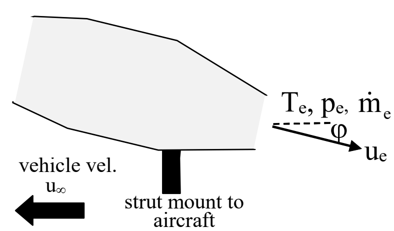
Problem Set #1: Review of Conservation & State Equations

- Homework solutions should be neat and logically presented, and **must follow** the format requirements at seitzman.gatech.edu/classes/ae4451/homeworkformat.html.
- For the following problems, YOU MUST include **a sketch** of the flow/system, and indicate clearly your choice of **control surface**. Also indicate on your figure your definitions of forces.
- Always indicate any **assumptions** you make - and be sure that your assumptions are both reasonable and necessary (i.e., you shouldn't make an assumption if the information provided in the problem does not require the assumption to be made). If you use any results or equations from the class notes or text in your solutions, please note and **reference** them (but you better be sure they are applicable to the problem at hand).
- Try to **solve** the problem **algebraically** first. Only use numbers/values in the final steps of your solution.

1. Variable Pitch Propulsor

An aircraft employs an electric propulsion system with a mass of 17.5 kg; an electrically driven fan inside the propulsor is used to accelerate the incoming air (and no other mass is added to the propulsor). Furthermore, the propulsion system can be rotated to produce both lift and forward thrust. When the aircraft and propulsor are operating in steady-state and moving horizontally (level flight, and to the left as shown in the figure) at 154 kph with ambient conditions of 284.0 K and 0.985 atm, the data recorded at the nozzle exit plane are: T_e= 286.8 K, u_e= 68.3 m/s, and $\dot{m}_e$ = 6.52 kg/s. Furthermore, the exit gases are moving at an angle (φ) of 40.0 degrees downward from the horizontal. For this problem, use γ=1.41 and a molecular weight of 28.7 for the air passing through the engine; do not use data from other sources for any air properties that would contradict these values.



a) Determine the vertical force exerted by the propulsion system on the aircraft – specify both the magnitude and direction (up or down) of the force.
b) Determine the horizontal force exerted by the propulsion system on the aircraft – specify both the magnitude and direction (left or right) of the force.
c) Find the amount of power that was added to the air flowing through the propulsor.

2. **Engine Nozzle**

   Consider the following properties of an aircraft engine operating at a cruise altitude. The engine's nozzle has a gas flow with a molecular weight of 28.3 and a specific heat ratio of 1.34. The gas enters the nozzle at a flowrate of 42.7 kg/s, and with a pressure of 0.5624 atm and a temperature of 597 C. The cross-sectional flow area at the nozzle inlet is 0.873 m^2. The gases expand through the nozzle, exiting at a temperature of 727.8 K and at the ambient pressure of 3.34 psia.
   (***Be careful with units***!!)

   a) Determine whether the gas flowing through the nozzle undergoes an isentropic process.

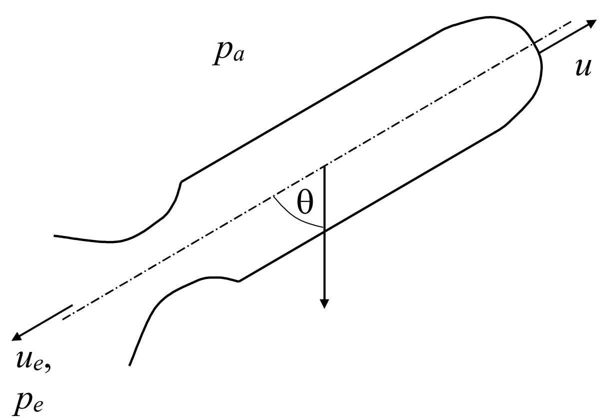
   b) Find the velocity of the gas at the nozzle's inlet and at its exit.

   c) For these conditions, calculate the force transmitted to the structure holding the nozzle. Provide **both direction and magnitude** of the force, and use a picture to help present your answer.

   d) Based solely on your results, determine whether *the force acting on the nozzle* would help speed up or slow down a vehicle that used this nozzle as part of a jet propulsion system.

3. **Rocket Acceleration**

   A rocket is traveling through the atmosphere. At some instant in time (t), the speed of the rocket in a direction parallel to the rocket axis is u, and the exit velocity and pressure are u_e and p_e. Also, the angle between the axis of the rocket and the gravity field is θ.



   Starting with the full integral form of the momentum conservation equation, **show** that the acceleration a of the rocket in the direction parallel to the rocket axis at time t is given by

$$a = \frac{\dot{m}}{M}\left\{u_e + \frac{(p_e - p_a)A_e}{\dot{m}}\right\} - g\cos\theta - \frac{D}{M}$$

   where M is the mass of the rocket at time t, g is the gravitational acceleration, and D is the aerodynamic drag force on the rocket.