Ideal Gas Mixtures

- Consider a mixture of gases containing more than one ideal gas
  - for example, gas C composed of gases A and B
- How to calculate properties of gas?
  - can use mixture averaged properties, e.g.,
  - or sum up partial pressure

\[ p = \sum_i p_i \]
\[ p_i = \rho_i R_i T \]
\[ p_i = \chi_i, p \]

overall gas is ideal
if each individual gas is ideal

Calculating Properties of Mixtures

- What about other properties?
  - internal energy \( U \) or enthalpy \( H \)
  - e.g.,
    \[ \Delta H_{C_{1,2}} = m_{mix} \int_{T_1}^{T_2} c_{p_{mix}} dT \]
    \[ c_{p_{mix}} = \sum_i Y_i c_{p_i} \]
    \[ Y_i = m_i/m_{mix} \]
  - or
    \[ \Delta H_{C_{1,2}} = \Delta H_{A_{1,2}} + \Delta H_{B_{1,2}} \]
    \[ \Delta H_{C_{1,2}} = m_A \left( h_A(T_2) - h_A(T_1) \right) \]
    \[ + m_B \left( h_B(T_2) - h_B(T_1) \right) \]
    \[ \Delta H_{C_{1,2}} = m_A \int_{T_1}^{T_2} c_{p_A} dT + m_B \int_{T_1}^{T_2} c_{p_B} dT \]
Ideal Gas Mixture Entropy

- What about entropy $S$?
  - use mixture averaged properties
  \[
  \Delta S_{c_1} = \int_{T_1}^{T_2} c_{p_{av}} \frac{dT}{T} - R_{mix} \ln \left( \frac{p_2}{p_1} \right)
  \]
  - or sum up components
  \[S_C(T, p) = S_A(T, ?) + S_B(T, ?)\]
  all components have same $T$, but what pressure should we use for each component? … their partial pressure

\[
\Delta S_{c_1_{A,B}} = m_A \int_{T_1}^{T_2} c_{p_A} \frac{dT}{T} - R_A \ln \left( \frac{p_{2A}}{p_{1A}} \right) + m_B \int_{T_1}^{T_2} c_{p_B} \frac{dT}{T} - R_B \ln \left( \frac{p_{2B}}{p_{1B}} \right)
\]

Example

- **Given:** air at 1 atm and 300 K compressed to 10 atm and 700 K
- **Find:** change in entropy per unit mass
- **Assume:** air is 79% N$_2$ and 21% O$_2$ (by mole)
  N$_2$, O$_2$ are TPG and CPG under these conditions

**Analysis:**

1) **mix. avg properties**

\[
MW_{air} = 0.79(28 \text{ kg/kmol}) + 0.21(32 \text{ kg/kmol}) = 28.85 \text{ kg/kmol}
\]
\[
R_{air} = \frac{\frac{8314 J}{\text{kmolK}}}{28.85 \text{ kg/kmol}} = 288 J/\text{kgK}
\]
\[
c_{p_{av}} \cong \frac{7}{2} R_{air} = \frac{7}{2}(1.01 \text{ kJ/kgK}) = 1.01 \text{ kJ/kgK}
\]

\[
\Delta s_{12} = c_{p_{air}} \ln \left( \frac{T_2}{T_1} \right) - R_{air} \ln \left( \frac{p_2}{p_1} \right) = 1.01 \frac{kJ}{\text{kgK}} \ln \left( \frac{7}{3} \right) - 0.288 \frac{kJ}{\text{kgK}} \ln \left( \frac{10}{1} \right)
\]
\[
= 0.191 \frac{kJ}{\text{kgK}}
\]

$S$ increased…why?
Example

- Analysis:
  2) summation

\[
\Delta S_{12,\text{air}} = \Delta S_{12,N_2} + \Delta S_{12,O_2}
\]

\[
m_{\text{air}} \Delta s_{12,\text{air}} = m_{N_2} \Delta s_{12,N_2} + m_{O_2} \Delta s_{12,O_2}
\]

\[
\Delta s_{12,\text{air}} = Y_{N_2} \Delta s_{12,N_2} + Y_{O_2} \Delta s_{12,O_2}
\]

\[
= 0.767 \frac{kg_{N_2}}{kg_{\text{air}}} \left(0.1968 \frac{kJ}{kg_{N_2} K}\right) + 0.233 \frac{kg_{O_2}}{kg_{\text{air}}} \left(0.1722 \frac{kJ}{kg_{O_2} K}\right)
\]

\[
= 0.191 \frac{kJ}{kgK}
\]

\[
Y_{N_2} = \frac{MW_{N_2}}{MW_{\text{air}}} = 0.79 \frac{28.01}{28.8} = 0.767
\]

\[
Y_{O_2} = 1 - Y_{N_2} = 0.233
\]

\[
\Delta s_{12,N_2} = \frac{8314 J}{28.01 kg_{N_2} K} \left[7 \ln\left(\frac{7}{3}\right) - \ln\left(\frac{7.9}{0.79}\right)\right]
\]

\[
= 0.1968 kJ/kg_{N_2} K
\]

\[
\Delta s_{12,O_2} = \frac{8314 J}{32.0 kg_{O_2} K} \left[7 \ln\left(\frac{7}{3}\right) - \ln\left(\frac{2.1}{0.21}\right)\right]
\]

\[
= 0.1722 kJ/kg_{O_2} K
\]