Turbine Overview

• Configurations (axial, radial, mixed), analysis and other issues similar to compressors

• Compared to compressors
  – higher loading $\Delta h_i/U^2$ (or specific work) and pressure ratio per stage - why?
    • favorable pressure gradient
  – usually much higher temperature inlet
    • higher temperature materials (strength) and/or blade cooling
Turbine Analysis

- Similar to compressor analysis
  - Euler turbomachinery (conservation) equations
    \[ T = m \left[ (rc_\theta)_{i+1} - (rc_\theta)_{i} \right] \]
    \[ \dot{W} = m \left[ u_{i+1} c_{\theta,i+1} - u_{i} c_{\theta,i} \right] \]
  - and cascade flow to find \( \Delta c_\theta \)
    - 1→2 nozzle
    - 2→3 rotor

Turbine Cascade Analysis

- Now blade moves upward (flip sign convention); again fixed \( r, u_1 = U \)
  \[ w_1 = c_z, \quad w_\theta + c_\theta = U, \quad \vec{c} = \vec{w} + \vec{u} \]
- Therefore for rotor, and constant \( c_z \)
  \[ c_{\theta_2} = c_z \tan \alpha_2 \]
  \[ c_{\theta_3} = U - c_z \tan \beta_3 \]
  \[ \frac{\Delta c_{\theta_{2,3}}}{U} = 1 - \frac{c_z (\tan \beta_3 + \tan \alpha_2)}{U} \]
  \[ = \Delta h_{\theta_{2,3}} / U^2 \]
- same form of blade loading eqn for turbine as compressor

Mechanics and Thermodynamics of Propulsion, Hill and Peterson

AE4451 Propulsion
Stage Pressure Ratio

- For adiabatic turbine with TPG/CPG

\[
Pr = \frac{P_{o1}}{P_{o3}} = \left[ 1 + \frac{1}{\eta_{st}} \left( \frac{T_{o3} - T_{o1}}{T_{o1}} \right) \right]^{1/\gamma} = \left[ 1 + \frac{1}{\eta_{st}} \left( \frac{\Delta h_{o,3}}{U^2} \gamma RT_{o1} \right) \right]^{1/\gamma} > 1 \text{ as written}
\]

- Stage pressure ratio still depends on
  1. \( \psi = f(U = r\Omega, \Delta c_\theta) \)
  2. blade \( M = f(r\Omega, T_{o1}) \)
  3. \( \eta_{st} \)

Axial Turbine Maps

- Larger stage pressure ratios and efficiencies then compressors
- Peak efficiency on-design
- For fixed RPM, larger pressure change (drop) at higher mass flowrate
  - more work extracted per unit mass
- At high (corrected) mass flowrate, nozzle becomes choked
Blade Design: Degree of Reaction

- We have TWO blade parameters to design
  - rotor trailing edge (match $\beta_3$)
  - nozzle trailing edge (match $\alpha_2$)

- How to do this?
  1. Degree of reaction, $R$
  2. Stage exit condition constraint ($\alpha_3$)

$$\Delta c_{\theta,2,3} = 1 - \frac{c_i}{U} \left( \tan \beta_3 + \tan \alpha_2 \right)$$

Similar issue for compressor; we just "ignored" designing $\alpha_1$

Degree of Reaction

- Recall
  $$R \equiv \frac{\Delta h_{\text{rotor}}}{\Delta h_{\text{stage}}}$$
  - allows us to distribute load (static pressure change) between rotor and nozzle (or stator)
  - how to relate static enthalpy change to azimuthal velocity changes?
    - $\Delta KE$!! $h_o = h + \frac{v^2}{2}$
    - for stationary blade, no work done
      $$\Delta h_o = 0 \Rightarrow \Delta h = -\Delta KE$$
  - e.g., nozzle blade if $c_z$ constant, and negligible $c_r$
    $$h_2 - h_1 = \left( \frac{c_{z,2}^2 + c_{\theta,2}^2}{2} \right) - \left( \frac{c_{z,1}^2 + c_{\theta,1}^2}{2} \right) = \left( c_{\theta,1}^2 - c_{\theta,2}^2 \right) / 2$$
Degree of Reaction (Turbine)

- Rotor blades??
  - are “stationary” in rotor’s reference frame
  \[ h_3 - h_2 = \left( w_{\theta_2}^2 - w_{\theta_3}^2 \right) / 2 \]

- Reaction

\[
R = \frac{h_3 - h_2}{h_3 - h_1} = \frac{h_3 - h_2}{(h_{o3} - c_3^2) + (h_{o1} - c_1^2)}
\]

if \( c_1 \approx c_3 \)

\[
R \approx \frac{h_3 - h_2}{h_{o3} - h_{o1}} = \frac{w_{\theta_2}^2 - w_{\theta_3}^2}{2U \left( c_{\theta_3} - c_{\theta_1} \right)}
\]

\[ \Delta c_{\theta_{3,1}} = 1 - \frac{c_3}{U} (\tan \beta_3 + \tan \alpha_2) \]

relates design blade angles to azimuthal KE change

Impulse Turbine

- \( R = 0 \)
  - all the pressure change occurs across the nozzle, or the nozzle creates high KE

\[ w_{\theta_2}^2 - w_{\theta_3}^2 = 0 \implies w_{\theta_3} = -w_{\theta_1} \]
\[ c_z \tan \beta_3 = -c_z \tan \beta_2 \]
\[ \implies \beta_3 = -\beta_2 \quad \Delta w_{\theta_3} = -2w_{\theta_1} \]

\[ \Delta c_{\theta_{3,1}} = 1 - \frac{c_3}{U} (\tan \beta_3 + \tan \alpha_2) \]

\[ \frac{\Delta c_{\theta_{3,1}}}{U} = 2 \left( 1 - \frac{c_3}{U} \tan \alpha_2 \right) \]
Impulse Turbine

- So for impulse turbine, blade loading coeff.

\[
\frac{\Delta h_{\text{stage}}}{U^2} = \frac{\Delta c_{\theta_3}}{U} = 2 \left( 1 - \frac{c_z}{U} \tan \alpha_2 \right)
\]

- Relates blade loading to nozzle exit angle

\[
\tan \alpha_2 = \frac{U}{c_z} \left( 1 - \frac{\Delta h_{\text{stage}}}{2U^2} \right)
\]

- From \( \Phi \) equation, rotor blade angles given by

\[
\tan \beta_3 = -\tan \beta_2 = \tan \alpha_2 - \frac{U}{c_z}
\]

Impulse Turbine

- To let largest power per unit mass flow rate \( \Rightarrow \) large \( \alpha_2 \)
  - tends to produce high velocities and \( p_0 \) losses
  - practical limit, \( \sim 70-75^\circ \)

- Further possible constraint
  - no exit swirl \( \Rightarrow c_{\theta_3} = 0 \)

\[
\frac{\Delta c_{\theta_3}}{U} = 2 \left( \frac{c_z}{U} \tan \alpha_2 \right) = \frac{c_{\theta_1}}{U}
\]

\[
\Rightarrow \frac{\Delta c_{\theta_3}}{U} = 2 \left( \frac{\Delta c_{\theta_1}}{U} \right) \Rightarrow \Delta h_{\text{stage}} = \frac{U^2}{U^2} = -2
\]

\[
\Rightarrow \tan \alpha_2 = 2(U/c_z), \quad \tan \beta_3 = U/c_z
\]
50% Reaction Turbine

- \( R = 0.5 \)
  - balanced p drop across stage
  \[ \frac{\Delta h_{stage}}{U^2} = \frac{\Delta c_{\theta_{1.3}}}{U} = \left( \frac{1}{c_2} - \frac{\tan \alpha_2}{U} \right) \]
  \[ \Rightarrow \tan \beta_2 = \tan \alpha_2 \]
  - if no exit swirl
  \[ \frac{\Delta h_{flow}}{U^2} = -1 \Rightarrow \alpha_2 = \beta_2 = \tan^{-1} \left( \frac{U}{c_2} \right) \]

half loading of impulse: less power/stage

Compressor-Turbine Matching

- Another part of design/operational requirement
- Need to "match" compressor and turbine stages on same spool
- Steady operation match
  1. \( N \) (RPM)
  2. \( \dot{m} \)
  3. \( \dot{W} \)
- Iterative procedure
Turbine Stresses/Operational Limits

- Turbine blades experience large stresses: bending, thermal and centrifugal (rotor: $10^4$-$10^5$ g)
- Materials exhibit significant loss of strength and enhanced creep at high $T$
  - low strength at modern engine $T_{o4}$ (high ST, $\eta_{th}$)
  $T_{o4} > 1400^\circ\text{C}$
  (2500°F)

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Turbine Inlet Temperature Evolution

- Solutions
  - high temperature materials
  - blade cooling
  - TBC (thermal barrier coatings)
**Turbine Blade Cooling**

- Usually use compressor (bleed) air
- Configurations
  - internal passages
  - external
    - film cooling
    - tip cooling
- Heat transfer designed to
  - focus on “hot” spots and initial stages
  - minimize stress concentration

**Rotor and nozzle cooling configurations**
Introduction to Heat Transfer

• Consider a simplified version of a (half) turbine blade
  • Inner cooling only
    – neglect film and tip cooling for now
    – hot gas (combustor products) flows over outer surface
    – “cold” gas (bleed air) flowing over inner surface
    – turbine blade “wall” in between
• How to analyze this “heat transfer” problem?

Conduction Heat Transfer

• Start with description of (conduction) heat transfer through the wall
  – assume one-dimensional
  – top side of wall uniform temp. \( T_{\text{outer}} \)
  – bottom side of wall uniform temp. \( T_{\text{inner}} \)
• Look at energy equation
  – differential CV
    \[ Q_{\text{in}} = \frac{d}{dt} \left( mc \frac{dT}{dx} \right) + Q_{\text{out}} \]
  – steady
    \[ \dot{Q}_{\text{in}} = \dot{Q}_{\text{out}} \]
• Need model for \( \dot{Q} \)
  – Fourier’s Law (1d)
    \[ \frac{\dot{Q}}{A} = -k \frac{dT}{dx} \]
Conduction and Thermal Conductivity

- For steady, uniform material
  - T gradient is a constant
    \[ \frac{dT}{dx} = -\frac{\dot{Q}}{kA} \]
  - so T varies linearly through wall

- Thermal conductivity
  - insulators like ceramics have much lower conductivities than metals

<table>
<thead>
<tr>
<th>Material</th>
<th>k (W / mK) at 1000°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nickel Super Alloys</td>
<td>20-30</td>
</tr>
<tr>
<td>Ceramic TBC’s</td>
<td>1-2</td>
</tr>
</tbody>
</table>

- so TBC will produce much lower heat flux for same temperature gradient

Effect of Adding TBC Coating

- Example Ni alloy with
  \[ \dot{Q} = -k \frac{dT}{dx} = -25 \frac{W}{mK} \quad 700K \quad 5mm = 3.5 \frac{MW}{m^2} \]

- Now add 500µm TBC
  \[ \dot{Q} = -k \frac{dT}{dx} = -k \frac{\Delta T}{\Delta x} \quad x \quad \Delta x = 0.5 \quad mm \]

  \[ \Rightarrow T_{mid} = \frac{\dot{Q}}{A} = \frac{\Delta x}{k_{TBC}} \frac{k_{TBC}}{k_{alloy}} \frac{\Delta x}{k_{alloy}} \frac{k_{alloy}}{k_{TBC}} \]

  \[ T_{mid} = 962K \]

  \[ \dot{Q} = -25 \frac{W}{mK} \quad 262K \quad 5mm = 1.3 \frac{MW}{m^2} \]

Most of the temperature drop occurs across TBC, much lower metal T and lower heat transfer
Convective Heat Transfer

- Examine heat transfer between gas flow and blade wall
- Convective heat transfer
  - due to fluid moving over surface
  - thermal boundary layer develops, like momentum boundary layer
- Model
  - so \( T_{\text{wall}} \) varies downstream
  - e.g., for laminar flow over flat plate

\[
\dot{Q}/A = h(T_{\text{gas},x} - T_{\text{wall}}) = h(Re_z, Pr)
\]

\[
h = 0.332 Pr^{-2/3} Re_z^{-1/2}
\]

Stanton Number

\[
\overline{h} = 0.664 Pr^{-2/3} Re_z^{-1/2}
\]

averaged over full length

Convective Heat Transfer - External

- Example
  - hot air
  - Analysis

- \( L = 4 \text{ cm} \)
- \( p = 10 \text{ atm} \)
- \( T_{\text{gas}} = 1850 \text{ K} \)
- \( T_{\text{wall}} = 1400 \text{ K} \)
- \( v = 250 \text{ m/s} \)
- \( Pr = 0.7 \)
- \( v = 3 \times 10^4 \text{ m}^2/\text{s} \)

\[
h_{\text{1mm}} = 0.332 \left( \rho_s c_p v \right) Pr^{-2/3} Re_{1mm}^{-1/2}
\]

\[
= 0.332 \left( \frac{1.013 \text{ MPa}}{1850 \text{ K}} \frac{1.28}{1.28 - 1} \frac{250 \text{ m/s}}{3 \times 10^{-4} \text{ m}^2/\text{s}} \right) 0.7^{-0.667} \left( \frac{250 \text{ m/s} (0.001 \text{ m})}{3 \times 10^{-4} \text{ m}^2/\text{s}} \right)^{-0.5}
\]

\[
= 0.332 \left( \frac{626 \text{ kW}}{m^2 \text{ K}} \right) 0.7^{-0.667} 833^{-0.5} = 9 \frac{\text{kW}}{m^2 \text{ K}}
\]

\[
\overline{h}_{\text{L}} = 2 h_{z=L} = 2.9 \frac{\text{kW}}{m^2 \text{ K}}
\]

\[
\dot{Q}_{\text{1mm}}/A = 9 \frac{\text{kW}}{m^2 \text{ K}} (1800 - 1400) \text{ K} = \frac{4 \text{ MW}}{m^2}
\]

\[
\dot{Q}_{\text{total}}/A = 1.3 \frac{\text{ MW}}{m^2}
\]

much higher heat load around leading edge
Turbine Blade Analysis

• In our two examples
  – conduction through TBC-coated alloy $\sim 1.3 \text{MW/m}^2$
  – convective heat transfer into blade $\sim 1.3 \text{MW/m}^2$
• So together they represent a single problem
  $$\dot{Q}/A_{\text{convection}} = \dot{Q}/A_{\text{conduction}}$$
• Next step is to investigate bleed air cooling requirement

Cooling – Convection Internal Flow

• In pipe/channel flow can’t assume infinite flow
  – boundary layers meet and central flow changes with axial distance
• Now
  $$\frac{Q}{A} = \frac{Q}{L \times \text{perimeter}} = h(T_{\text{inner}} - T_{\text{bulk, coolant}})$$
• New expressions for $h$, e.g., for round tubes
  – turbulent flow, profile still developing
  – averaged over channel length
  $$h = 0.036 \frac{k}{d} Re^{0.8} Pr^{1/3} \left( \frac{d}{L} \right)^{0.055}$$
Turbine Blade Analysis

- Assuming same information in previous examples AND 2mm height channels with span = 80% of blade chord, with 500K, 30 m/s inlet bleed air, negligible spacing between channels

\[
\frac{\dot{Q}_{\text{cool}}}{A} = \frac{1}{2} h(T_{\text{inner}} - T_{\text{bleed}})
\]

\[
h = 191 \text{W/m}^2
\]

\[
\frac{\dot{Q}_{\text{cool}}}{A} = 20 \text{kW/m}^2
\]

- Much less than the cooling requirement from previous parts of the analysis – need to enhance the cooling \(\Rightarrow\) film cooling