Ideal Gas Mixtures

- Most propulsion applications do not involve one “pure” gas – involve gas mixtures
  - for example: air (O₂, N₂…), engine exhaust (products of combustion)

- How to calculate properties of ideal gas mixture?
  - can use mixture averaged properties, e.g.,
    \[ p = \rho R_{\text{mix}} T \quad R_{\text{mix}} = \frac{R}{MW_{\text{mix}}} \quad MW_{\text{mix}} = \sum_i \chi_i MW_i \quad \chi = n_i/n_{\text{mix}} \]
  - or sum up partial pressure
    \[ p = \sum_i p_i \quad p_i = \rho_i R T \quad \text{if each individual gas is ideal} \]

Calculating Properties of Mixtures

- What about other properties?
  - internal energy \( U \) or enthalpy \( H \)
  - e.g., gas C composed of part A and B

\[ \Delta H_{C,1:2} = m_{\text{mix}} \int_{T_1}^{T_2} c_{p_{\text{mix}}} dT \]

- mixture averaged properties
  - or

\[ \Delta H_{C,1:2} = \Delta H_{A,1:2} + \Delta H_{B,1:2} \]

\[ \Delta H_{C,1:2} = m_A \left(h_A(T_2) - h_A(T_1)\right) + m_B \left(h_B(T_2) - h_B(T_1)\right) \]

\[ \Delta H_{C,1:2} = m_A \int_{T_1}^{T_2} c_{p_A} dT + m_B \int_{T_1}^{T_2} c_{p_B} dT \]

- simple summation

\[ Y_i = m_i/m_{\text{mix}} \]
Ideal Gas Mixture Entropy

- What about entropy \( S \)?
  - use mixture averaged properties
  \[
  \Delta S_{c_{12}} = \int_{T_1}^{T_2} c_{p_{av}} \frac{dT}{T} - R_{mix} \ln \left( \frac{p_2}{p_1} \right)
  \]
  - or sum up components
  \[
  S_C(T, p) = S_A(T, ?) + S_B(T, ?)
  \]
  all components have same \( T \), but what pressure should we use for each component? … their partial pressure

\[
\Delta S_{c_{12}} = m_A \left\{ \int_{T_1}^{T_2} c_{p_A} \frac{dT}{T} - R_A \ln \left( \frac{p_{2A}}{p_{1A}} \right) \right\} + m_B \left\{ \int_{T_1}^{T_2} c_{p_B} \frac{dT}{T} - R_B \ln \left( \frac{p_{2B}}{p_{1B}} \right) \right\}
\]

Example

- Given: air at 1 atm and 300 K compressed to 10 atm and 700 K
- Find: change in entropy per unit mass
- Assume: synthetic air is 79% \( N_2 \) and 21% \( O_2 \) (by mole); \( N_2, O_2 \) are TPG and CPG under these conditions
- Analysis:

  1) mix. avg properties
  \[
  MW_{av} = 0.79(28 \text{ kg/kmol}) + 0.21(32 \text{ kg/kmol}) = 28.85 \text{ kg/kmol}
  \]
  \[
  R_{av} = \frac{R}{MW_{av}} = \frac{8314 \text{ J/kmolK}}{28.8 \text{ kg/kmol}} = 288 \text{ J/kgK}
  \]
  \[
  c_{p_{av}} \approx 7 \frac{R}{MW_{av}} = 1.01 \text{ kJ/kgK} \]
  \[
  \Delta s_{12} = c_{p_{av}} \ln \left( \frac{T_2}{T_1} \right) - R_{av} \ln \left( \frac{p_2}{p_1} \right) = 1.01 \frac{\text{kJ}}{\text{kgK}} \ln \left( \frac{7}{3} \right) - 0.288 \frac{\text{kJ}}{\text{kgK}} \ln \left( \frac{10}{1} \right)
  \]
  \[
  S_{increased}...why?
  \]
Example

• Analysis:
  2) summation

\[ \Delta S_{12,\text{air}} = \Delta S_{12,N_2} + \Delta S_{12,O_2} \]

\[ m_{\text{air}} \Delta s_{12,\text{air}} = m_{N_2} \Delta s_{12,N_2} + m_{O_2} \Delta s_{12,O_2} \]

\[
\begin{align*}
\Delta s_{12,\text{air}} &= Y_{N_2} \Delta s_{12,N_2} + Y_{O_2} \Delta s_{12,O_2} \\
&= 0.767 \frac{kg_{N_2}}{kg_{\text{air}}} \left( 0.1968 \frac{kJ}{kg_{N_2}K} \right) \\
&\quad + 0.233 \frac{kg_{O_2}}{kg_{\text{air}}} \left( 0.1722 \frac{kJ}{kg_{O_2}K} \right) \\
&= 0.191 \frac{kJ}{kgK} 
\end{align*}
\]

\[
\begin{align*}
Y_{N_2} &= \frac{MW_{N_2}}{MW_{\text{air}}} = 0.79 \frac{28.01}{28.8} = 0.767 \\
Y_{O_2} &= 1 - Y_{N_2} = 0.233 \\
\Delta s_{12,N_2} &= \frac{8314}{28.01 \frac{kJ}{kg_{N_2}K}} \left[ \frac{7}{2} \ln \left( \frac{7}{3} \right) - \ln \left( \frac{7.9}{0.79} \right) \right] \\
&= 0.1968 \frac{kJ}{kg_{N_2}K} \\
\Delta s_{12,O_2} &= \frac{8314}{32.0 \frac{kJ}{kg_{O_2}K}} \left[ \frac{7}{2} \ln \left( \frac{7}{3} \right) - \ln \left( \frac{2.1}{0.21} \right) \right] \\
&= 0.1722 \frac{kJ}{kg_{O_2}K} 
\end{align*}
\]