## Problem Set #2: Conservation Equations and Chemical Energy

- Homework solutions should be neat and logically presented, and **must follow** the format requirements at <u>seitzman.gatech.edu/classes/ae4451/homeworkformat.html</u>.
- For the following problems, YOU MUST include **a sketch** of the flow/system, and indicate clearly your choice of **control surface**.
- Always indicate any **assumptions** you make and be sure that your assumptions are both reasonable and necessary (i.e., you shouldn't make an assumption if the information provided in the problem does not require the assumption to be made). If you use any results or equations from the class notes or text in your solutions, please note and **reference** them (but be sure they are applicable to the problem at hand).
- Try to **solve** the problem **algebraically** first. Only use numbers/values in the final steps of your solution.

## Background

Most liquid rocket engines (LRE) use pumps to raise the pressure of the liquid propellants; the propellants are stored in tanks at "low" pressure and must be pressurized to enter the rocket's high-pressure combustion chamber. Since a pump

requires power to operate, it is standard to connect the liquid pump to a turbine by a shaft; this combination is called a **turbopump**. The turbine produces the shaft power that the pump needs by expanding a hot, high pressure gas.

There are various options for producing the hot turbine-inlet gas. In a "staged-combustion" cycle, the turbine inlet gas is produced by a **preburner/precombustor**, a combustor that burns either a very fuel-rich or very oxygen-rich mixture. The following problems relate to a



turbopump and oxygen-rich preburner in an LRE using a staged-combustion cycle.

## 1. LRE Turbopump Turbine

The liquid oxygen pump for a high thrust rocket engine will require 4.92 MW of shaft power to operate. The preburner that feeds the turbopump's turbine will produce a gas with a molecular weight of 31.9, a specific heat ( $c_p$ ) of 1153 J/kgK, a stagnation temperature of 1187 K, and a stagnation pressure of 443 atm. The turbine is expected to be adiabatic, with an adiabatic efficiency of 74.3%.

- a) If we want to make sure that the stagnation pressure exiting the turbine is not lower than 289 atm, but also that we use as low a mass flowrate of gas as possible, what would be the stagnation temperature and stagnation pressure of the gas exiting the turbine, and the mass flowrate of gas passing through the turbine?
- **b)** What would be the stagnation temperature and stagnation pressure of the gas exiting the turbine if it produced the same required power and had the same mass flowrate you found in part a), but operated reversibly?

## 2. LRE Preburner

The oxygen-rich preburner used to produce the turbine inlet conditions for the turbopump described above will run on a mixture of oxygen and RP-1 fuel. The RP-1 will enter the preburner at 298 K and 443 atm, while the oxygen enters at 90.2 K and 443 atm.

For this problem, assume RP-1 (which is a mixture of hydrocarbon fuels) has a carbon:hydrogen ratio of 2.5:4.9 (i.e., like it had a "chemical formula" of  $C_{2.5}H_{4.9}$ ). Also, obtain any other necessary species properties only from the table below – **do not use any other sources** (also **do not use** the molecular weight or specific heat information given in Problem 1). You should assume that the combustor is adiabatic (this temperature is known as the *adiabatic flame temperature*).

	MW (g/mol)	$\frac{\overline{h}_{298K,443atm} - \overline{h}_{90.2K,443atm}}{\text{(kJ/mol)}}$	$\Delta ar{h}_{\!_{f,298K}}$ (kJ/mol)	<i>c̄<sub>p,298-1500K</sub></i> * (kJ/kmol⋅K)
RP-1	34.9	not available	-57.032	not available
O2	32.0	10.01		36.8
CO <sub>2</sub>	44.0	33.76	-395.32	56.9
H <sub>2</sub> O	18.0	11.25	-242.81	54.7

\*These GAS specific heat values have been chosen so that you can assume the gases are thermally and calorically perfect over the given temperature range (298-1500K); in reality, the gases are not calorically perfect.

For this oxygen-rich (fuel lean) mixture; assume the gas exiting the preburner is composed **solely** of a mixture of  $CO_2$ ,  $H_2O$  and  $O_2$ .

- a) What should be the oxygen-fuel mass ratio (O/F) supplied to the preburner to produce the required preburner exit stagnation temperature of 1187 K?
- **b)** What would be the stoichiometric O/F ratio for this fuel and oxidizer?
- c) What would be the mole fraction of O<sub>2</sub> in the gas exiting the preburner?

Warning – for this problem, do not use the simplified combustor model presented in the class lecture that assumes the combustion products have the same properties as the oxidizer gas.