## **Problem Set #1: Review of Conservation & State Equations**

- Homework solutions should be neat and logically presented, and **must follow** the format requirements at <u>seitzman.gatech.edu/classes/ae4451/homeworkformat.html</u>.
- For the following problems, YOU MUST include **a sketch** of the flow/system, and indicate clearly your choice of **control surface**. Also indicate on your figure your definitions of forces.
- Always indicate any **assumptions** you make and be sure that your assumptions are both reasonable and necessary (i.e., you shouldn't make an assumption if the information provided in the problem does not require the assumption to be made). If you use any results or equations from the class notes or text in your solutions, please note and **reference** them (but you better be sure they are applicable to the problem at hand).
- Try to **solve** the problem **algebraically** first. Only use numbers/values in the final steps of your solution.

## 1. Rocket Acceleration

A rocket is traveling through the atmosphere. At some instant in time (*t*), the speed of the rocket in a direction parallel to the rocket axis is *u*, and the velocity and pressure at the exit plane of the rocket nozzle are  $u_e$  and  $p_e$ . Also, the angle between the axis of the rocket and the gravity field is  $\theta$ .

Starting with the full integral form of the momentum conservation equation, show that the acceleration a of the rocket in the direction parallel to the rocket axis at time t is given by

 $p_a$ 

$$a = \frac{\dot{m}}{M} \left\{ u_e + \frac{\left(p_e - p_a\right)A_e}{\dot{m}} \right\} - g\cos\theta - \frac{D}{M}$$

where M is the mass of the rocket at time t, g is the gravitational acceleration, and D is the aerodynamic drag force on the rocket.

## 2. Electric Ducted Fan

An electric propulsion (EP) system for a small aircraft consists of a fan inside a duct (i.e., a ducted fan) that is driven by an electric motor. The fan is used to raise the stagnation pressure of the incoming air and

accelerate it through a nozzle (no other mass leaves the nozzle). The total mass of the propulsion system is 56.4 lb<sub>m</sub>; additionally, the propulsion system can be rotated to produce both lift and forward thrust.



When the aircraft connected to the propulsor is operating at a constant air speed of 169 kph in steady, level flight, and at an altitude where the ambient conditions are 282.0 K and 0.958 atm, the data recorded at the exit plane of the nozzle are:  $T_e$ = 285.9 K,  $u_e$ = 174.2 m/s, and  $\dot{m}_e$ = 13.2 kg/s. Furthermore, the exit gases are moving at an angle ( $\delta$ ) of 33.0 degrees downward from the horizontal. For this problem, use  $\gamma$ =1.39 and a molecular weight of 28.9 for the air passing through the propulsor; do not use data from other sources for any air properties that would contradict these values.

- a) Determine the vertical force exerted by the propulsion system on the aircraft (via the engine pylon); specify both the magnitude and direction (up or down) of the force.
- b) Determine the horizontal force exerted by the propulsion system on the aircraft; specify both the magnitude and direction (left or right) of the force.
- c) Find the amount of power that was added to the air flowing through the propulsor by the fan.
- d) Determine if the air underwent a reversible process as it passed through the propulsion system.

## 3. Engine Nozzle

An aircraft engine is cruising where the ambient conditions are 0.301 atm and 231 K. The gas flowing through the engine nozzle has a molecular weight of 28.4 and a specific heat ratio of 1.32. The gas enters the nozzle at a flowrate of 65.3 kg/s, and with a pressure of 0.5426 atm and a temperature of 573 C. The cross-sectional flow area at the nozzle inlet is 0.973 m<sup>2</sup>. The gases expand through the nozzle, exiting at a temperature of 764.3 K and a pressure of 4.781 psia. You should assume the nozzle is adiabatic. (**Be careful with units**!!)

- a) Find the velocity of the gas at the nozzle inlet and the velocity at the nozzle exit.
- b) Determine the nozzle's adiabatic efficiency.