## Problem Set #5: Ideal Rocket Analysis

- Homework solutions should be neat and logically presented, see format requirements at <a href="http://seitzman.gatech.edu/classes/ae4451/homeworkformat.html">http://seitzman.gatech.edu/classes/ae4451/homeworkformat.html</a>.
- If appropriate, include **a sketch** of the flow/system, and indicate clearly your choice of **control surface**.
- Always indicate any **assumptions** you make. If you use any results or equations from the class notes or text in your solutions, please note and **reference** them (but you better be sure they are applicable to the problem at hand).
- Try to **solve** the problem **algebraically** first. If possible, only use numbers/values in the final steps of each solution.
- All plots/graphs should be computer created, look professional and be easily readable and interpretable – which means think carefully about your axis scaling and limits. Do not provide tables of the data shown in the graph; however be sure to provide the equations used to compute the data and explanations of how the calculations were done (note: copies of code or spreadsheet expressions are not acceptable ways to provide the equations).

## Introduction

For this problem set, you have been asked to help with the preliminary design of a

hydrazine thruster, a common type of thruster for spacecraft attitude control systems.

The propellant, hydrazine  $(N_2H_4)$ , is stored as a liquid, but before reaching the nozzle, it passes through a catalyst bed. In the bed, the hydrazine vaporizes, and it also **exothermically** decomposes (thereby raising the temperature significantly) initially into 80% ammonia (NH<sub>3</sub>) and 20% nitrogen (N<sub>2</sub>). However, some of the ammonia also



From Sutton and Biblarz, Rocket Propulsion Elements, 2010

**endothermically** decomposes into  $H_2$  and  $N_2$ . The catalyst bed is designed to be sufficiently short that the hydrazine completely decomposes, without converting too much of the ammonia, which would drop the temperature.

Based on previous experience, you should expect the decomposed gas leaving the catalyst bed and entering the nozzle to have a composition of:  $\chi_{NH3}=33.0\%$ ,  $\chi_{N2}=27.8\%$ ,  $\chi_{H2}=39.2\%$ ; with a stagnation temperature of 1422 K and a specific heat ratio  $\gamma=1.242$ .

## 1. Nozzle Exit Conditions and Thruster Coefficients

The hydrazine thruster is to be operated at a stagnation pressure of 300.0 psi in the thrust chamber on a satellite in low Earth orbit (LEO) where the ambient pressure is 0.10  $\mu$ Pa. You may assume the thruster's nozzle has an efficiency of 100%. Determine and make plots of:

- a) pressure (in psia), temperature and velocity at the **exit** of the thruster as a function of nozzle exit area ratio ( $\epsilon$ ), with  $\epsilon$  ranging from 10 to 500;
- b) thrust coefficient and characteristic velocity (on a single graph, which may require using separate left and right vertical axes) as function of  $\epsilon$  for a 10-500 range.

## 2. Thruster Performance and Design

For the same conditions as in the previous problem:

- a) determine and plot (on a single graph, which may require using separate left and right vertical axes), the specific impulse (in seconds) and the thrust per nozzle throat area ( $\tau/A_t$ , in N/cm<sup>2</sup>) vs  $\epsilon$  for  $\epsilon$  ranging from 10-500;
- b) determine and plot (on a single graph, which may require using separate left and right vertical axes) the throat diameter (Dt) and propellant mass flowrate (m) required to produce a thrust of 1.00 N (~0.225 lbf) vs ε for 10-500;
- c) determine which of these 4 performance parameters:  $c_{\tau}$ ,  $c^*$ ,  $I_{sp}$ , and  $\tau/A_t$ , would change the most if the thrust chamber stagnation pressure was decreased by a factor of 2 (holding  $\epsilon$  and  $T_0$  fixed);
- d) determine how the required throat diameter and propellant mass flowrate of our thruster would change (pick one for each: increase, decrease, stay the same) if we could increase the catalyst bed exit temperature to 1500 K (**without** changing the thrust,  $p_0$  or  $\epsilon$ )? Your must justify your answer but numerical values for D<sub>t</sub> and m are not required.