Laminar Nonpremixed Combustion: Evaporating Droplets

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Liquid Fueled Combustion

- Pool flames
- Atomized sprays
- Droplet evaporation vs. burning
- Simplify
  - single, spherical droplet (surface tension), moving with flow
Evaporation Process

- For evaporation (no burning)
  - mass loss
  - heating

\[ \dot{m} = \frac{d}{dt} \left( \dot{m} c_p T \right) \]

Evaporation: Droplet Evolution

- Mass loss, evaporation must be balanced by heat transfer to drop
- Instantly place drop in hot gas
- After short time, heating and (small amount) of evap.
- Eventually quasi-steady state
  - not T_{bp}
Evaporation Rate

- How fast does droplet lose mass \( \dot{m} = ? \)
- Assumptions
  1. Quiescent, infinite medium
  2. Quasi-steady
  3. Uniform droplet
  4. Single component liquid (not soluble for gas)
  5. Binary (Ficks Law) diffusion
  6. Constant thermophysical properties \( (c_p, \rho D, \lambda, \ldots) \)
  7. No viscous dissipation, buoyancy,….

Evaporation Rate – Conservation Eq.

- Mass
  \[ \dot{m}(r) = \text{constant} \]
  \[ \dot{m} = \dot{m}_F = 4\pi r^2 \dot{m}' = \text{const} \quad (1) \]
- Species
  \[ \dot{m}'_s = Y_f \dot{m}' - \rho D \nabla Y_F \]
  \[ \dot{m} = -4\pi r^2 \frac{\rho D}{1 - Y_F} \frac{dY_F}{dr} \]
  \[ \frac{dY_F}{1 - Y_F} = \frac{\dot{m}}{4\pi \rho D} \]
  \[ \frac{\dot{m}^*}{4\pi \rho D} = \frac{\dot{m}}{4\pi \rho D} \]
Evaporation Rate – Conservation Eq.

• Species (con’t)
  - integrate from $r_s$ to arb. $r$
  
  \[ Y_F(r) = 1 - \frac{(1 - Y_{F,\infty})e^{-r_s/r}}{e^{-r_s/r}} \]  
  
  (2)
  
  - use $\infty$ B.C. in (2)
  
  \[ \dot{m} = -4\pi Dr_s \ln(1 + B_s) \]  
  
  (4) $Y_{F,s}$ can’t be 1; $B_s \to \infty$

• This is evaporation rate – with 2 unknowns

Evaporation Rate – Conservation Eq.

• Energy ($c_p = \text{const}$)
  
  \[ \dot{m}c_p \frac{dT}{dr} = \frac{d}{dr} \left( \frac{\lambda A 4\pi^2 r^2 \frac{dT}{dr}}{dr} \right) \]  
  
  \[ ? r_s^* = 4\pi^2 r^2 \frac{dr}{dr} \]  
  
  (5)
  
  - integrate
  
  \[ ? \text{const} + r_s^* T = r_s^2 \frac{dT}{dr} \]  
  
  - at droplet surface, no convection - evaporation

\[ r_s^2 \frac{dT}{dr} \bigg|_{r_s} = \frac{\dot{m}}{4\pi^2} h_{fg} \lambda \frac{dT}{dr} \bigg|_{r_s} = \dot{m}_s (h_{\text{ap}} - h_{\text{liq}}) \]  

heat flux at surface ($\nabla T_s$) drives evaporation/regression
Evaporation Rate – Conservation Eq.

- **Energy** (constant)
  - use surface grad in (5)
  \[ r^2 \frac{dT}{dr} = r_s^* \left( T - T_s + \frac{h_{fg}}{c_p} \right) \]
  - integrate (like species eq.) and use \( T_\infty \) B.C.
  \[ \frac{r_s^*}{r} = \ln \left[ \frac{c_p (T_\infty - T_s) + h_{fg}}{c_p (T - T_s) + h_{fg}} \right] \]

\[ T(r) = \left( r_s^* - \frac{h_{fg}}{c_p} \right) + \left( T_\infty - T_s + \frac{h_{fg}}{c_p} \right) e^{-\frac{h_{fg}}{c_p} r} \]

\[ m = -4\pi \rho \alpha r_s \ln (1 + B_h) \] (8)

Spalding Transfer # (for energy)

Mass and Species Summary

- Compare (4) and (8)
  \[ m = -4\pi \rho Dr_s \ln (1 + B_h) \]

- For \( Le = 1 \)
  \[ B_y = B_h \]
  \[ \frac{Y_{F,s} - Y_{F,\infty}}{1 - Y_{F,s}} = \frac{c_p (T_\infty - T_s)}{h_{fg}} \] (9)

- So need relation for \( Y_{F,s} \) vs. \( T_s \)
  - then can find evaporation rate (and \( Y_{F,s}, T_s \))
Wet Bulb Temperature

- Assume **phase equilibrium** between liquid and vapor at droplet surface
- **Clausius Clapeyron equation** determines vapor pressure of interface

\[
\frac{dp}{dT} = \frac{h_f}{T v_f} > 0 \\
\frac{dp_{vap}}{dT} = \frac{h_f}{T v_f} \\
\frac{dp_{vap}}{p_{vap}} = \frac{h_f}{T v_f} \\
\frac{p_{vap}}{p_{vap}} = \frac{T}{v_f} \\
\Rightarrow \ln p_{vap} = -\frac{h_f}{RT} + C
\]

\[
P_{vap} \approx Ce^{-h_f(T_{vap})/RT_{vap}}
\]

\[
C = pe^{-h_f(T_{vap})/RT_{vap}}
\]

Wet Bulb Temperature

- In terms of mass fraction

\[
Y_{F,s} = Y_{F,vap}(T_s) = \chi_{F,vap}(T_s) \frac{W_F}{W_{mix}}
\]

\[
Y_{F,s} = \frac{p_{vap}(T_s)}{p} \frac{W_{mix}}{W_F} = \frac{(p_{vap}/p)W_F}{(1-p_{vap}/p)W_{ax}}
\]

\[
Y_{F,s} = \left[1 + \left(\frac{p}{Ce^{-h_f(T_s)/RT_s}} - 1\right)\frac{W_{ax}}{W_F}\right]^{-1}
\]

\[\text{(10)}\]

equilibrium relation between \(T_s\) and \(Y_{F,s}\)
Evaporation Rate Summary

• For given droplet size \( r_s \), quasi-steady result

\[
\frac{\dot{m}}{r_s} = -4\pi \rho D \ln \left(1 + B_y\right) = -4\pi \rho \alpha \ln \left(1 + B_h\right)
\]  
\[
B_y \equiv \frac{Y_{F,s} - Y_{F,\infty}}{1 - Y_{F,s}} \quad B_h \equiv \frac{c_s(T_\infty - T_s)}{h_{fs}}
\]  

• Use phase equilibria to find \( Y_{F,s}(p, T_s) \) from (10)

• If constant \( \rho D, \rho \alpha \) and constant \( T_s, Y_{F,s} \) \( \Rightarrow B_y, B_h \) constant

\[
\frac{\dot{m}}{r_s} = \text{constant}
\]

Droplet Lifetime

• How does droplet diameter change with time?
• How long until droplet evaporated (diam=0)?
  – droplet lifetime

• Droplet mass

\[
m_d = \rho_s \pi d_s^3 / 6
\]

• Mass loss rate

\[
\frac{\dot{m}_d}{d_s} = \frac{\dot{m}}{d_s} = \rho_s \frac{\pi}{2} \frac{d_s \dot{d}_s}{dt} = \rho_s \frac{\pi}{4} \frac{d(d_s^2)}{dt}
\]

  – compare to quasi-equil. result

\[
\frac{\dot{m}_d}{r_s} = \text{constant}; \quad \frac{d(d_s^2)}{dt} = \frac{2}{\rho_s \pi} \frac{\dot{m}_d}{d_s^2/2} = -8\rho_s \alpha \ln \left(1 + B_h\right) \equiv K
\]  

Evaporation Constant
Droplet Lifetime

"D² Law" \[ \frac{d(d_s^2)}{dt} = -K \Rightarrow d_s^2(t) = d_s^2(0) - Kt \] (12)

- rate that \( d^2 \) changes is constant
- Lifetime
  - \( d_s = 0 \) \( t_d = \frac{d_s^2(0)}{K} \) (13)
  - neglects transient
  - for HC in hot air, \( K \approx 10^{2-3} \mu m^2/\text{ms} \)
  - e.g., \( K = 1000 \mu m^2/\text{ms} \)
  \[ \Rightarrow t_d(200 \mu m) = 40 \text{ ms} \]
  \[ t_d(2 \mu m) = 4 \mu s \]

small droplets evaporate much faster after Lefebvre

Calculating \( K \)

\[ K = 8 \frac{\rho_s}{\rho_l} \alpha \ln(1 + B_h) = 8 \frac{1}{\rho_l c_p} \ln \left( 1 + \frac{c_p(T_w - T_s)}{h_{fs}} \right) \]

- Law and Williams suggest for HC/air systems
  - use \[ c_p = c_{p,F} \left( T \right) \quad T = T_{wb} + T_w \]
  \[ \lambda_s = 0.4\lambda_F \left( T \right) + 0.6\lambda_\infty \left( T \right) \quad T_{wb} \rightarrow T_{bp} \]

- Note
  - \( K \approx 1/\rho_l \), \( K \downarrow \rho_l \uparrow \) - lighter liquids evaporate faster
  - \( K \downarrow h_{fs} \uparrow \) - volatile liquids evaporate faster
**Additional Comments**

- Similar results for other liquid to gas phase change
- Also for phase change of solid to gas
  - sublimation of solids, ablation heat transfer, solid propellant combustion
- Heat flux to surface determines regression rate
  \[
  \left. \lambda \frac{dT}{dr} \right|_{\text{surface}} = \dot{q}''_{\text{surface}} = \dot{r} \rho_{\text{condens}} \left( \Delta h_{\text{phasechange}} + \Delta h_{\text{sens}} \right)
  \]
  *in absence of radiation, controlled by T grad. at surface*
- Other important effects
  - convective cooling (slip velocity) – dense sprays
  - internal circulation – multicomponent
  - supercritical behavior (high pressure) – fuels
  - transient time (high heat rates) – solubility

**Other Effects**

- Evaporation increased by convection (slip velocity)
  \[
  K_{\text{conv}} \approx K \left( 1 + 0.22 \sqrt{\text{Re}_D} \right)
  \]
- Include transient effects (conv. + transient)
  \[
  K_{\text{eff}}(T_{bp}, \bar{u}, d) \to
  \]