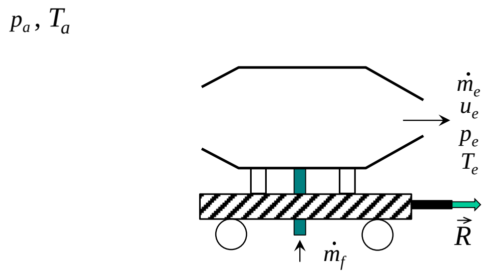
Example #1: Static Jet Engine Thrust

- **Given:** Stationary jet engine on test stand



- **Find:** How does force required to hold stand depend on ambient and exhaust conditions

- **Assume:** steady-state (d/dt = 0), inviscid flow, uniform exhaust

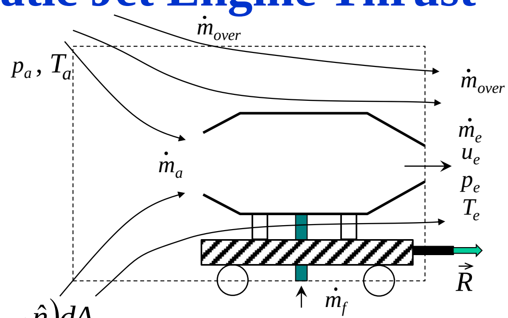
Example #1: Static Jet Engine Thrust

- **Analysis:**

1st need to define CV/CS

HOW?

Where you know (or want to know) values!



Mass

$$0 = \cancel{\frac{\mathrm{d}}{\mathrm{d}t}\int_{CV} \rho dV} + \int_{CS} \rho(\vec{u}_{rel} \cdot \hat{n})dA$$

$$0 = 0 + \int_{CS} \rho(\vec{u}_{rel} \cdot \hat{n})dA = \sum \dot{m}_{out} - \sum \dot{m}_{in}$$

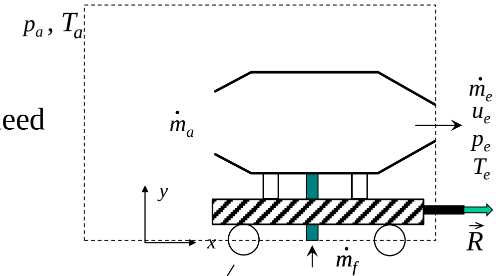
$$= (\dot{m}_e + \dot{m}_{over}) - (\dot{m}_a + \dot{m}_{over} + \dot{m}_f) \Rightarrow \boxed{\dot{m}_e = \dot{m}_a + \dot{m}_f}$$

Example #1: Static Jet Engine Thrust

- **Analysis:**

Momentum

already defined CS, but need reference frame



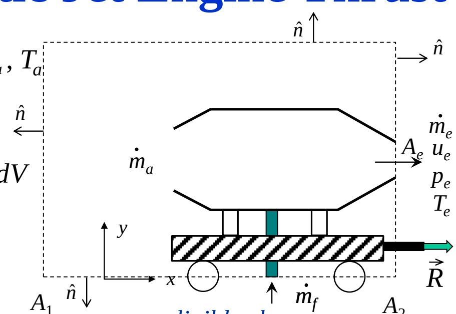
$$\underset{\vec R}{\cancel{\vec F_{\substack{solid\,body\\ on\,fluid}}}} - \int_{CS} p\hat n dA + \underset{0}{\cancel{\int_{CS}\vec\sigma_{shear} dA}} + \int_{CV}\rho\vec f dV = \underset{0}{\cancel{\frac{d}{dt}\int_{CS}\rho\vec u dV}} + \int_{CS}\rho\vec u\left(\vec u_{rel}\cdot\hat n\right)dA$$

$$\vec R = \int_{CS}\rho\vec u\left(\vec u_{rel}\cdot\hat n\right)dA + \int_{CS}p\hat n dA - \int_{CV}\rho\vec f dV$$

Example #1: Static Jet Engine Thrust

- **Analysis:**

Momentum



$$\vec R = \int_{CS}\rho\vec u\left(\vec u_{rel}\cdot\hat n\right)dA + \int_{CS}p\hat n dA - \int_{CV}\rho\vec f dV$$

R in positive x-direction

$$R = \int_{CS}\rho u_x\left(\vec u_x\cdot\hat n\right)dA + \int_{CS}p\hat n dA - 0$$

assume negligible change

$$R = \int_{left}\rho\underset{0}{\cancel{u_x}}\left(\vec u_x\cdot\hat n\right)dA + \int_{nozzle}\rho u_x\left(\vec u_x\cdot\hat n\right)dA + \int_{over}\cancel{\rho u_x}\left(\vec u_x\cdot\hat n\right)dA + \int_{left}p\hat n dA + \int_{right}p\hat n dA$$

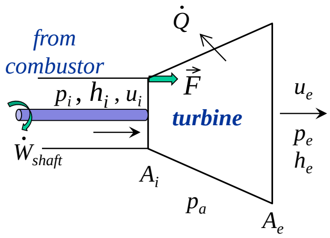
$$R = \dot m_e(+)u_e + \left[p_a(-)A_1\right] + \left[p_e(+)A_e + p_a(+)A_2\right] + [0] \qquad + \int_{top,bottom}p\hat n dA$$

$A_2 = A_1 - A_e$ $\qquad \boxed{R = \left(\dot m_a + \dot m_f\right)u_e + \left(p_e - p_a\right)A_e}$

Is this the same as the static thrust?

Example #2: Turbine in Jet Engine

- **Given:** Jet engine turbine connected to the rest of the engine



  with inlet and out properties shown, and F≡force turbine structure exerts on rest of engine

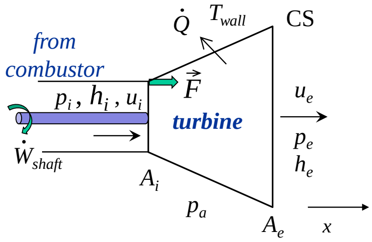
- **Find:** Conservation equation relations for turbine

- **Assume:** steady-state (d/dt = 0); inviscid flow; uniform at i, e

Example #2: Turbine in Jet Engine

- **Analysis:**

CV?



Mass

Momentum

Energy

Entropy