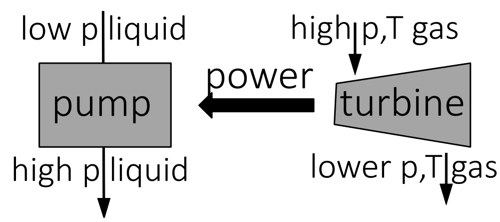
Problem Set #2: Conservation Equations and Chemical Energy

- Homework solutions should be neat and logically presented, and **must follow** the format requirements at seitzman.gatech.edu/classes/ae4451/homeworkformat.html.
- For the following problems, YOU MUST include **a sketch** of the flow/system, and indicate clearly your choice of **control surface**.
- Always indicate any **assumptions** you make - and be sure that your assumptions are both reasonable and necessary (i.e., you shouldn't make an assumption if the information provided in the problem does not require the assumption to be made). If you use any results or equations from the class notes or text in your solutions, please note and **reference** them (but be sure they are applicable to the problem at hand).
- Try to **solve** the problem **algebraically** first. Only use numbers/values in the final steps of your solution.

Introduction

Most liquid rocket engines (LRE) use pumps to raise the pressure of the liquid propellants; the propellants are stored in tanks at low pressure and must be pressurized to enter the rocket's high-pressure combustion-chamber. Since a pump requires power to operate, it is standard to connect the liquid pump to a turbine by a shaft; this combination is called a **turbopump**. The turbine produces the shaft power that the pump needs by expanding a hot, high pressure gas, i.e., by decreasing the gas's pressure and temperature as it flows through the turbine.



There are various options for producing the hot turbine-inlet gas. In a "staged-combustion" cycle, the turbine inlet gas is produced by a **preburner/precombustor**, a combustor that burns either a very fuel-rich or very oxygen-rich mixture. The following problems relate to a turbopump and oxygen-rich preburner in an LRE using a staged-combustion cycle.

1. LRE Turbopump

Consider a pump that will be used to raise the pressure of liquid oxygen for a very large, first-stage rocket engine; it is expected that the pump will require 5.25 MW of shaft power to operate. The preburner that feeds the turbopump's turbine is expected to produce 45.69 kg/s of a gas with a stagnation temperature of 1214 K and a stagnation pressure of 434 atm. The turbine is expected to have an adiabatic efficiency of 73.7% and no heat losses.

For this problem, assume that the gas passing through the turbine has a molecular weight of 31.7 and a specific heat c_p of 1109 J/kgK.

a) After exiting the turbine, what will be the stagnation temperature and stagnation pressure of the gas?

b) What would those values be if the turbine operated reversibly and still produced the required power?

2. LRE Preburner

Now consider the oxygen-rich preburner that will be required to produce the turbine inlet conditions for the turbopump described above. The preburner will run on a mixture of oxygen and RP-1 fuel. Furthermore, the RP-1 enters the preburner at 298 K and 434 atm, while the oxygen enters at 90.2 K and 434 atm.

For this problem, assume RP-1 (which is a mixture of hydrocarbon fuels) has a carbon:hydrogen ratio of 12.5:24.5 (i.e., like it had a “chemical formula” of $C_{12.5}H_{24.5}$). Also, obtain any other necessary species properties only from the table below – **do not use any other sources** (also **do not use** the molecular weight or specific heat information given in Problem 1). You should assume that the combustor is adiabatic (this temperature is known as the *adiabatic flame temperature*).

	MW (g/mol)	$\bar{h}_{298K,434atm} - \bar{h}_{90.2K,434atm}$ (kJ/mol)	$\bar{c}_{p,298-1500K}$ * (kJ/kmol·K)	$\Delta\bar{h}_{f,298K}$ (kJ/mol)
RP-1	175	---	---	-285.16
O_2	32.0	11.98	33.5	
CO_2	44.0	33.76	48.0	-395.32
H_2O	18.0	12.94	38.5	-242.81

*These GAS specific heat values have been chosen so that you can assume the gases are thermally and calorically perfect over the given temperature range (298-1500K); in reality, the gases are not calorically perfect.

For this oxygen-rich (fuel lean) mixture; assume the gas exiting the preburner is composed **solely** of a mixture of H_2O, CO_2, and O_2.

a) What should be the oxygen-fuel mass ratio (O/F) supplied to the preburner to produce the required preburner exit temperature of 1214 K?

b) What is the equivalence ratio (ϕ) of this fuel-oxidizer mixture?

c) What fraction of the gas exiting the preburner will be O_2 (report both mole **and** mass fractions)?

Warning – for this problem, do not use the simplified combustor model presented in the class lecture that assumes the combustion products have the same properties as the oxidizer gas.