Nonequilibrium Flows and Radiation

- Homework solutions should be neat and logically presented, see format requirements (seitzman.gatech.edu/classes/ae6050/homeworkformat.html).
- To receive credit, show ALL work in the format described above. If you use equations from the notes, the class textbook or another book, please cite the reference.

1. Nonequilibrium (Inviscid) Shock Flow

A high speed (M=12), 2-d (wedge-shaped) body is traveling through air with ambient conditions of 300 K and 0.002 atm. The front (nose) of the wedge is shown in the figure below.

a) Make a qualitative sketch of the leading edge of the shock that you expect to occur. Include an estimate of the shock angle near the tip of the nose.

b) Make a qualitative plot of the entropy of the fluid along a cut through the flow on the line n (dashed in the figure), which is normal to the body surface. Start close to the body (i.e., the origin of n should be there). **You must explain the reasoning to support your answer.** (Do not make unnecessary assumptions.)

2. Shear Stress and Random Molecular Motion

Consider a shear layer flow of xenon such as that shown in the figure to the right. This shear layer initially (i.e., at x=0) consists of a higher speed, uniform flow above, and a lower speed, uniform flow below. At x=0, both flows have the same temperature.

a) Sketch the molecular velocity probability distributions for \( c_x \) and \( c_y \) that would exist at x=0 (i.e., before the flows begin to interact) and in a small vertical slice near y=0 (i.e., averaged for the region between the two horizontal dashed lines). Note, \( c \) represents the “absolute” velocity of a molecule (not just the random velocity). Draw separate sketches for the \( c_x \) and \( c_y \) distributions (i.e., use different sets of axes for each) BUT use the same scaling for the axes of your two plots.

b) Sketch the \( c_x \) and \( c_y \) probability distributions averaged between the dashed lines but now at a downstream x in the “near-field”, i.e., at location N shortly after the flows begin to interact. Again draw separate sketches for \( c_x \) and \( c_y \) and use the same scaling as the plots drawn in part (a).

c) Repeat part (b) at a downstream location in the “far-field”, i.e., at location F very far after the flows began to interact. Put these two new sketches ON THE SAME GRAPHS as the sketches from part (b), i.e., both \( c_x \) sketches should appear on the same plot for comparison, and likewise for the two \( c_y \) sketches.
d) Explain how your results from parts (b) and (c) produce the effects we attribute to shear stress.

3. Equation of Radiative Transfer
Consider a non-scattering, uniform, equilibrium gas flowing through a square duct of height \( h = 0.1 \text{m} \) and length \( L = 1 \text{m} \). The gas temperature is 2000 K, the pressure is 0.1 bar and the molecular weight is 10. The absorption coefficient of this gas at a wavelength of 2 \( \mu \text{m} \) and at these conditions is 0.001 \( \text{cm}^{-1} \). Assume the walls of the duct are completely transparent and non-emitting, and that at the given conditions Doppler broadening dominates the lineshape.

a) What is the optical depth (or optical thickness) for radiation at 2 \( \mu \text{m} \) traversing in the vertical direction from the top to the bottom of the duct?

b) Estimate the spectral intensity (at 2 \( \mu \text{m} \)) of the radiation passing through the bottom wall of the duct that is also traveling normal to the bottom wall.

c) The radiant spectral power (e.g., W/\( \mu \text{m} \)) emitted by a gas of volume \( V \) is

\[
q_{\lambda} = \int \int a_{\lambda}^i(x, T) \nu_{ab}(T(x)) dV d\Omega
\]

What is the emitted spectral power at 2 \( \mu \text{m} \) for the gas in the duct?

d) Repeat part (b) assuming the pressure of the flowing gas was reduced to 0.01 bar without changing the gas temperature (or composition).

4. Radiative Properties
Consider a post-shock gas flow at 4900 K and 0.01 atm composed mostly of He and 1\% (mole fraction) of atom Z. The following table shows various properties (degeneracies, energies, and Einstein A coefficients for transitions to the ground state) for each of the electronic energy levels of Z.

<table>
<thead>
<tr>
<th>Energy Level - ( i )</th>
<th>( g_i )</th>
<th>( \varepsilon_i ) (J)</th>
<th>( A_{i,0} ) (sec(^{-1}) sr(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>0</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>2.20 \times 10^{-20}</td>
<td>10^{2}</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2.82 \times 10^{-19}</td>
<td>10^{4}</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>5.12 \times 10^{-19}</td>
<td>10^{5}</td>
</tr>
</tbody>
</table>

a) What are the (vacuum) wavelengths (in nm) for radiative transitions from the given excited electronic energy levels to the ground level?

b) At which wavelength would you find the greatest photon emission rate (e.g., photons/sec) at the given conditions.

c) At which wavelength would you find the greatest emissive power (e.g., W/cm\(^3\))? 

d) We can define a characteristic time for radiative electronic energy loss for the Z atoms using the spontaneously emitted power and the electronic energy of the Z atoms. What would this characteristic time be for the given conditions, based only on the transitions from the given levels to the ground level.