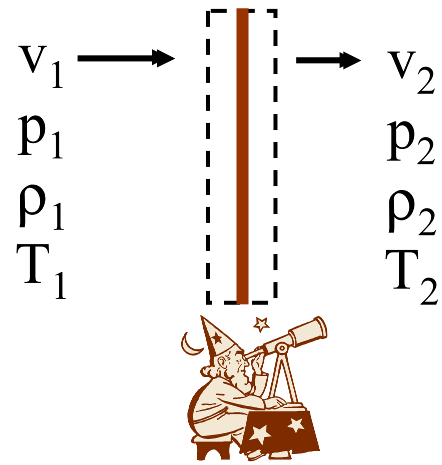
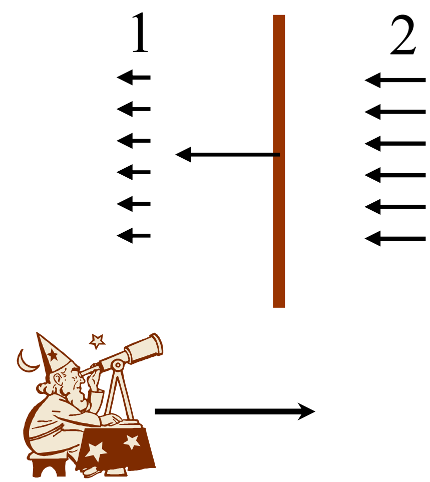
Moving Normal Shocks

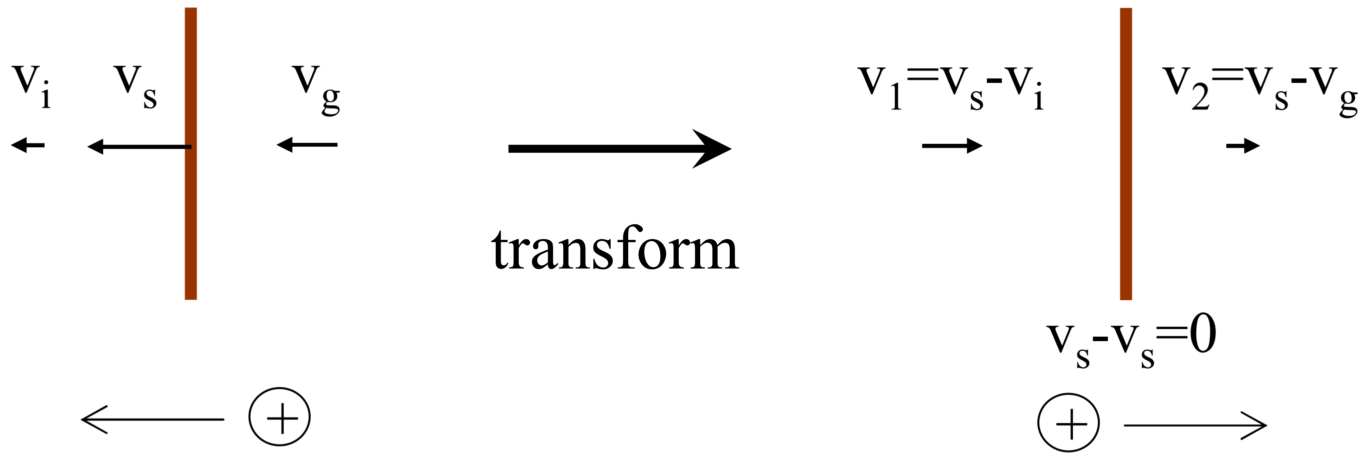
- So far, considered changes across shock wave for the case of the shock not moving
  - observer "sitting" on the shock, moving with shock
- What happens to properties if we consider the shock to be moving
  - observer not moving at same speed as shock





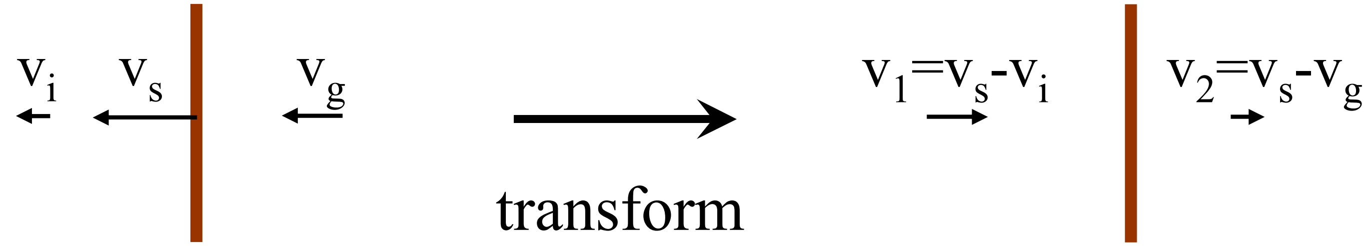
Coordinate Transformation

- First, convert moving shock to stationary shock
  - Galilean transform
  - switch directions (+) and add shock speed, v_s



- Now shock problem looks same as stationary (steady) problem that we already solved

Shock Property Ratios

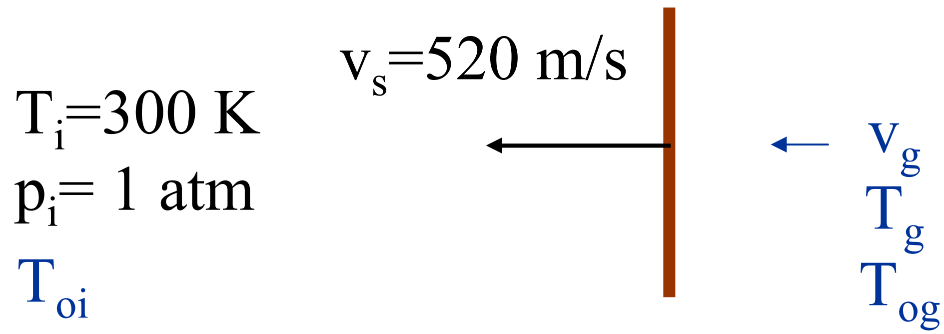


- **Static properties**
  - property you would measure if moving with flow
  - so, unaffected by transformation
  - e.g., still use $T_2/T_1 = \left(1+\frac{\gamma-1}{2}M_1^2\right)\Big/\left(1+\frac{\gamma-1}{2}M_2^2\right)$

    with $M_1=v_1/a_1$; $a_1^2=\gamma RT_1$; $T_1=T_i$; etc.
- **Stagnation properties**
  - depend on velocity; not same after transform
  - find using static properties and M_i, M_g

Example: Known Shock Speed

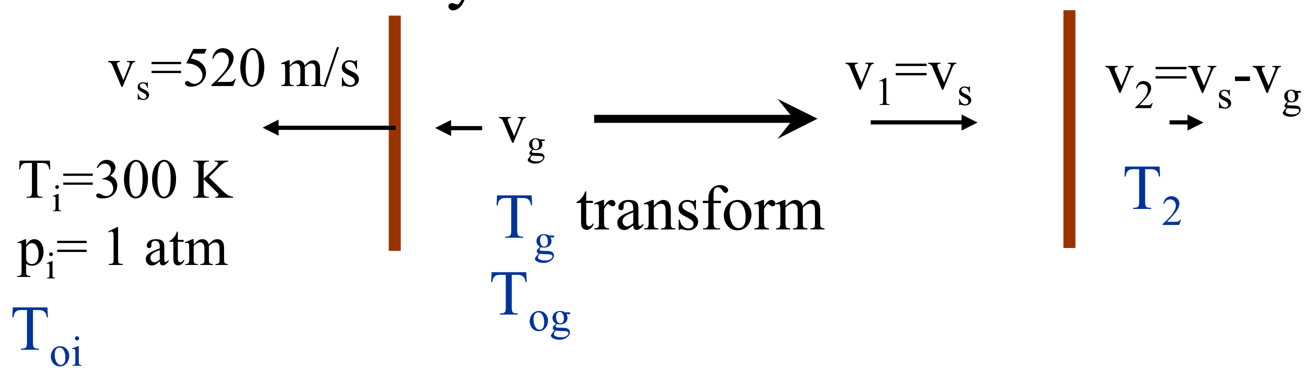
- **Given:** Normal shock moving at 520 m/s **into still air** (300 K, 1 atm)



- **Find:**
  1. Temperature behind shock (after shock passes)
  2. Velocity of gas behind shock (in "lab" reference frame)
  3. Stagnation temperature before and after shock (in lab ref. frame)
- **Assume:** Air TPG/CPG with γ=1.4

Solution: Known Shock Speed

- **Analysis:** Transform to stationary shock



  - find M_1 in stationary frame

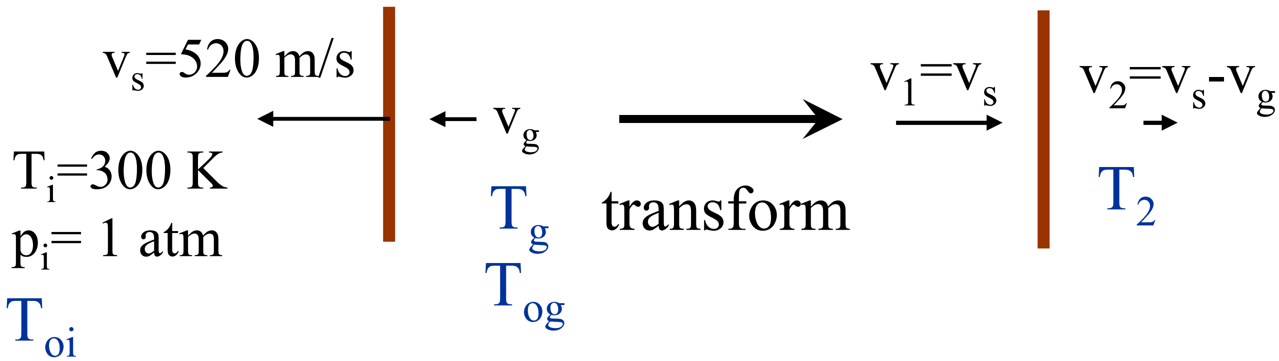
$$M_1 = \frac{v_1}{a_1} = \frac{v_1}{20\sqrt{T_1}} = \frac{v_1}{20\sqrt{T_i}} = \frac{520}{20\sqrt{300}} = 1.50$$

  - M_2 from B.1 or (VII.11)

$$M_2 = \sqrt{\left(M_1^2 + \frac{2}{\gamma-1}\right) \Big/ \left(\frac{2\gamma}{\gamma-1} M_1^2 - 1\right)} = 0.70$$

Solution: Known Shock Speed

- **Analysis (con't):**



  - T_2 from B.1 or (VII.9)

$$T_g = T_2 = 300\text{K}\left(1+\frac{1.4-1}{2}1.5^2\right)\Big/\left(1+\frac{1.4-1}{2}0.7^2\right) = 396\text{K}$$

  - v_g from B.1 or (VII.8)

$$v_g = v_s - v_2 = v_s\left(1-\frac{v_2}{v_1}\right) = v_s\left(1-\frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}}\right) = 241\text{m/s}$$

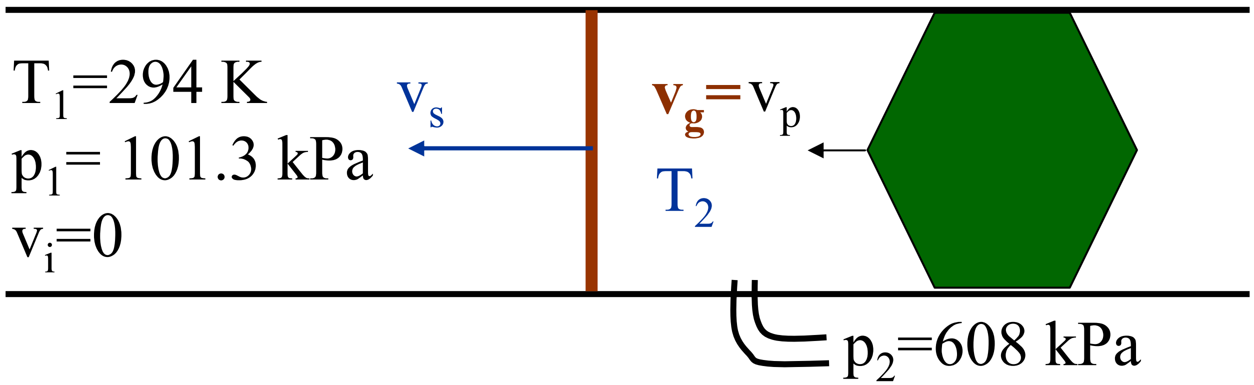
  - T_{oi}, T_{og}

$$T_o = T + \frac{v^2}{2c_p};\, T_{oi} = 300\text{K};\, T_{og} = 396\text{K} + \frac{(241\text{m/s})^2}{2(1005\text{J/kgK})} = 425\text{K}$$

T_o not same for moving shock

Example: Known Postshock Pressure

- **Given:** Supersonic projectile (or equivalently piston) pushing gas ahead in tube filled with **initially still air**
  - leading shock produced
  - p_2 measured

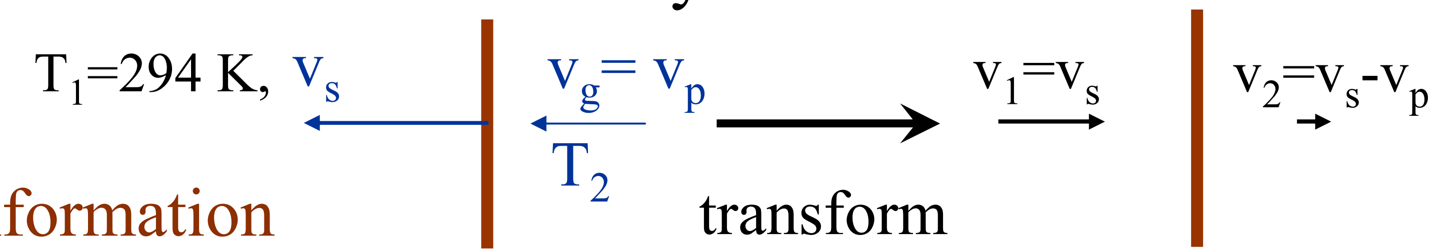


- **Find:**
  1. shock speed (v_s) (lab reference frame)
  2. projectile speed (v_p) (lab reference frame)

- **Assume:** Air TPG/CPG with γ=1.4

Solution: Known Postshock Pressure

- **Analysis:** Transform to stationary shock
  - use p information



$$\frac{p_2}{p_1} = \frac{608}{101.3} = 6.00 \Rightarrow M_1 = 2.30,\ M_2 = 0.534$$

B.1/ (VII.12)

in shock's ref. frame, **subsonic behind shock**

$$v_s = v_1 = a_1 M_1 = 20\sqrt{294}\,\text{m/s}\,(2.3) = 789\,\text{m/s}$$

M's + B.1 or (VII.9)

$$\Rightarrow T_2/T_1 = 1.947$$

$$T_2 = 1.947(294\text{K}) = 572\text{K}$$

$$v_p = v_s - v_2 = v_s - a_2 M_2 = v_s - 20\sqrt{T_2}\,M_2$$

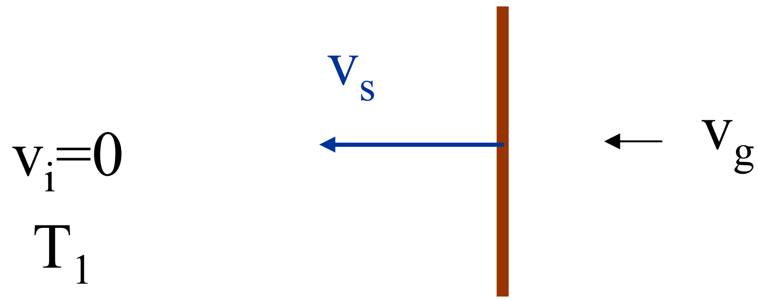
$$v_p = 789 - 20\sqrt{572}(0.534)\,\text{m/s} = 533\,\text{m/s}$$

$$M_g = \frac{v_p}{a_2} = \frac{533\,\text{m/s}}{20\sqrt{572}\,\text{m/s}} = 1.12$$

in lab reference frame, **supersonic behind shock**

Example: Postshock Speed Known

- **Given:** Normal shock **moving into still gas** (at T_1) produces known gas speed (v_g) behind shock

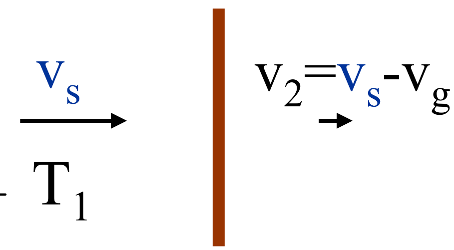


- **Find:**
  - Expression for shock speed v_s in terms of v_g

- **Assume:**
  - TPG/CPG

Solution: Postshock Speed Known

- **Analysis:** In stationary ref. frame



  – velocity ratio

$$\frac{v_1}{v_2} = \frac{M_1}{M_2}\sqrt{\frac{T_1}{T_2}} = \frac{M_1}{M_2}\sqrt{\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}} = \sqrt{\frac{\frac{1}{M_2^2}+\frac{\gamma-1}{2}}{\frac{1}{M_1^2}+\frac{\gamma-1}{2}}} = \ldots = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}$$

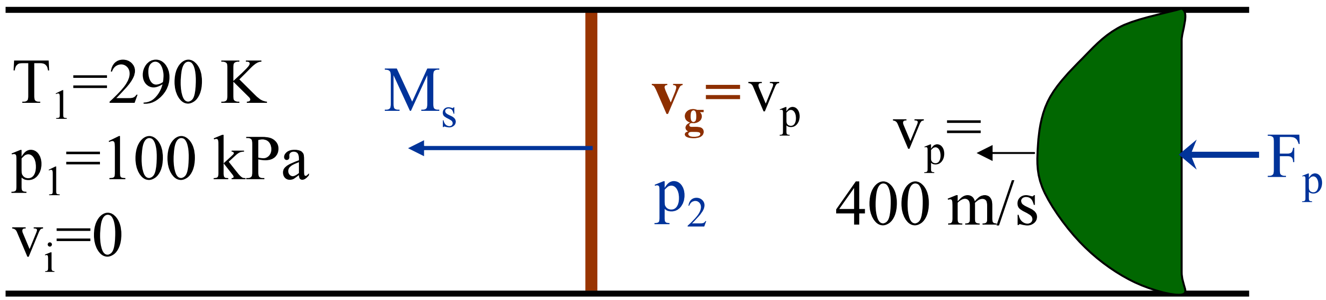
(VII.18)
another normal shock relation

$$\frac{v_s}{v_s - v_g} = \frac{(\gamma+1)(v_s/a_1)^2}{(\gamma-1)(v_s/a_1)^2+2} \Rightarrow v_s^2 - \left(v_g \frac{\gamma+1}{2}\right)v_s - a^2 = 0$$

$$v_s = \frac{\gamma+1}{4}v_g + \frac{1}{2}\sqrt{\left(\frac{\gamma+1}{2}\right)^2 v_g^2 + 4\gamma R T_1}$$ **(VII.19)**

Numerical Example: Known v_g

- **Given:** Piston impulsively set into motion at 400 m/s in 25cm^2 tube filled with **initially still air** @ 290K, 100 kPa
  - leading shock produced

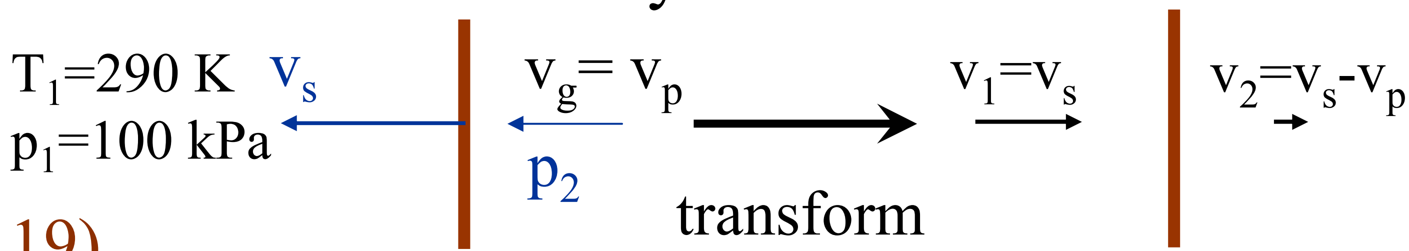


- **Find:**
  1. Mach number of shock, M_s (relative to unshocked gas)
  2. Force, F_p, required to keep piston moving

- **Assume:** Air TPG/CPG with γ=1.4, no friction on piston

Solution: Known v_g

- **Analysis:** Transform to stationary shock



  - use (VII.19)

$$v_s = \frac{\gamma+1}{4} v_g + \frac{1}{2}\sqrt{\left(\frac{\gamma+1}{2}\right)^2 v_g^2 + 4\gamma R T_1}$$

$$= 0.6(400\,\text{m/s}) + \frac{1}{2}\sqrt{1.44(400\,\text{m/s})^2 + 5.6\left(287\,\text{m}^2/\text{s}^2\text{K}\right)290\text{K}}$$

$$v_s = 657\text{m/s} \Rightarrow M_s = v_s/a_1 = 657/\sqrt{1.4(287)290} = 1.93$$

$M_1 = M_s$ ($v_i = 0$) → B.1 or (VII.12)

$$\Rightarrow p_2/p_1 = 4.18$$

$$p_2 = 418\text{kPa}$$

$$F_p = p_2 A_p$$

$$= 418\text{kPa}\left(0.0025\text{m}^2\right) = 1045\text{N}$$

235 lb_f to move 2.2 inch diam. piston w/o friction