Equilibrium/Frozen Flow Modeling and Nonequilibrium

Problem Introduction

Your goal is to predict the flow field properties around a simple airfoil shape flying at 0° angle of attack at hypersonic conditions. The figure below provides the geometry of the airfoil, which has a 2 m chord. The airfoil is traveling at 6200 m/s, and the ambient temperature and pressure ahead of the airfoil are 240 K and 0.001 atm (conditions similar to those at an altitude between 60 and 70 km).

![Airfoil Diagram]

Starting Assumptions

For the purposes of THIS PROJECT, you should assume the following:

- our "air" contains only N and O species with an N/O nuclei ratio of 1.2:1 (by mole);
- N₂, O₂, NO, and O are the only species “possible” in the mixture;
- the airfoil is two-dimensional; and
- the flow is inviscid (except within shocks) and adiabatic.

Molecular Data

You should use the molecular parameters for N₂, O₂, NO and O from Vincenti and Kruger and the class notes (essentially the data that was used in homework set #3). To avoid calculating negative values for enthalpies/energies in this problem (a convenience only), you may want to consider setting an appropriate system energy zero.

Goals

You need to perform the following calculations.

1. **Approach Flow**
   Determine the Mach number of the flow approaching the airfoil, as well as its enthalpy and entropy.

2. **Leading Edge Oblique Shock**
   Determine the shock angle and the following flow properties (i.e., post shock properties at point A) for the shock produced by the flow turn at the front of the airfoil:
- Shock angle
- Pressure
- Temperature
- Density
- Velocity
- Composition (mass fractions, $Y_i$)

Calculate these shock properties for three flow assumptions:
- **equilibrium** flow (using a *partition function-based* approach like that used in homework set #3);
- **chemically frozen** flow, i.e., all other energy modes in equilibrium (using a *partition function-based* approach like that used in homework set #3); and
- **frozen** flow (i.e., all molecular energy modes are frozen except translation and rotation).

3. **Expansion**

Determine the following flow properties as a function of turn angle as the flow goes through the expansion turn around the bottom of the airfoil:

- Velocity
- Temperature
- Density
- Mach number
- Pressure
- Composition (mass fractions, $Y_i$)

Just before the turn, assume the oncoming, post-shock conditions are based solely on the equilibrium flow assumption result from part (2), but for the expansion results, perform the calculations for the three separate flow assumptions as the flow goes through the turn:
- **equilibrium** flow;
- **frozen chemistry** flow; and
- **frozen** flow.

4. **Detached Bow Shock Estimates**

For the three flow assumption cases, what is the maximum angle of attack at which this airfoil could be flown (for the same 6200 m/s flight velocity) before a detached bow shock would form in front of the airfoil.

If a detached bow shock did form, what would be the (immediate) post-shock temperature and pressure near its center, where the bow shock could be modeled as a normal shock? Again provide an answer for each of the flow assumptions.
5. Vibrational Nonequilibrium Estimates

Your next goal is to provide information to determine the likelihood for nonequilibrium to exist for the vibrational mode of O₂. Specifically, we want to know if the O₂ vibrations are likely to be out of equilibrium with the translational energy.

To make estimates of the relaxation time constants, use the model described in class, based on the paper by Millikan and White,¹ for the vibrational relaxation time, \( \tau_v \) (sec), for a vibrating species \( i \) with a collision partner \( j \)

\[
p \tau_{v,i-j} = \exp \left[ 1.16 \times 10^{-3} \mu_{ij}^{1/2} \theta_v^{4/3} \left( T^{-1/3} - 0.015 \mu_{ij}^{1/4} \right) - 18.42 \right]
\]

where, \( p \) is the total pressure (in atm), \( \theta_v \) is the characteristic vibrational temperature (in K), \( T \) is the (translational) temperature (in K) of the gas, and \( \mu \) is the unitless reduced mass for a given pair,

\[
\mu_{ij} = \frac{m_i m_j}{m_i + m_j}.
\]

The total vibrational relaxation time for a given species in a mixture of gases is given by summing over all the collision partners, i.e.,

\[
\frac{1}{\tau_{v,i}} = \sum_j \frac{\chi_j}{\tau_{v,i-j}}.
\]

where \( \chi_j \) is the mole fraction of species \( j \) in the mixture.

Nonequilibrium Estimates Required

a) Oblique Shock: Estimate the characteristic vibrational relaxation time constant for O₂ behind the oblique shock (at the zero angle of attack condition) based on the flow conditions (temperature, pressure, and composition) determined using our three simplified flow assumptions. Specifically, provide three estimates of the vibrational relaxation time constant \( \tau_{v,O_2} \): each one based on the post-shock conditions predicted by one of the flow assumptions.

Then use these three time constants to estimate how far a distance after the leading edge it will take the O₂ vibrational energy mode to achieve a near equilibrium distribution AND if this is likely to occur before the flow reaches the expansion turn. Hint: use the airfoil dimensions to help determine this.

b) Expansion: Estimate the characteristic vibrational relaxation time constant for O₂ through the expansion at the bottom of the airfoil as a function of turn angle through the expansion, based on the flow conditions (temperature, pressure,

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and composition) determined using each of the three flow assumption results from part 3.

Then provide an estimate of when (i.e., at what angle through the turn) the O$_2$ is likely to no longer be in vibrational equilibrium as it goes through the expansion along a streamline that passes through point A (see Figure 1), which is $\delta=10$ cm from the airfoil surface.

**Cooperation/Teaming**

You can work on this project either individually or in a group of two members. If you work in a group, the group should turn in a single report.

**Project Report**

You need to turn in a hardcopy report and submit a PDF on Canvas (just the PDF on Canvas for DL students). Note, this report is NOT to be written in the standard homework format. There should be a SINGLE report for all the parts (goals) of this assignment that must include the following items. Please use the section titles indicated below (except for the cover page – which should not be titled).

**Cover Page**

Should include (at least): a report title, author name(s), date

**Introduction**

A BRIEF overview of all the goals/issues of this assignment in your own words.

**Approach**

Describe (and include) the equations you are solving and a clear description of your solution method/approach - not just what computer tool you used, but a brief but descriptive outline of the methods you used. Include the approaches used for ALL parts of this assignment (subsections for each different goal might be helpful).

**Results and Discussion**

Present ALL your results in this section (you may have subsections if you like). You should present your results primarily as tables or plots, whichever is more appropriate for a given result. For plots and tables, present your results so that it is easy to compare the different flow assumptions, e.g., by plotting all the flow cases for a given flow variable on a single graph. Also, make sure the graph does not “lose information,” e.g., consider semi-log or log-log scaling if needed to expand the dynamic range of the plot. Include a running discussion/interpretation of your results as you present them.

**Conclusions**

Summarize your results and draw interesting and relevant conclusions from them.