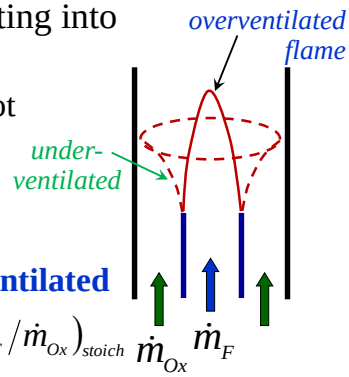
Laminar Jet Flames

- Reacting jet will be "similar" to nonreacting/mixing flow ***except***
  1. Now have source and sink terms in the species conservation (and energy) equations
     - fuel and oxidizer will react to create products and convert chemical to thermal energy
  2. Must add diffusion of products (and intermediates/ radicals) to fuel and oxidizer diffusion
  3. Heat release will produce non-uniform temperature
     - ρ = constant probably **not** a good assumption, will also lead to buoyancy effects (body force term)
     - T dependent diffusivities will also vary with position

Jet Flames: Overall Fuel/Ox. Ratio

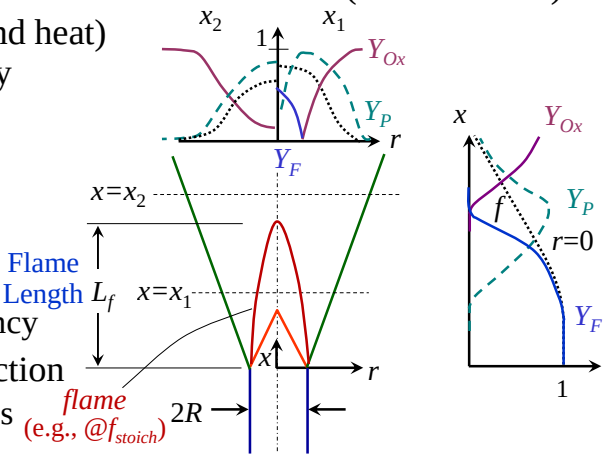
- Previously examined fuel jet exiting into infinite environment
  - important to recognize that is not always the case
- Based on total available fuel and oxidizer in system
  - overall excess oxidizer, **overventilated** (always the case for open air flame) $\dot{m}_F/\dot{m}_{Ox} < (\dot{m}_F/\dot{m}_{Ox})_{stoich}$
  - overall excess fuel, **underventilated**

$$\dot{m}_F/\dot{m}_{Ox} > (\dot{m}_F/\dot{m}_{Ox})_{stoich}$$



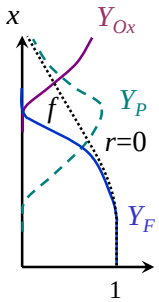
Overventilated Jet Flame Description

- Fuel, oxidizer diffuse toward flame (reaction zone)
- Products (and heat) diffuse away
- Flame tip
  - final fuel burnout
  - location impacted by buoyancy
- Mixture fraction
  - continuous



Mixture Fraction

- Recall f def'n. (IX.5) $$f \equiv \frac{mass\ jet\ fluid\ @\ point}{total\ mass\ @\ point}$$
- So what is the meaning of this definition when species originating in the jet are being consumed (e.g., to make products)
  - *mixture fraction represents fraction of the mass at any point that originated in the jet*
  - e.g., CO_2 molecule's C might originate from the fuel in the jet, while O's came from the oxidizer
  - similarly N_2 part of "products" for fuel jet exiting into air



Stoichiometric Mixture Fraction

- Assuming fuel is in the jet, f_{stoich} represents fraction of product mass that originated from the fuel stream (IX.15)
  - mass conservation
    $$\nu_P \overline{W}_P = \nu_F \overline{W}_F + \nu_{Ox} \overline{W}_{Ox}$$

$$\nu_F F + \nu_{Ox} Ox \rightarrow \nu_P P$$

$$f_{stoich} = \frac{\nu_F \overline{W}_F}{\nu_P \overline{W}_P} = \frac{\nu_F \overline{W}_F}{\nu_F \overline{W}_F + \nu_{Ox} \overline{W}_{Ox}} = \frac{1}{1 + \left(\nu_{Ox} \overline{W}_{Ox} / \nu_F \overline{W}_F\right)}$$

- Example methane jet exiting into air (O_2 and diluent)

$$CH_4 + 2\left(O_2 + 3.76 N_2\right)$$

$$f_{stoich} = \frac{16}{16 + 9.52(28.9)} = 0.055 = \frac{1}{1 + 2 \times 2 \times \frac{1}{0.233}}$$

In terms of the actual fuel and O_2 in the two streams:

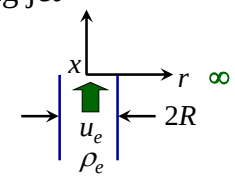
(IX.16)

$$f_{stoich} = \frac{1}{1 + \frac{\nu_{O_2} \overline{W}_{O_2}}{\nu_{fuel} \overline{W}_{fuel}} \frac{Y_{fuel,jet}}{Y_{O_2,Ox}}}$$

Conservation Equations

- Problem setup: rewrite conserv. eqns. IV.5,7,14,15 for simplifying assumptions
  1. same as nonreacting case: laminar, steady, p const., axisymmetric, quiescent/infinite reservoir, Fickian *binary* diffusion, no axial diffusion (but **$\rho \neq$const**)
  2. plus for flame: vertical jet, normal thermal diffusion, no radiative transfer, negl. viscous dissipation, Le=1
  - use similar normalizations to nonreacting jet

$$x^* = x/R \qquad u_x^* = u_x/u_e$$
$$r^* = r/R \qquad u_r^* = u_r/u_e \qquad \rho^* = \rho/\rho_e$$



Conservation Equations

- **Mass**

$$\frac{\partial}{\partial x^*}\left(\rho^* u_x^*\right)+\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\rho^* u_r^*\right)=0 \qquad \text{(IX.17)}$$

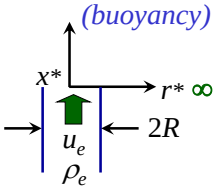
- **Axial momentum** (IX.18)

$\equiv 1/Fr$, Froud #

$$\frac{\partial}{\partial x^*}\left(r^*\rho^* u_x^* u_x^*\right)+\frac{\partial}{\partial r^*}\left(r^*\rho^* u_r^* u_x^*\right)-\frac{\partial}{\partial r^*}\left(r^*\frac{1}{Re}\frac{\partial u_x^*}{\partial r^*}\right)=r^*\frac{Rg}{u_e^2}\left(\rho_\infty^*-\rho^*\right)$$

axial conv. *radial conv.* *radial diffusion* *body force (buoyancy)*

$$Re=\frac{\rho_e u_e R}{\mu} \longleftarrow \textit{local}$$



– **if $Fr \ll \Delta\rho/\rho_e \Rightarrow$ *buoyancy controlled***

– **if $Fr \gg \Delta\rho/\rho_e \Rightarrow$ *buoyancy negligible***

Conservation Equations

- **Species/Energy**
  - next we could write species conservation eq'ns. and energy equation in terms of T
  - BUT run into problems with boundary conditions at flame/rxn zone = source/sink
  - for example

$$\dot{m}_e''=\rho_e u_e$$

$$\frac{\partial}{\partial x^*}\left(r^*\rho^* u_x^* Y_i\right)+\frac{\partial}{\partial r^*}\left(r^*\rho^* u_r^* Y_i\right)-\frac{\partial}{\partial r^*}\left(r^*\frac{1}{Re}\frac{1}{Sc}\frac{\partial Y_i}{\partial r^*}\right)=\frac{r^*}{\dot{m}_e''}\dot{m}_i''$$

axial conv. *radial conv.* *radial diffusion* *source/sink*

- source/sink terms = 0 except in flame (rxn) zone, **but where is the flame located?**
- e.g., for flame sheet approx., locates itself to satisfy reactant mass flux ratio (IX.4) $\frac{\dot{m}_i''}{\dot{m}_j''}=\frac{\nu_i}{\nu_j}\frac{\overline{W}_i}{\overline{W}_j}$

Conserved Scalars

- Solution to this problem - write equations in terms of ***scalars that have no sources or sinks in the flow***
  - scalar that "exists" on both sides of flame and whose "integral" is constant (like J_e and $\dot{m}_e$ in nonreacting case)
- **Examples**
  - **total enthalpy**, $h_{sens}+h_{chem}$, if negligible: radiation, viscous dissipation, body force work (+ no conduction to bodies)
  - **mixture fraction**, f
  - **elemental** (atom) **mass fraction** $Z_i = \sum_{j=1}^{N} \mu_{ij} Y_j$

μ_{ij} = mass proportion of element i in species j

$$\mu_{H,CH_4} = 4/16$$

Conserved Scalar Equations

- Species→ **Mixture fraction**

$$\frac{\partial}{\partial x^*}\left(r^*\rho^* u_x^* f\right)+\frac{\partial}{\partial r^*}\left(r^*\rho^* u_r^* f\right)-\frac{\partial}{\partial r^*}\left(r^*\frac{1}{Re}\frac{1}{Sc}\frac{\partial f}{\partial r^*}\right)=0 \qquad \text{(IX.19)}$$

- Energy→**Total enthalpy**

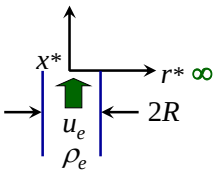
$$\frac{\partial}{\partial x^*}\left(r^*\rho^* u_x^* h^*\right)+\frac{\partial}{\partial r^*}\left(r^*\rho^* u_r^* h^*\right)-\frac{\partial}{\partial r^*}\left(r^*\frac{1}{Re}\frac{Le}{Sc}\frac{\partial h^*}{\partial r^*}\right)=0 \qquad \text{(IX.20)}$$

(IX.21) $$h^* \equiv \frac{h-h_\infty}{h_e-h_\infty}$$

=0 in pure ambient

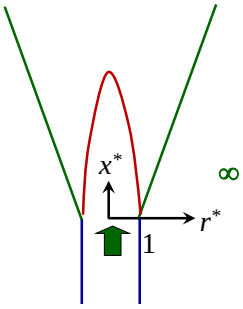
=1 in pure jet fluid

- same normalization as f



Boundary Conditions

$$\eta = u_x^*, f, h^*$$



- General axisymmetric jet
  - @ $r^* = 0$, any x^* $\quad \partial\eta/\partial r = 0,\ u_r^* = 0$
  - @ $r^* = \infty$, $\quad$ ″ $\quad \eta = 0$
  - @ $x^* = 0$, $r^* > 1$ $\quad \eta = 0$
- For top hat/uniform exit profile
  - @ $x^* = 0$, $r^* \le 1$ $\quad \eta = 1$

Conserv. Eqn./BC Summary

- Same boundary conditions on normalized variables
- Mom./species/energy eqns (IX.17-19) similar

$$\left.\right) = r^* \frac{Rg}{u_e^2}\left(\rho_\infty^* - \rho^*\right) \quad \textit{axial mom.}$$

$$\frac{\partial}{\partial x^*}\left(r^*\rho^* u_x^* \eta\right) + \frac{\partial}{\partial r^*}\left(r^*\rho^* u_r^* \eta\right) - \frac{\partial}{\partial r^*}\left(r^* \frac{\partial \eta}{\partial r^*} \frac{1}{Re} \quad \frac{1}{Sc}\right) = 0 \quad \textit{species}$$

$$\left.\frac{Le}{Sc}\right) = 0 \quad \textit{energy}$$

  - no source/sink terms
    - ***except*** in mom. if buoyancy effects not negligible (Fr small)
  - Schmidt and Lewis number dependence
- So if Le=1, h^*=f ; if Fr large and Sc=1 then $u_x/u_e = f$
- Still need to relate f, h^* to ρ, T *unique relations if we assume fast chemistry (Da >>1)*

State Relationships

- Assuming ideal gases

$$\rho = p\overline{W} / \overline{R}T$$

- p=*const* a given; so to find ρ, need T and $\overline{W}$
  - general process, starting with value for f

$$\begin{tikzcd}
 & Y_i \arrow[r, "{(\sum Y_i/\overline{W}_i)^{-1}}"] \arrow[d, "\sum Y_i c_{p_i}"] & \overline{W} \\
f \arrow[ur] \arrow[dr] & c_p & \\
 & h \arrow[r] & T
\end{tikzcd}$$

e.g. for Le=1, h=f*

- So we need (state relations) $f \rightarrow Y_i$

Y_i State Relationships

- For flame sheet approximation ($\nu_F F + \nu_{Ox} Ox \rightarrow \nu_P P$), linear relationships between f and Y_F, Y_{Ox}, Y_P
- Example of fuel jet exiting into air

$$f_{stoich} = \frac{\overline{W}_F \nu_F}{\overline{W}_P \nu_P}$$

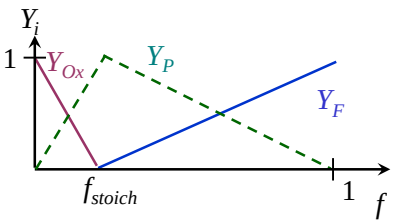
  - for $f > f_{stoich}$; on fuel side of flame
    - $Y_{Ox} = 0$
    - $Y_F(f=1)=1$; $Y_F(f_{stoich}) = 0$

$$\Rightarrow Y_F = (f - f_{stoich})/(1 - f_{stoich})$$

    - $Y_P = 1 - Y_F$

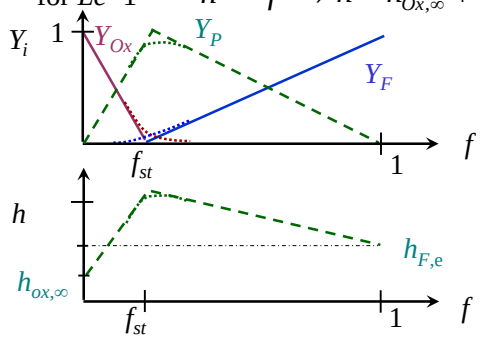
$$\Rightarrow Y_P = (1 - f)/(1 - f_{stoich})$$

  - similarly for $f < f_{stoich}$
    - $Y_F = 0 \qquad Y_P = f / f_{stoich} \qquad Y_{Ox} = 1 - f / f_{stoich}$



State Relationships

- ***h*=?**
  - for Le=1 $\quad h^* = f \Rightarrow h = h_{Ox,\infty} + f\left(h_{F,e} - h_{Ox,\infty}\right)$

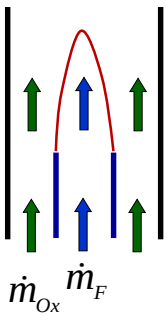


Simple, linear relations for
- flame sheet approx.
- Le_i=1
- T if c_p=constant

- **Multistep chemistry** (flame sheet assumes single step)
  - for fast chemistry (i.e., chemical equilibrium) can still use state relation, but now fuel and oxidizer can coexist
  - broadens "flame"

History of Solution Approaches

- **Burke & Schumann (1928)**
  - "classical" analytic solution (e.g., see texts by Lewis&von Elbe; Kuo)
  - assumed u_x=const, u_r=0
    - so mom. equation not needed (can't account for buoyancy)
  - assumed flame sheet, single D
  - reasonable results for L_f of circular flames



- **Roper (1977)**
  - removed u_x=const. limitation
  - results improved for buoyant and non-circular jets
- **Numerical solutions** (e.g., Kee & Miller, *AIAA J.* **16**,1978)
  - can include finite rate kinetics, differential diffusion

Simplified Solutions

- Start with simplest case
- ***Sc*=1 (*Le*=1), Constant Density, Const. "Properties", Flame Sheet**
  - ρ, $\mu = \rho\nu = \rho D = \rho\alpha$ = constant
  - then all conservation equations become

$$\frac{\partial}{\partial x^*}\left(r^* u_x^* \eta\right) + \frac{\partial}{\partial r^*}\left(r^* u_r^* \eta\right) - \frac{\partial}{\partial r^*}\left(r^* \frac{1}{Re_j}\frac{\partial \eta}{\partial r^*}\right) = 0$$

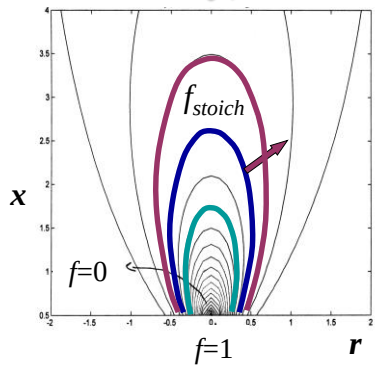
  - which gives identical solution as for nonreacting jet (IX.11,12)

Solutions (Constant Density)

- **Flame Location**
  - at $f = f_{stoich}$
  - from (IX.16)

$$f_{stoich} = \frac{1}{1 + \left(\frac{O_2}{fuel}\right)_{stoich} \frac{Y_{fuel,e}}{Y_{O_2,\infty}}}$$

stoich. oxygen/ fuel mass ratio



  - if you **dilute oxidizer** $f_{stoich} \downarrow$ *(flame gets bigger)*
  - if you **dilute fuel** $f_{stoich} \uparrow$ *(flame gets smaller)*

Discussion: why?

Solutions (Constant Density)

- **Flame Width**
  - r_{flame} where $\xi=\xi(f_{stoich})$
  - from (IX.11)

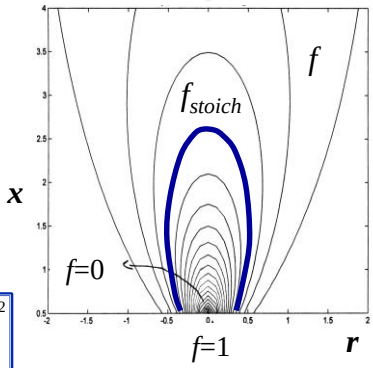
$$\xi_{stoich} = 2\left[\left(\frac{3}{8}\frac{\mathrm{Re}_j R}{f_{stoich} x}\right)^{1/2} - 1\right]^{1/2}$$

  - $\xi \rightarrow r$ from (IX.10)

$$\boxed{\frac{r_{flame}}{x} = \frac{8}{\sqrt{3}}\frac{1}{Re_j}\left[\left(\frac{3}{8}\frac{Re_j}{f_{stoich}}\frac{R}{x}\right)^{1/2} - 1\right]^{1/2}} \quad \text{(IX.22)}$$

  - consistent with previous flame size result

    $r_{flame}\uparrow$ for $f_{stoich}\downarrow$



Solutions (Constant Density)

- **Flame Length**
  - flame tip reached when $r_{flame}=0$
    - get $x_{flame}=L_f$ from (IX.22)

(IX.23) $$\boxed{L_f = \frac{3}{8} Re_j \frac{R}{f_{stoich}}}$$

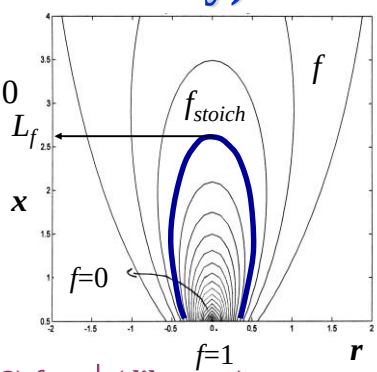
vol. flowrate $= \pi R^2 u_e$

$$= \frac{3}{8\pi}\frac{1}{f_{stoich}}\frac{Q_e}{\nu}$$

$$L_f \propto \frac{Q_e/D}{f_{stoich}}$$

$L_f\uparrow$ for 1) $Q_e/D\uparrow$ or 2) $f_{stoich}\downarrow$ (dilute ox.) *Discussion: why?*

  - for given mass flowrate $L_f \propto \dot{m}_{jet}/\mu \neq$ *function* (p)
    - reduce combustor length by having distributed nozzles



Solutions (Burke-Schumann)

- Earliest (approximate) solution
  - **includes density variation**
  - but assumes constant u_x and const. properties
    - but can get same result if one assumes ρD=const (though actually $\rho D \propto T^{0.5\text{-}0.8}$ for simple gases)
- **Species equation** (pseudo fuel = conserved scalar)

$$\cancel{\frac{1}{r}}\frac{\partial}{\partial x}\left(\cancel{r}\rho u_x Y_F'\right)+\cancel{\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho u_r Y_F'\right)}-\frac{1}{r}\frac{\partial}{\partial r}\left(r\rho D\frac{\partial Y_F'}{\partial r}\right)=0$$

$$u_{x,ref}\frac{\partial}{\partial x}\left(Y_F'\right)=D_{ref}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Y_F'}{\partial r}\right)$$

Solution by Bessel Functions (series solution)

Solutions (Variable Density)

- Fay extended Burke-Schumann approach
  - still Sc=Le=1 but with momentum equation, $\rho \neq const$
  - limited to Fr large (no buoyancy)

$$L_f \propto L_{f,\mu_{ref},\rho_{ref}=const}\times\frac{\rho_F\rho_\infty}{\rho_{ref}^2}\frac{1}{I_u\left(\rho_\infty/\rho_f\right)}$$

ρ_F= density in cold fuel
ρ_∞= density in cold oxidizer
ρ_f= density at flame

I_u= mometum integral (>1)

ρ_∞/ρ_f	ρ_∞/ρ_{ref}	$I_u(\rho_\infty/\rho_f)$
1	1	1
3	2	2.4
5	3	3.7
7	4	5.2
9	5	7.2

after Turns Table 9.2

$$\Rightarrow L_f > L_{f,\rho=const}$$

$\rho_\infty/\rho_{ref} \sim \rho_F/\rho_{ref}$

- Roper
  - includes buoyancy (see Turns)

Non-Analytic Solutions

- Four important issues not included in most analytic solutions $\Rightarrow$ need for numerical solutions
  1. Finite Rate Chemistry
     - can't apply state relations, e.g., for $f \rightarrow Y_i$
     - solution will depend on chemical time scales and flow time scales (diffusion, residence times)
  2. Differential Diffusion
     - analytic solutions assumed binary diffusion
     - e.g., product and intermediate species can diffuse at different rates
  3. Axial Diffusion
     - can't neglect in near-field or for low u_e (e.g., low Re_{jet})
  4. Temperature dependent properties