IV. Rocket Propulsion Systems

D. Chemical Rocket Cycle Analysis

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for AE 4451 Jet and Rocket Propulsion

Ideal Rocket Cycle Analysis

• Just like with air-breathing engines, we are interested in predicting rocket thrust and other performance parameters
• Start by considering simplified, ideal rocket cycle with 1) combustion chamber and 2) nozzle
• Idealizing assumptions
  1. steady flow, thermally and calorically perfect gas, constant properties (MW, \( \gamma \))
  2. chemical reaction equivalent to constant pressure heating (reversible)
  3. nozzle expansion is 1-d, reversible and adiabatic (isentropic)
**Ideal Rocket Cycle**

- **Combustion Chamber**
  - energy conservation (adiabatic, no work)
  
  \[
  \dot{m} \Delta h_R = \dot{m} (h_{e2} - h_{o1}) = \dot{m} c_p (T_{o2} - T_{o1})
  \]
  
  \[
  T_{o2} = T_{o1} + \Delta h_R / c_p
  \]  
  \(\text{(1)}\)

- **Nozzle**
  - energy conservation (adiabatic, no work)
  
  \[
  \dot{m} h_{o2} = \dot{m} (h_e + u_e^2 / 2)
  \]
  
  \[
  u_e = \sqrt{2(h_{o2} - h_e)} = \sqrt{2 c_p T_{o2} \left(1 - \frac{T_e}{T_{o2}}\right)} = \sqrt{\frac{2 \gamma}{\gamma - 1} \frac{R T_{o2}}{P_{o2}} \left[1 - \left(\frac{P_e}{P_{o2}}\right)^{\gamma-1/\gamma}\right]}
  \]  
  \(\text{(2)}\)
Ideal Rocket Cycle

- Nozzle
  - choked throat \((M_t=1)\)

\[
\dot{m} = \rho u A = \frac{p_{o2}}{R} \sqrt{\frac{1}{\gamma} \frac{2}{\gamma+1}} \sqrt{\frac{\gamma}{\gamma+1}} A_t \sqrt{\frac{\gamma}{\gamma+1}} T_{o2}^{\frac{\gamma+1}{\gamma-1}} \tag{3}
\]

Maximizing Specific Impulse

- Already saw \(I_{sp} \sim u_e\)
- To maximize \(u_e\)
  1. low MW propellant
  2. large \(p_c/p_e\) (high C.C. pressure, nozzle area ratio)
  3. large \(T_o\) (high \(\Delta h_f/c_p\) chemical energy)
- Thrust \(\sim m u_e\); to get high mass flow rate
  1. large throat area, \(A_t\)
  2. high chamber p, \(p_o\)
  3. low combustion T, \(T_o\)
  4. high propellant MW

\[
u_e = \left[ \frac{2\gamma}{\gamma-1} R T_{o2} \right]^{\frac{\gamma-1}{\gamma}} \left[ 1 - \left( \frac{p_c}{p_{o2}} \right)^{\frac{\gamma-1}{\gamma}} \right]
\]

\[
\dot{m} = \frac{p_{o2}}{\sqrt{R T_{o2}/MW}} A_t \sqrt{\frac{\gamma}{\gamma+1}} T_{o2}^{\frac{\gamma+1}{\gamma-1}}
\]