**Laminar Nonpremixed Combustion: Jet and Counterflow Flames**

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**Review**

- Conservation equations for simple jet

\[
\frac{\partial}{\partial x} \left( \rho^* u_r^* \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r^* \rho^* u_r^* \right) = 0
\]

- Momentum (axial)

\[
\frac{\partial}{\partial x} \left( r^* \rho^* u_r^* \eta^* \right) + \frac{\partial}{\partial r} \left( r^* \rho^* u_r^* \eta^* \right) - r^* \frac{1}{Re} \frac{\partial \eta^*}{\partial r} = r^* \frac{gR}{u_c^2} (\rho^* - \rho_r)
\]

- Species, Energy (Le=1)

\[
\frac{\partial}{\partial x} \left( r^* \rho^* u_r^* \eta^* \right) + \frac{\partial}{\partial r} \left( r^* \rho^* u_r^* \eta^* \right) - r^* \frac{1}{Re \cdot Sc} \frac{\partial \eta^*}{\partial r} = 0
\]
Review

- State Relations
  \[ \rho = \frac{\rho W}{RT} \]

- Boundary Conditions
  - simple jet
    - 0 far away
    - 1 at exit
    - no gradient at centerline

Need to get \( T, \overline{W}, \rho, (\sum Y_i/\overline{W})^i \)

Solutions

- Start with simplest case
- \( Sc=1 \) (\( Le=1 \)), Constant Density, Const. “Properties”
  - \( \rho, \mu = \rho D = \rho \alpha = \text{const} \)
  - then all conservation equations become
    \[ \frac{\partial}{\partial x} \left( r' u' \eta \right) + \frac{\partial}{\partial r} \left( r' u' \eta \right) - \frac{\partial}{\partial r} \left( r' \frac{1}{Re_j} \frac{\partial \eta}{\partial r} \right) = 0 \]
  - which gives identical solution as for nonreacting jet
    \[ \frac{u_x}{u} = f \frac{x}{x_1} = h \frac{x}{x_1} \frac{1}{R Re_j} = 3 \left[ 1 + \frac{5}{4} \left( 1 + \frac{3}{64} \frac{C_i^2}{Re_j^2} \right) \right]^{-2} \]
    \[ = \frac{3/8}{1 + (3/64) \frac{C_i^2}{Re_j^2} \left( r^2/x^2 \right)} \]
Solutions (Constant Density)

• Flame Location
  - at \( f = f_{\text{stoich}} \)
  - for diluted oxygen, pure fuel
    \[ f_{\text{stoich}} = \frac{1}{1 + \left( \frac{Q}{F} \right)_{\text{stoich}} Y_{\text{atm}}} \]
  - \( f_{\text{stoich}} \downarrow \) if you dilute oxidizer (flame gets bigger)
  - similarly \( f_{\text{stoich}} \uparrow \) as dilute fuel (flame gets smaller)

Discussion: why?

• Flame Length
  - use flame width
    \[ r_{\text{flame}}(f = f_{\text{stoich}}) = \frac{8}{\sqrt{3}} \frac{1}{R} \left[ \left( \frac{3}{8} \frac{Re_f}{f_{\text{stoich}}} \right)^{1/2} \right] - 1 \]
  - when \( r_{\text{flame}} = 0 \)
    \[ x_{\text{flame}} = L_f = \frac{3}{8} Re_f \frac{R}{f_{\text{stoich}}} = \frac{3}{8} \pi \frac{Q_e}{V} \]
    \[ L_f \propto \frac{Q_e/D}{f_{\text{stoich}}} \]

  - for given mass flowrate \( L_f \propto \dot{m} / \mu \neq \text{function (p)} \)
  - reduce combustor length by having distributed nozzles

Discussion: why?
Solutions (Burke-Schumann)

- Earliest (approximate) solution
  - allowed density to vary
  - assumed no radial velocity
  - assumed constant $u_r$ and const. properties (no mom. eq.)
    - but can get same result if one assumes $pD=\text{const}$
      (though actually $pD\propto T^{1/2}$ for simple gases)

- Species equation (pseudo fuel = conserved scalar)
  \[
  \frac{1}{r} \frac{\partial}{\partial x} (rp\rho Y_r') + \frac{1}{r} \frac{\partial}{\partial r} (rp\rho Y_r') - \frac{1}{r} \frac{\partial}{\partial r} \left( rpD \frac{\partial Y_r'}{\partial r} \right) = 0
  \]
  \[
  u_{r,\text{ref}} \frac{\partial}{\partial x} (Y_r') = D_{\text{ref}} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial Y_r'}{\partial r} \right)
  \]
  Solution by Bessel Functions (series solution)

Solutions (Variable Density)

- Fay extended Burke-Schumann approach
  - still $Sc=Le=1$ but with momentum equation, $\rho\neq\text{const}$
  - limited to $Fr$ large (no buoyancy)

\[
L_f \approx L_{f,\rho=\text{const}} \times \frac{\rho_c \rho_w}{\rho_\text{ref}} \frac{1}{I_u(\rho_\text{ref}/\rho_f)}
\]

- $\rho_c$ = density in cold fuel
- $\rho_w$ = density in cold oxidizer
- $\rho_f$ = density at flame
- $I_u$ = momentum integral (>1)

\[
\Rightarrow L_f > L_{f, \rho=\text{const}}
\]

- Roper
  - includes buoyancy (see Turns)

<table>
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<th>$\rho_c/\rho_f$</th>
<th>$\rho_w/\rho_\text{ref}$</th>
<th>$I_u(\rho_\text{ref}/\rho_f)$</th>
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AE/ME 6766 Combustion
Failure of Flame Sheet Approx.

- Multistep Chemistry
  - even with fast chemistry (equilibrium chemistry) will still have fuel and oxidizer coexisting
  - changes $f \rightarrow Y, h$ state relations
- Finite Rate Chemistry
  - solution will depend on chemical time scales and flow time scales (diffusion, residence time)

Scalar Dissipation

- Definition
  $$\chi = 2D \left( \frac{\partial f}{\partial x} \right)^2$$
  - units of $1/s$
- Essentially inverse of characteristic diffusion time in the flame
- Small scalar dissipation ($\rightarrow 0$) is like homogeneous chemical reactor (no diffusion)
- High scalar dissipation can lead to flame extinction
Finite Rate Kinetic Effects

- Can see effect of $Da = \tau_{flow}/\tau_{chem}$ in opposed jet flow
  - higher velocity gives less flow time (time to react)

Partially Premixed Combustion

- Practical systems involve combination of nonpremixed and premixed combustion
  - partially premixed combustion
  - stratified combustion
- Example, premixed flames usually anchor diffusion flames
  - finite time for reactions required
  - triple point flame
Nonpremixed Lifted Flames

- Higher flowrate, longer distance to ignition point
  - less diffusion flame, more premixed
- As get to too high a jet velocity
  - can reach point where either nonflammable
  - or even stoichiometric flame too slow to propagate
  ⇒ blowoff of nonpremixed flame

Soot in Diffusion Flames

- Typical results, ethylene-air
Soot – What is it?

• Mostly graphitic carbon
• Nucleation → growth → dehydrogenation → graphitization
• Branchy aggregates of primary particles
• Primary particle diameter $d_p$

Soot Formation

• Soot primarily forms due to “heating” (pyrolysis) of HC fuel species in oxygen deficient regions
• Large ring HC condense
• Particles grow and agglomerate
• Soot oxidizes in high T, oxygen regions
Soot Emissions

- So soot primarily produced in regions of
  - “high” fuel concentration (less oxidizer: O₂, OH, ...)
  - high temperature (promotes pyrolysis and growth)
- Soot oxidized in regions of
  - high temperature
  - high oxidizer concentration
- So most soot made in combustor is subsequently destroyed (oxidized) unless
  - rich region remains unmixed
  - gases cooled (mixing, walls, soot radiation)